

**Remanufacturing Products with Limited  
Component Durability and Finite Life Cycles**

by

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**2003/54/TM**

Working Paper Series

# ***Remanufacturing Products with Limited Component Durability and Finite Life Cycles***

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## **Abstract**

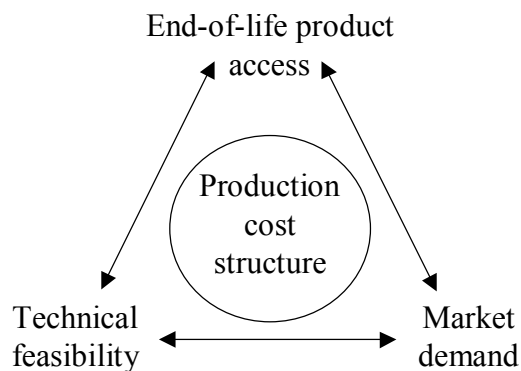
Remanufacturing recovers value from end-of-life products by reusing their durable components for the manufacturing of a product with the original functionality. The examples of remanufacturing in industry suggest that its economic success as a corporate strategy depends on the way the costs of all involved operations are matched with the collection rate of end-of-life products, the component durability and the market demand for remanufactured products. We assess the economic impact of these interdependencies when the remanufactured product is a perfect substitute for the all-new one, i.e. market demand for the remanufactured product is only limited by the length of the product life cycle. The quantitative results of our systemic and non-linear modeling approach indeed show that remanufacturing is profitable only if component durability and length of the product life cycle are in tune with the costs and the efficiency of the production and collection system. The length of the product life cycle and the component durability also need to be coordinated with each other but we find that this can be done independently of the collection rate.

# **1 Introduction**

Remanufacturing recovers value from collected end-of-life products by reusing their durable components for the manufacturing of a product with the original functionality. There are well-documented examples in industry showing that remanufacturing can be very profitable (Ayres et al. 1997). Remanufacturing toner cartridges is a \$3 billion industry and a network of several thousand third party remanufacturers now competes with the original equipment manufacturers (Ginsburg 2001). For the manufacturers of single use cameras the economic net effect of reusing components has clearly been positive (Behrendt et al. 1999, Kodak 1999), and Xerox has saved almost \$200 million in materials and parts costs in less than five years through remanufacturing its copiers (Davis 1996).

The general economic viability of such asset recovery systems based on end-of-life product take-back remains very controversial, though. Remanufacturing is long-standing practice to produce automotive spare parts (Steinhilper 1998) but the German car industry tried very hard to avoid mandatory take-back of end-of-life vehicles under the European ELV directive (Munsberg 1999, Handelsblatt 1999). The computer industry is more positive towards product take-back but at present concentrates mainly on organized disposal (Hewlett-Packard 2001, Ferrer 1997a). While original equipment manufacturers of mobile phones conclude that take-back is uneconomic (Clift & Wright 2000), third party remanufacturers demonstrate that it can be highly profitable (Guide & Van Wassenhove 2001, Shields 2002).

The case studies of all the above-mentioned examples suggest that the economic success not just depends on the costs of all the involved operations. Constraints like accessibility of end-of-life products, technical feasibility of remanufacturing and market demand for remanufactured products crucially impact the economics of remanufacturing and the way these constraints interact determines its profitability (See Figure 1). This calls for an analysis



**Figure 1: Economics of remanufacturing**

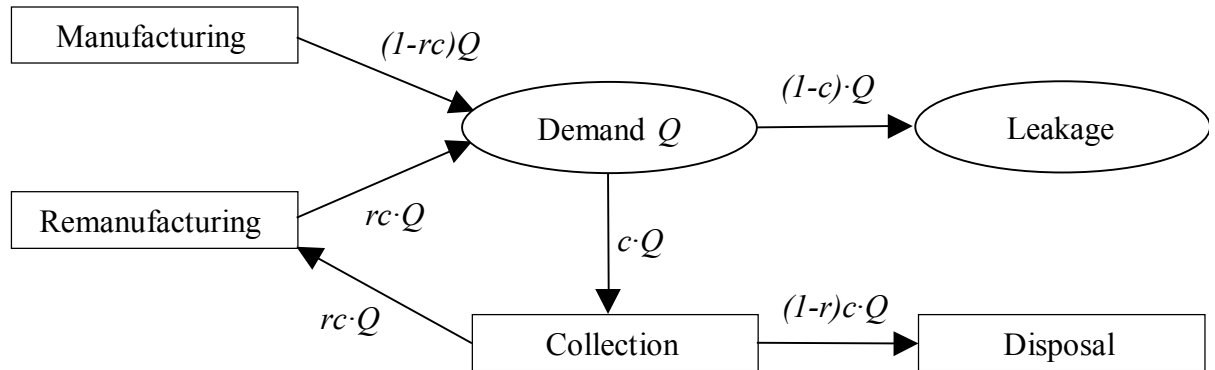
framework, which enables the systematic study of the economics of remanufacturing under the impact of these three constraints. The modeling methodology we present in this paper delivers such a framework for quantitative analysis.

As a natural starting point of our modeling efforts we consider an original equipment manufacturer who has the option to collect and remanufacture her products after the end of their use phase. The remanufactured product is a perfect substitute of the all-new one, which means that at identical price customers derive the same utility from both. Section 2 develops a simple economic model for such a production and consumption system, which accounts for the above mentioned constraints in a very general fashion. In Section 3 we identify the limited durability of the product components relative to the time the products are in use as the main technical constraint for remanufacturing and introduce a simple model for it. In Section 4 this is combined with the economic model, which allows us to analyze the economic impact of the interactions between cost structure, collection rate and limited component durability. In Section 5 we establish that the market demand for remanufactured products is limited even when it is a perfect substitute of the all-new one. This constraint on the product demand is created by the finite length of product life cycles, which is defined as the total time span between product launch and market withdrawal. If the life cycle of a product model is short relative to its average use phase, a large percentage of the end-of-life products will return to the company at a time when there is no longer demand for this product model. A simple model enables us to express this in a closed-form quantitative way. Section 6 combines this with the economic model from Section 2, which reveals how the interactions between cost structure, collection rate and product life cycle impact the economics of remanufacturing. Section 7 fi-

nally investigates the relationship between limited component durability and finite product life cycles. In Section 8 we conclude our paper with a summary of our findings and a brief discussion of the way our model could and should be extended to accommodate different remanufacturing environments, especially those where the remanufactured product competes with the all-new one on the market.

## 1 Basic economic model of product remanufacturing

In this section we introduce a basic economic model of production systems with product recovery and remanufacturing of a perfect substitute. The model distinguishes between four different groups of operations, i.e. manufacturing, collection, remanufacturing and disposal, which have different types and quantities of product inflows and outflows as shown in Figure 2.



**Figure 2: Product flow diagram of a production system with reuse and remanufacturing**

The take-back scheme of the manufacturer collects a certain percentage  $c$  of all marketed products  $Q$  after they reach the end of their useful lives. In most cases the collection rate  $c$  will be smaller than one for the following reasons:

- The collection network does not cover all areas where products could be collected.
- The user of the product does not return the item or simply disposes of it.
- There are third parties, who also collect the end-of-life products.

Part of the collection process is an inspection, which may involve partial disassembly, and only a percentage  $r$  of all collected and inspected products can be remanufactured and remarketed. The remanufacturing yield  $r$  is limited by technical constraints, the most important of which will be discussed in section 3, and also dependent on sufficient market demand, which will be the subject of section 5. The other part of the collected and inspected end-of-life products cannot be remanufactured and creates disposal costs for the company, which might be somewhat reduced through materials or energy recovery. The disposal of uncollected end-of-life products has to be paid by the end user, which is the business reality for practically all goods at present. In Europe, North America and Japan governments are introducing or considering product take-back legislation (OECD 1996) though, which would considerably change the economics of product remanufacturing. Examples are the European ELV and WEEE directives on end-of-life vehicles and waste electric and electronic equipment. The implications of such legislation are not considered at present, but can be easily included in the model.

The product between collection rate  $c$  and remanufacturing yield  $r$ , called remanufacturing rate  $rc$ , indicates the efficiency of product recovery management based on component reuse. The remanufacturing process brings the end-of-life product back to a like-new condition and the remanufactured product provides the customer with the same utility as the newly manufactured one. They are perfect substitutes on the market and the total demand  $Q$  can therefore be satisfied with any mix of manufactured and remanufactured products. In this quasi-static description of the system the flow variables show no time dependency. For dynamic modeling purposes we express the total demand as the time integral of the demand rate,  $Q = \int_0^T q(t)dt$ ,  $T$  being the end of the planning horizon or the product life cycle. All other costs and flow variables can then also be expressed as time dependent values.

Cost parameter	Operations
$C_{man}$	<ul style="list-style-type: none"> <li>– Purchase of components and materials</li> <li>– Manufacture of components</li> <li>– Assembly and testing of a product</li> <li>– Distribution of the manufactured product</li> </ul>
$C_{coll}$	<ul style="list-style-type: none"> <li>– Collection and inspection of an end-of-life product</li> </ul>
$C_{reman}$ ( $< C_{man}$ )	<ul style="list-style-type: none"> <li>– Disassembly of a remanufacturable end-of-life product</li> <li>– Cleaning and repair of reusable components</li> <li>– Purchase or manufacture of missing components</li> <li>– Assembly and testing of the remanufactured product</li> <li>– Distribution of the remanufactured product</li> </ul>
$C_{disp}$	<ul style="list-style-type: none"> <li>– Disposal of a non-remanufacturable end-of-life product</li> </ul>

**Table 1: Unit costs in the basic model of a production system with remanufacturing**

This paper is not concerned with the operational details of product recovery and re-manufacturing. There is ample literature covering issues like reverse logistics, disassembly strategies, production planning and inventory control of remanufacturing systems. Good overviews are provided by (Fleischmann et al. 1997), (Guide et al. 1998) and (Guide 2000). It is also of no importance for our analysis whether the company itself or subcontracted third parties carry out the collection and remanufacturing activities (see e.g. Savaskan et al. 1999 and Guide & Van Wassenhove 2001) since we do not deal with supply chain coordination issues here. Only the cost functions of the four groups of operations need to be known since the model uses this information as input for further analysis and is not designed to help optimizing any of the above-listed operations. For most part of our first order analysis the operations are assumed to have constant unit costs but the unit costs could easily be modeled as functions of the output quantities or other variables to reflect effects like economies of scale or learning. Table 1 lists the acronyms of the unit cost parameters and summaries of the activities they comprise.

Without any product take-back and remanufacturing the total production cost is simply  $C_{man}^{total} = Q \cdot C_{man}$ . If the company decides to collect and remanufacture end-of-life products as shown in Figure 2 the total cost of satisfying market demand  $Q$  with a mix of manufactured

and remanufactured products is  $C_{reman}^{total} = (1 - rc)QC_{man} + rcQC_{reman} + cQC_{coll} + (1 - r)cQC_{disp}$ .

To see when remanufacturing reduces costs for the company it is convenient to calculate the difference in average production cost,

$$\Delta C := (C_{man}^{total} - C_{reman}^{total})/Q = rc(C_{man} - C_{reman} + C_{disp}) - c(C_{coll} + C_{disp}).$$

Product recovery and remanufacturing leads to cost savings, if the remanufacturing yield is above a critical value,

$$\Delta C > 0 \Leftrightarrow r > \frac{C_{coll} + C_{disp}}{C_{\Delta} + C_{disp}}, \text{ where } C_{\Delta} := C_{man} - C_{reman} > 0.$$

In other words, the value of the take-back option,  $r(C_{\Delta} + C_{disp})$ , must exceed the cost of the option,  $(C_{coll} + C_{disp})$ . This very general condition has to be interpreted with care since it is likely that not all unit costs and flow variables are independent of each other. For instance, the unit collection cost may depend on the collection rate. In the remainder of the paper we will explore some of these interdependencies and their impact on the cost effectiveness of remanufacturing.

## 2 Modeling limited component durability

As mentioned earlier not all of the collected products can be remanufactured. One important reason for this is the limited durability of the reusable components. In order to quantify this durability constraint to remanufacturing a suitable parameter is introduced. It denotes the average number of times a component can be used for the same kind of product, is called average number of lives  $n$  and defined as

$$n := \left\lfloor \frac{\text{average component life}}{\text{average product use}} \right\rfloor.$$

The average component life is the mean of the distribution of component life times, which is a function of component design, product design and the characteristics of the product use and maintenance. The total lifetime of a component should include lifetime extensions via repair and refurbishment. This is done for example with electric motors in industrial products



(Klausner et al. 1998), where the motor is disassembled, cleaned and the commutators are re-surfaced. For our analysis we assume that a reusable component from an end-of-life product can be brought back to a like-new condition. Modeling deterioration (see e.g. Swan 1970) of the component is beyond the scope of this paper but would be a useful extension of the model.

Product	Component	Average component life	Average product use	$n$	Source
Car tire	Casing	500,000km	150,000km	3	(Ferrer 1997b)
Computer	Chip	80,000hrs	20,000hrs	4	(Keeble 1998)
Single use camera	Camera core	6 cycles	1 cycle	6	(Kodak 1999)
Glass bottle	Whole product	25 cycles	1 cycle	25	(UBA 1996)
Wooden pallet	Whole product	50 cycles	1 cycle	50	(UBA 1996)
Crates for bottles	Whole product	120 cycles	1 cycle	120	(UBA 1996)

**Table 2: Average number of lives of some product components**

The average product use denotes the average total usage between product sale and end-of-life and is derived from a customer use distribution. Average component life and average product use are given in the quantity best describing the product use (e.g. total use time, mileage, cycles, etc.). Table 2 lists some products and the average number of lives  $n$  of their reusable parts. If the variances of the component life and customer use distributions are large, the resulting distribution of the number of lives  $n$  will also have a substantial variance. In such a case the company will have to use large safety margins to reduce the statistical volatility and thus ensure reliability of the reused components.

The so-defined average number of lives  $n$  will now be employed to quantify the durability constraint that limits the remanufacturing yield  $r$ , which denotes the percentage of collected products that are remanufactured. From Figure 2 it follows that total demand  $Q$  is satisfied with  $(1-rc)Q$  newly manufactured products and  $rcQ$  remanufactured ones, which already completed one or more cycles in the production system. We assume for the moment that there is always enough market demand to absorb all products that can be remanufactured. Therefore products leave the system only because they are either not collected at the end of their final

use phase, or their reusable components have achieved their maximal number of lives  $n$ . The minimal amount of products that has to be newly manufactured is therefore  $(1-c)Q + c^n(1-rc)Q$ , where  $(1-c)Q$  is to compensate for leakage through imperfect collection and  $c^n(1-rc)Q$  replaces the losses through limited durability, which are all the products that contain components returning for the  $n^{\text{th}}$  time. From  $(1-rc)Q \geq (1-c)Q + c^n(1-rc)Q$  follows that

$$rc \leq (rc)_n := \frac{c - c^n}{1 - c^n} \quad \text{or} \quad r \leq (r)_n := \frac{1 - c^{n-1}}{1 - c^n}.$$

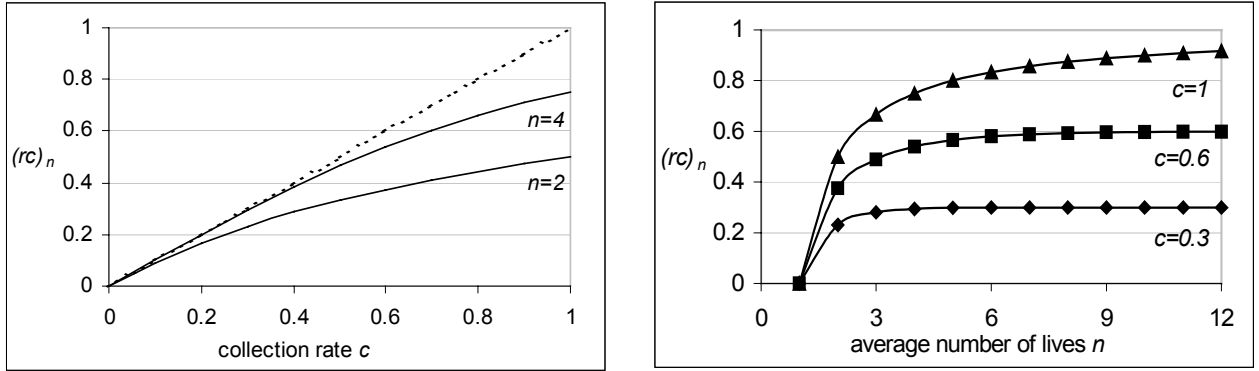
This result is an upper bound for all take-back and remanufacturing systems with unconstrained product demand and limited component durability. For  $c \neq 1$  the durability limit to the remanufacturing rate can also be written as  $(rc)_n = \frac{c - c^n}{1 - c} \cdot \frac{1 - c}{1 - c^n} = \sum_{i=1}^{n-1} c^i / \sum_{i=0}^{n-1} c^i$ , and it has the following properties:

$$\lim_{n \rightarrow \infty} (rc)_n = \lim_{n \rightarrow \infty} \left( \frac{c - c^n}{1 - c^n} \right) = c \quad (\text{unlimited component durability})$$

$$\lim_{c \rightarrow 1} (rc)_n = \lim_{c \rightarrow 1} \left( \frac{c - c^n}{1 - c^n} \right) = \frac{n-1}{n} \quad (\text{perfect collection})$$

$$\frac{d}{dc} (rc)_n = \left( \sum_{i=1}^{n-1} i c^{i-1} \right) / \left( \sum_{i=0}^{n-1} c^i \right)^2 > 0$$

$$\text{in particular } \frac{d}{dc} (rc)_n (c=0) = 1 \text{ and } \lim_{c \rightarrow 1} \left( \frac{d}{dc} (rc)_n \right) = \frac{n-1}{2n}$$



**Figure 3: Durability limit  $(rc)_n$  as a function of the collection rate  $c$  and the average number of lives  $n$**

Two examples illustrate these results (Figure 3).

First, consider the electric motors, which are used in photocopiers and can be reused for copier remanufacturing. If the copiers are leased, i.e.  $c=1$ , and the electric motors can be reused 3 times ( $n=4$ ), then maximally 75% of total demand could be satisfied with copiers containing reused motors. If the motors could only be reused once ( $n=2$ ) the maximal remanufacturing rate would drop to 50%. If the copiers were sold and the collection rate was only  $c=0.3$  the more durable electric motor ( $n=4$ ) could achieve a remanufacturing rate of 29.4%. For the motor with less durable design ( $n=2$ ) the maximal remanufacturing rate would drop by 6.3% down to 23.1%.

The collection system of Kodak's single-use cameras yields a current collection rate of approximately 60% worldwide (Kodak 2001). The camera frame, metering system and flash circuit are designed to be used up to six times ( $n=6$ ) (Kodak 1999). As a result Kodak could satisfy a maximum of 58% of the market demand with cameras containing reused components. As discussed above, no matter how often the camera components could be reused, the maximal possible remanufacturing rate could never exceed the collection rate of 60%, which is only 2% more than the current figure. On the other hand, if  $n$  would be three instead of six

the maximal remanufacturing rate would drop below 49%, so from a design perspective Kodak's durability choice  $n$  is consistent with the collection rate  $c$ .

### 3 The economic impact of limited component durability

Limited component durability reduces the maximal possible remanufacturing rate to the durability limit  $(rc)_n$ , which in turn creates an upper limit for the average production cost difference:

$$\Delta C \leq (\Delta C)_n := \frac{c - c^n}{1 - c^n} (C_{man} - C_{reman} + C_{disp}) - c(C_{coll} + C_{disp})$$

This upper limit is a non-linear function of the collection rate  $c$  and the number of component lives  $n$ . Figure 4 shows that the economically optimal collection rate for given cost structure and  $n$  can be anywhere between zero and one, even though we assume constant marginal collection cost. The reason for this slightly counter-intuitive result is that the marginal cost of the take-back option,  $(C_{coll} + C_{disp})$ , is constant for all collection rates, whereas the marginal value of the take-back option,  $r(C_{\Delta} + C_{disp})$ , decreases with increasing collection. The decreasing marginal value is due to the fact that more and more non-remanufacturable products are being collected and disposed of, especially if  $n$  is low.

The optimal collection rate for  $n=2$  can be easily calculated analytically and is

$$c_n^{opt}(n=2) = \text{Max} \left\{ 0, \text{Min} \left\{ \sqrt{\frac{C_{\Delta} + C_{disp}}{C_{coll} + C_{disp}}} - 1, 1 \right\} \right\}.$$

For  $n > 2$  no closed-form solution exists, since a general polynomial equation of degree  $2n$  would have to be solved formally, which Evariste Galois (1811-1832) proved to be impossible for degrees larger than four in his theory of algebraic equations. Optimal collection rates for  $n > 2$  therefore have to be determined numerically.

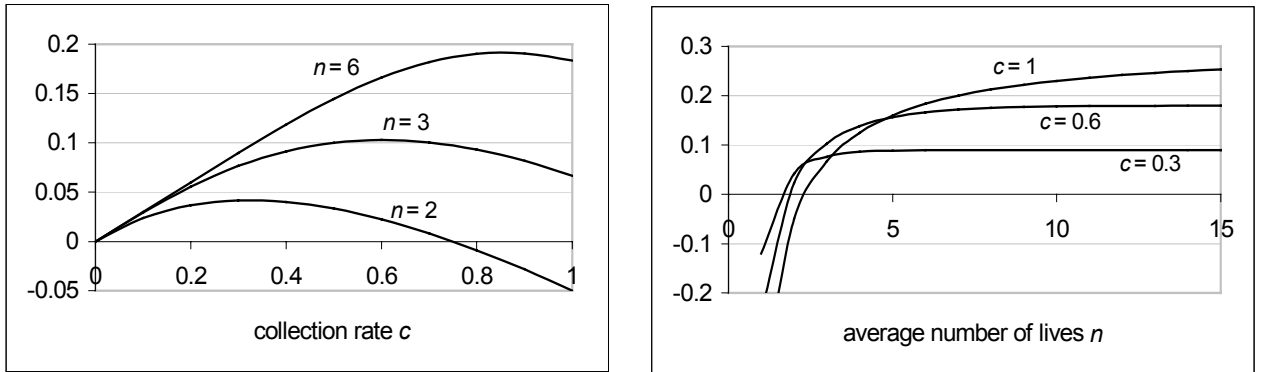
Using the results from section 3 we calculate for which average number of lives  $n$  the optimal collection rate  $c_{opt}$  equals one:

$$\lim_{c \rightarrow 1} \left( \frac{d}{dc} (\Delta C)_n \right) = \frac{n-1}{2n} (C_{\Delta} + C_{disp}) - (C_{coll} + C_{disp}) = 0 \Leftrightarrow n = \frac{C_{\Delta} + C_{disp}}{C_{\Delta} - C_{disp} - 2C_{coll}}$$

Since the average production cost difference is strictly convex in  $c$  and  $n$ , the optimal collection rate  $c_{opt}$  also equals one when  $n$  exceeds this value and is below one when  $n$  is below it:

$$c_n^{opt} < 1 \Leftrightarrow n < \frac{C_{\Delta} + C_{disp}}{C_{\Delta} - C_{disp} - 2C_{coll}} \quad \text{and} \quad c_n^{opt} = 1 \Leftrightarrow n \geq \frac{C_{\Delta} + C_{disp}}{C_{\Delta} - C_{disp} - 2C_{coll}}$$

For practical management purposes it is more useful to work numerically. The analytical result above states that, for example, for the cost structure from Figure 4 the optimal collection rate  $c_{opt}$  will always be below zero, no matter how durable the components, whereas numerical analysis reveals that for  $n=10$  the optimal collection rate is already  $c_{opt}=0.95$ .



**Figure 4: Average cost saving per product (in% of  $C_{man}$ ) with limited component durability and constant unit costs  $(C_{man}, C_{reman}, C_{coll}, C_{disp}) = (1, 0.5, 0.2, 0.2)$  as a function of the collection rate  $c$  (left) and the average number of lives  $n$  (right)**

The assumption of constant unit collection cost is a reasonable first order approach for many cases (Fleischmann 2000). Kodak pays a fixed amount for each returned camera to the developing laboratories (Toktay et al. 2000, Socolow et al. 1994), and Xerox supplies its customers with prepaid return packaging for all toner containers, print cartridges and laser printer supplies (Green Futures 2001). To further refine the model appropriate functional forms of the

collection cost need to be derived and inserted into the equations above. For higher values of the collection rate  $c$  the unit collection cost is typically modeled as monotonically increasing with  $c$  since a further increase of already high collection rates should be disproportionately costly to achieve (Savaskan et al. 1999). This will increase the convexity of the average production cost difference as a function of  $c$  (as shown in Figure 4) and therefore lower the optimal collection rate.

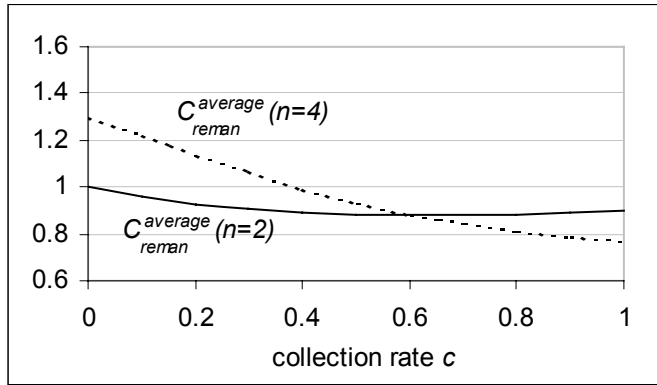
Our economic model is based on the assumption that the reusability of the components is only detectable after the collection cost has occurred, which is the reason for the decreasing marginal value of the take-back option. To increase the cost-efficiency of the take-back scheme a company may wish to collect only remanufacturable end-of-life products and refuse all others. This may not be possible, though, and obtaining the necessary information about the remanufacturability of the end-of-life products will also come at a cost (Klausner et al. 1998). The total value of this information would be  $(1-r)cQ(C_{coll} + C_{disp})$ , which are the cost savings if none of the non-remanufacturable products would have to be collected and disposed of.

Companies deciding to engage in remanufacturing often realize the need to change the product design to optimize the resulting financial return, and the durability level of the reusable components is one of the key decisions to be made (Murray 1995, Kodak 1999). The parameter  $n$  enters the average production cost difference as a power, which turns it into an asymptotic and highly non-linear function of the number of component lives  $n$ :

$$\begin{aligned} (\Delta C)_n &\rightarrow c(C_{\Delta} - C_{coll}) \quad \text{for } n \rightarrow \infty \\ \frac{d}{dn}(\Delta C)_n &= \frac{\ln c \cdot c^n \cdot (c-1)}{(1-c^n)^2} (C_{man} - C_{reman} + C_{disp}) > 0 \\ \frac{d}{dn}(\Delta C)_n &\rightarrow 0 \quad \text{for } n \rightarrow \infty \end{aligned}$$

The marginal cost difference quickly approaches zero because, whereas the cost of the take-back option is independent of  $n$ , its value grows strongly with the first few increases of  $n$  but

then levels off quickly. The option value is initially reduced by the limited remanufacturing yield  $(r)_n$ , i.e. the probability of components being collected after  $n$  uses and having to be disposed of. This probability is  $c^n$  and rapidly approaches zero, i.e.  $(r)_n$  approaches one, as  $n$  increases. Increasing  $n$  beyond a certain level would have virtually no economic benefit, even though we assume that the unit manufacturing cost is independent of  $n$ , i.e. durability is a costless attribute of the component. This level is lower for smaller collection rates.



**Figure 5: Average production cost for two different electric motor designs for reuse**

It is very likely, though, that improving component durability comes at a cost, which means that the resulting economic benefits created by the increased remanufacturing yield need to be traded off against the higher unit manufacturing cost. Our modeling approach is well equipped to quantify this trade-off and facilitate the optimal durability choice, as shown in the following example. Assume that a power tool manufacturer engaging in product take-back and remanufacturing has the production cost structure  $(C_{man}(n=2), C_{man}(n=4), C_{reman}, C_{coll}, C_{disp}) = (1, 1.3, 0.3, 0.2, 0.1)$  and needs to choose the durability of the reusable component, the electric motor. Figure 5 shows the average production cost of both durability choices as a function of the collection rate,  $(1 - (rc)_n)C_{man}(n) + (rc)_nC_{reman} + cC_{coll} + (c + (rc)_n)C_{disp}$ . It can be seen that the benefits of increased remanufacturing due to improved durability only outweigh the higher unit manufacturing cost if a collection rate of 60% or more can be achieved.

## 4 Modeling finite product life cycles

The available amount of remanufacturable products may exceed their market demand. In the case of perfect substitutes this is not due to low customer valuation but the timing between product demand and availability of end-of-life products, which may only return when no demand is left for the particular product model. Typically, a product model is not withdrawn from the market because the type of product is no longer in demand but because it is replaced by new product generations, which result from competitive pressures and technological innovations. For competitive reasons companies increasingly shorten the time for new product development and the time between market introductions of consecutive product generations, thereby shortening the life cycles of the product models (Billington et al. 1998). This can have a strong impact on product take-back and remanufacturing, which we now demonstrate and quantify.

The marketing concept of the product life cycle describes the sales of a product model from market introduction until market withdrawal (Mahajan et al. 1993) and is formalized as a function  $q(t)$ , with  $q > 0$  for  $0 < t < T$  and  $q = 0$  for  $t \leq 0$  or  $t \geq T$ , where  $T$  is the length of the product life cycle. A certain percentage of the sold products will return to the company according to a residence time distribution  $d(\Delta t)$ , which denotes the frequency of all customers (or the likelihood of one customer) to return the product after time interval  $\Delta t$ . The returned products at time  $t$  are now  $v(t) = \int_0^t q(s) \cdot d(t-s) ds$ , which can be easily integrated numerically and will generally result in a curve that, relative to  $q(t)$ , is shifted to the right by the mean of  $d(\Delta t)$  and spread according to its variance and higher moments.

For our purposes we require a closed form solution for  $v(t)$  that is simple enough to allow for analytical treatment but still contains all the essential features of time dependent



product sales and returns. In order to reveal the principal interactions of the system the model should use a minimal set of system parameters. The product life cycle is therefore modeled as an isosceles triangle and completely described by its length  $T$  and the total demand

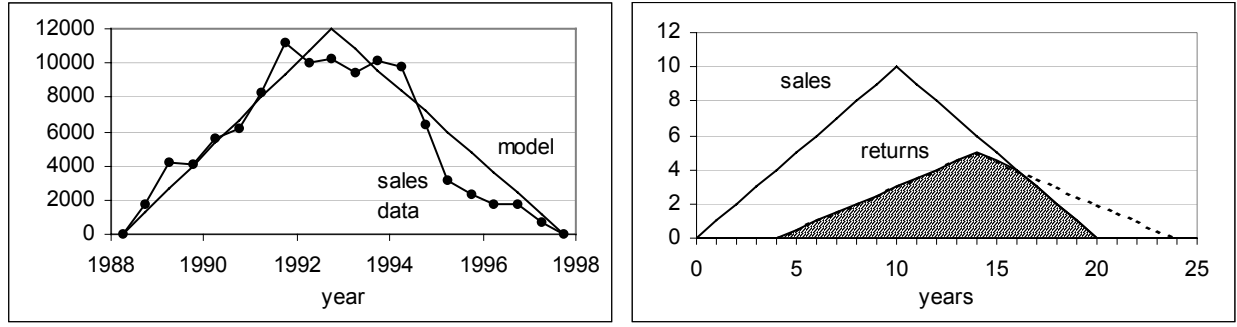
$$Q = \int_0^T q(t)dt :$$

$$q(t) = \begin{cases} 4Q \cdot t / T^2 & 0 < t \leq T/2 \\ 4Q \cdot (T-t) / T^2 & T/2 < t < T \\ 0 & \text{elsewhere} \end{cases}$$

Assuming a constant residence time  $t_\Delta$  between the sale of a particular product and its return to the manufacturer and a constant collection rate  $c$ , which is equivalent to assuming a constant return probability over time, the returns of end-of-life products over time are:

$$v(t) = \begin{cases} 4cQ \cdot (t - t_\Delta) / T^2 & t_\Delta < t \leq t_\Delta + T/2 \\ 4cQ \cdot (T + t_\Delta - t) / T^2 & t_\Delta + T/2 < t < t_\Delta + T \\ 0 & \text{elsewhere} \end{cases}$$

A complete description of product life cycle and product returns is given by the four parameters  $Q, c, T$  and  $t_\Delta$ . Any shape more complex than an isosceles triangle would require additional parameters, which, in this first analysis, would not help in developing intuition. The sales data for a photocopier model in Figure 6 left) show that our assumptions are not unreasonable, since the shortening of the product life cycle mainly reduces the maturity stage of stagnant sales, which turns the traditional trapezoidal shape more and more into a triangle.

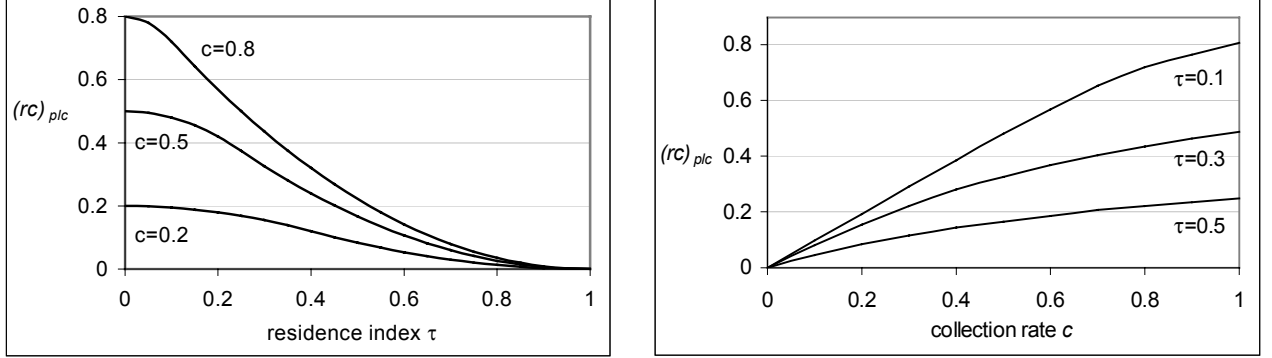


**Figure 6: left) Product life cycle of a photocopier model (Source: Marx-Gomez & Rautenstrauch 1999) right) Model of a product life cycle and its product returns**

Assuming that the durability of the components is infinite ( $n = \infty$ ) the shaded area in Figure 6 right) represents all returned products that can be remanufactured and re-marketed. There is not enough market demand to accommodate the rest of the returned products, and they will have to be disposed of instead. It is important to be aware that the product life cycle  $q(t)$  denotes the joint sales of all-new and remanufactured products. If product components are sold  $m$  times, they appear in Figure 6 right) as one sale of an all-new product and  $(m-1)$  sales of remanufactured products (located in the shaded area). The components then return again (for the  $m^{th}$  time) outside the product life cycle and have to be disposed of due to lack of market demand. The integral over the shaded area in Figure 6 right) divided by total demand  $Q$  gives an upper bound for the remanufacturing rate  $rc$ . In analogy to the durability limit it will be called remarketing limit  $(rc)_{plc}$  and is calculated as  $(rc)_{plc} = \int_{\Lambda}^T |q(t) - v(t)| dt$ . Using elementary geometry the closed form solution can be calculated as

$$rc \leq (rc)_{plc} := \begin{cases} c - \frac{2c}{1-c} \tau^2 & \text{for } \tau \leq \frac{1-c}{2} \\ \frac{2c}{1+c} (1-\tau)^2 & \text{for } \tau \geq \frac{1-c}{2} \end{cases}, \text{ where } \tau := \frac{t_{\Lambda}}{T}.$$

Figure 7 shows that  $(rc)_{plc}$  is a continuous function of the collection rate  $c$  and  $\tau$ , which we will call the residence index. The remarketing limit to the remanufacturing yield is  $r_{plc} = (rc)_{plc} / c$ .



**Figure 7: Remarketing limit  $(rc)_{plc}$  as a function of  $\tau = \frac{t_d}{T}$  and  $c$  (for  $\tau = 0.3$ )**

The remarketing limit decreases monotonically with increasing residence time and goes to zero as the residence time approaches the length of the product life cycle. For a residence index equal or larger than one all returned products have to be disposed of without any value-added recovery through component reuse. This would typically be the case for many electric appliances and electronic goods, where current life cycles are often under one year (Billington et al., 1998). In Section 3 we established that computer chips have an average number of lives of four and are therefore a promising candidate for reuse. When most chips become available for reuse, though, they are one or two generations old, and the PCs they were built in are no longer on the market. The chips are either disposed of with the obsolete PCs or sometimes reused in other products, like electronic toys, for which, one could argue, they are highly overqualified.

## 5 The economic impact of finite product life cycles

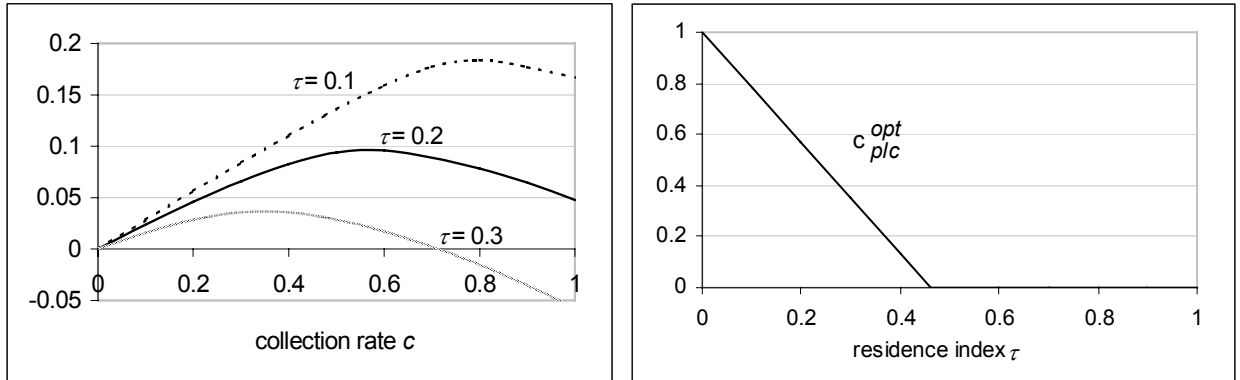
In analogy to Section 4 we now combine our result for the remarketing limit with the general

equation of the economic model to quantify an upper bound on the average production cost difference. Assuming that all returned products could technically be remanufactured ( $n = \infty$ ) the maximal average production cost difference is

$$\Delta C \leq (\Delta C)_{plc} := \begin{cases} \left( c - \frac{2c}{1-c} \tau^2 \right) (C_{\Delta} + C_{disp}) - c(C_{coll} + C_{disp}) & \text{for } \tau \leq \frac{1-c}{2} \\ \frac{2c}{1+c} (1-\tau)^2 (C_{\Delta} + C_{disp}) - c(C_{coll} + C_{disp}) & \text{for } \tau \geq \frac{1-c}{2} \end{cases}$$

Since the production cost difference is now a monotonically decreasing function of  $\tau$ , re-manufacturing saves production cost as long as  $\tau$  is below a certain threshold:

$$(\Delta C)_{plc} > 0 \Leftrightarrow \tau < \begin{cases} \sqrt{\frac{1-c}{2} \frac{C_{coll} - C_{\Delta}}{C_{\Delta} - C_{disp}}} & \text{for } \tau \leq \frac{1-c}{2} \\ 1 - \sqrt{\frac{1+c}{2} \frac{C_{coll} + C_{disp}}{C_{\Delta} + C_{disp}}} & \text{for } \tau \geq \frac{1-c}{2} \end{cases}$$



**Figure 8: left) Average cost savings per product (in % of  $C_{man}$ ) as a function of  $c$  for different  $\tau$  and the cost structure  $(C_{man}, C_{reman}, C_{coll}, C_{disp}) = (1, 0.5, 0.2, 0.2)$  right) Optimal collection rate  $c_{plc}^{opt}$  as a function of  $\tau$  for the same cost structure**

Figure 8 left) shows the average production cost difference as a function of the collection rate  $c$  for different values of  $\tau$  and constant unit costs. The economically optimal collection rate with regards to the finite product life cycle,  $c_{plc}^{opt}$ , can have any value between zero and one. The reason for this is similar to the one causing the results in Section 4. If  $\tau$  is large,

then many of the collected end-of-life products return when demand is too small to accommodate all remanufacturable products and some have to be disposed of instead.  $c_{plc}^{opt}$  optimizes the trade-off between savings through remanufacturing and the costs of collecting and disposing excess end-of-life products when  $\frac{\partial(\Delta C)_{plc}}{\partial c} = 0$  and  $\frac{\partial^2(\Delta C)_{plc}}{\partial c^2} < 0$ . The closed-form solution of  $c_{plc}^{opt}(\tau)$  is shown below and exemplified in Figure 8 right) for a given cost structure.

$$c_{plc}^{opt} = \begin{cases} 0 & \text{for } f(\tau) < 0 \\ f(\tau) & \text{for } f(\tau) \in [0, 1] \text{ with } f(\tau) = \begin{cases} I - \sqrt{2 \frac{C_{\Delta} + C_{disp}}{C_{\Delta} - C_{coll}}} \cdot \tau & \text{for } C_{\Delta} \leq 2C_{coll} + C_{disp} \\ \sqrt{2 \frac{C_{\Delta} + C_{disp}}{C_{coll} + C_{disp}}} (1 - \tau) - 1 & \text{for } C_{\Delta} \geq 2C_{coll} + C_{disp} \end{cases} \\ 1 & \text{for } f(\tau) > 1 \end{cases}$$

If the unit collection cost is not constant but increases monotonically with the collection rate the convexity of the average production cost difference as a function of  $c$  increases and the economically optimal collection rate is lower. Of course, it may not always be easy for the manufacturing company to control the collection rate. One assumption we made throughout the paper is for the collection rate to be constant over time. If the company could influence the collection rate over time, e.g. with financial incentives like rebates on new sales, it would be beneficial to collect more products at the beginning of the life cycle and fewer at the end. Where take-back is offered as a general customer service (customers do not have to dispose of the product themselves) controlling the collection rate may prove to be more difficult.

## 6 Coordinating limited durability and finite product life cycles

We now investigate a production system with remanufacturing considering both the effect of limited component durability and the impact of finite product life cycles. We assume that,

apart from those two constraints, the maximal possible amount of collected end-of-life products is remanufactured. There are two different cases to consider:

- The component durability is the effective limit of the remanufacturing rate,  $(rc)_n \leq (rc)_{plc}$
- The finite product life cycle is the effective limit of the remanufacturing rate,  $(rc)_{plc} < (rc)_n$

As long as  $(rc)_n \leq (rc)_{plc}$ , remanufacturing is only limited by the component durability. The company needs to manufacture  $(1 - (rc)_n)Q$  all-new products and the reusable components will, on average, be used  $1/(1 - (rc)_n)$  times. This is the maximal component reuse for the given product design and collection rate, and an increase in component durability could be considered. The length of the product life cycle has no impact on the economic viability of reuse and remanufacturing, which is entirely governed by the relationship between production cost structure, collection rate and component durability as investigated in Section 4.

As soon as  $(rc)_{plc} < (rc)_n$ , the production cost difference is bounded by limited market demand. The economic analysis for this case has been carried out in the previous section. The lack of demand limits the remanufacturing rate to the extent that value-added in the form of collected remanufacturable products is lost. The durability of the reusable components is higher than is justified by the ratio between the length of the product life cycle and the average residence time of the product.

Durability choice exactly matches market demand when  $(rc)_n = (rc)_{plc}$ . Both limits are functions of the collection rate  $c$  in different non-linear ways. So, theoretically, matching pairs of number of lives  $n$  and residence indices  $\tau$  should be calculated individually for each collection rate  $c$ . However, as functions of  $c$ ,  $(rc)_n$  and  $(rc)_{plc}$  (Figure 3 left and Figure 7 right) are very similar and we find that if we exactly match both remanufacturing limits for perfect collection ( $c=1$ ) they are very well matched for all other collection rates:

$$(rc)_n(c=1) = \frac{n-1}{n}(1-\tau)^2 = (rc)_{plc}(c=1) \Rightarrow \tau = 1 - \sqrt{\frac{n-1}{n}} \Leftrightarrow n = \frac{1}{1-(1-\tau)^2}$$

$n$	2	3	4	5	6	7	8
$\tau = 1 - \sqrt{(n-1)/n}$	0.2929	0.1835	0.1340	0.1056	0.0871	0.0742	0.0646

**Table 3: Matching pairs of number of component lives  $n$  and residence indices  $\tau$**

Remarkably, for these matching pairs of  $n$  and  $\tau$  not only the functional values are equal at  $c=1$  but also their first derivatives,

$$\frac{d}{dc}(rc)_n(c=1) = \frac{n-1}{2n} = \frac{1}{2}(1-\tau)^2 = \frac{d}{dc}(rc)_{plc}(c=1).$$

For  $c=0$  the durability limit has the functional value 0 and the first derivative 1. The remarketing limit is also 0 for  $c=1$  and its first derivative has the value  $1-2\tau^2$ , which is in the range  $[0.83,1]$  since  $0 \leq \tau \leq 0.2929$  in order to match any number of component lives (see Table 3). So, since we know that for pairs of  $n$  and  $\tau$  with  $(rc)_n(c=1) = (rc)_{plc}(c=1)$  both functions start at  $c=0$  with value 0 and first derivative equal or close to 1, are strictly convex and end at  $c=1$  with the same value (by design) and the same derivative, it is now less surprising that for all other collection rates the difference  $|(rc)_n - (rc)_{plc}|/(rc)_n$  is mostly considerably lower than 5%. From a remanufacturing perspective component durability and product life cycle are nearly optimally matched (regardless of the collection rate) when the average number of lives  $n$  and residence index  $\tau$  fulfill

the following equation:

$$(rc)_{plc} \approx (rc)_n \Leftrightarrow \tau = 1 - \sqrt{\frac{n-1}{n}}$$

This correspondence between the two different constraints has been derived using simplified models for component durability and product life cycles but is very useful as a first-order indicator. To give an example of its application, reuse of computer chips ( $n=4$ ) is already limited by the product life cycle as soon as  $\tau > 1 - \sqrt{3/4} = 0.134$ , regardless of the collection rate.

The amount of component lives that will remain unused if  $(rc)_{plc} < (rc)_n$  can be computed as follows. The durable components of each of the  $(1 - (rc)_{plc})Q$  newly manufactured products can, on average, be used  $1/(1 - (rc)_n)$  times. After the end of the product life cycle,  $Q$  of these  $\frac{1 - (rc)_{plc}}{1 - (rc)_n}Q$  component lives will have been used. The remaining component lives are lost due to the fact that remanufacturable products, i.e. end-of-life products that contain reusable components, return outside the product life cycle. The total number of component lives that will remain unused due to lack of demand is therefore  $\frac{1 - (rc)_{plc}}{1 - (rc)_n}Q - Q = \frac{(rc)_n - (rc)_{plc}}{1 - (rc)_n}Q$ . If  $\tau = 1 - \sqrt{\frac{n-1}{n}}$  the component durability and the length of the product life cycle are matched such that the number of remaining component lives at the end of the product life cycle is practically zero.

## 7 Conclusions

In this paper we introduced a modeling methodology that allows the economic assessment of production systems with or without product take-back and remanufacturing. We developed a simple economic model of an original equipment manufacturer, who has the option to collect at least part of her products at the end of their use phase to remanufacture them into perfect substitutes of the all-new version. Assuming that the unit cost saving from remanufacturing is larger than the unit collection cost it follows that remanufacturing is profitable if the remanufacturing yield, i.e. the fraction of collected products that is remanufactured and remarketed, is above a critical value.

The remanufacturing yield is limited by technical and market constraints. Modeling limited component durability as a major technical constraint introduces non-linearities into



our model. The production cost is now a function of the cost structure, the collection rate and the component durability, which need to be coordinated to create economically optimized remanufacturing systems. The same is true when finite product life cycles are modeled as the main constraint in a market, where all-new and remanufactured products are perfect substitutes. Cost structure, collection rate and the length of the product life cycle need to be coordinated to yield optimal economic results.

We finally investigated remanufacturing systems with both limited component durability and finite product life cycles. Both constraints need to be matched as well to realize the cost savings potential of remanufacturing. We find that this can be achieved independent of the collection rate despite their different nonlinear dependence on collection. We expect this result to be robust and not specific to our chosen modeling approach but it was beyond the scope of this paper to show this.

In the present paper it is assumed that the product containing reused components is a perfect substitute for the all-new version. If the actual or perceived quality of the remanufactured product is not identical with the all-new version it will have to be marketed as a mere substitute that is competing with the all-new version, like the ‘green line’ copiers that Xerox introduced in 1992 (VanThiel 1994). Theoretically these products might even be able to capture a price premium for their environmental benefits but it is more likely that they have to be marketed as a low cost alternative to the all-new version (Earl & Clift 1999). This will have two effects. It will cannibalize the sales of the all-new products to a certain degree but it may also be able to extend the customer base of the company attracting environmentally or price conscious customers, who previously did not consider purchasing a product of that brand (VanThiel 1994). The second effect implies that there may still be market demand for the low cost version of the product even though the life cycle of the higher priced all-new one has already ended. We believe that our modeling approach can be readily extended to accommodate

a remanufacturing environment, where collection rate, component durability, product life cycles and product differentiation interact with each other to determine the viability of product recovery and remanufacturing. This will be the subject of future research.

There are two more fundamentally different component reuse options, which should be mentioned briefly. For a company that produces durable goods of high value, like copiers and cars, repair and maintenance will be quite an important part of their business. Here, returning end-of-life products can be a welcome source of spare parts, especially after the company stopped producing the model. The car industry has a long tradition in remanufacturing automotive parts as spares (Steinhilper 1998), and the first practiced component reuse option at Xerox was to cannibalize the returning end-of-lease copiers for spare parts. It can be expected, though, that the volume of required spares is rather limited (Thierry et al. 1995) and that therefore component reuse for spare parts alone is not enough to be able to deal with the amount of product returns created by a take-back system. If the reusable components cannot be employed for the manufacturing of the original product and are also not needed as spare parts, the company may still be able to sell them on secondary markets. Since about 1993 a viable secondary industry has emerged around the reuse of computer chips in a wide variety of low-tech uses in electronic appliances and toys (Keeble 1998, Davis 1996). Generally, though, it is much more difficult to find or create secondary markets for components with high innovation rates and increased customization, like computer chips, than for standardized components with mature technology.

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