Category Captainship: Outsourcing Retail Category Management
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Abstract

Retailers in the consumer goods industry often rely on a leading manufacturer for category management, a form of manufacturer-retailer collaboration referred to as category captainship. There are reported success stories about category captainship, but also a growing debate about its potential for anti-competitive practices by category captains. Motivated by conflicting viewpoints, the goal of our research is to deepen our understanding of the consequences of such collaboration initiatives between the retailer and only one of its manufacturers. To this end, we develop a game theoretic model of two competing manufacturers selling through one retailer that captures the basic tradeoffs of using category captains for category management. We consider two scenarios that are in line with traditional retail category management and category captainship. In the first scenario, the retailer is responsible for managing the category and determines retail prices and assortment. In the second scenario, we assume that the retailer delegates part or all retail category management decisions to one of the manufacturers in return for a target category profit, and implements its recommendations. We compare these two scenarios to investigate the impact of the transition on all stakeholders in the supply chain. We conclude with design recommendations on the scope and structure of category captainship.

Key words: category management, category captainship, retailing, supply chain collaboration.

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1 Introduction

A product category is defined as a group of products that consumers perceive to be interrelated and/or substitutable. Soft drinks, oral care products and breakfast foods are some examples of retail categories. Category management is a process for managing entire product categories as business units. Unlike the traditional brand-by-brand or SKU-by-SKU focus, category management emphasizes the management of a product category as a whole, allowing retailers to take into account the customer response to decisions made about substitutable or interrelated products. In particular, retail category management involves decisions such as product assortment, pricing, and shelfspace allocation to each product on the basis of category goals. Taking into account the interdependence between products increases the effectiveness of these decisions. However, category management requires that significant resources be dedicated to understanding the consumer response to the assortment, pricing and shelf placement decisions of products within a category.

Recently, retailers have started to outsource retail category management to their leading manufacturers, a practice often referred to as category captainship. Factors such as the increase in the number of product categories offered by retailers, combined with the scarcity of resources to manage each category effectively have given rise to this new trend. In a typical category captain arrangement, the retailer shares pertinent information such as sales data, pricing, turnover, and shelf placement of the brands with the category captain. The retailer also provides the category captain with a profitability target for the category. The category captain, in return, conducts an analysis about the category and provides the retailer with a detailed plan that includes recommendations about which brands to include in the category, how to price each product, how much space to allocate to each brand, and where to locate each brand on the shelf. The retailer is free to use or discard any of the recommendations provided by the category captain. In practice, retailer response ranges from adoption of all recommendations to a process for filtering and selectively adopting manufacturer recommendations.

Several retailers and manufacturers who practice category captainship report positive benefits. For example, Carrefour, a French retailer, implements category captainship with
Colgate in the oral care category, with both companies reporting positive benefits (ECR Conference, 2004). Similarly, retailers such as Wal-Mart and Metro practice category captainship in some of their product categories. Retailers usually assign manufacturers such as Kraft Foods, P&G, and Danone to serve as category captains because of their established brands in the market and their resource availability.

At the same time, conflicts of interest may be an issue. First, there may be a conflict of interest between the retailer and the category captain because the retailer’s objective is to maximize category profit whereas the category captain’s objective is to maximize its own profit (Gruen and Shah 2000). Second, there may be a conflict of interest between the category captain and the non-captain manufacturers. In particular, the category captain may take advantage of its position to disadvantage competitors. Fierce competition is a predominant aspect of the consumer goods industry, spurred by the explosion in the number of new products introduced combined with the much slower increase in the total shelfspace available to offer these products to the consumers. In Store Wars, Corstjens and Corstjens (1995) describe contemporary national brand manufacturers as being in a continuous battle for shelfspace at the retailer.

In this context, it is not surprising that there is an emerging debate on whether or not category captainship poses some antitrust challenges such as “competitive exclusion,” where the category captain takes advantage of its position to disadvantage other manufacturers (Steiner 2001, Desrochers et al. 2003). This phenomenon is illustrated by a recent antitrust case in the smokeless tobacco category. United States Tobacco Co. (UST), the category captain, was sued by Conwood, its largest competitor (Greenberger 2003). Conwood claimed that UST was using its position as category captain to exclude competition and to provide advantage to its own brand. The court ruled that UST’s practices resulted in unlawful monopolization, and condemned UST to pay a $1.05 billion antitrust award to Conwood. Similarly, many other category captainship arrangements have been taken to court regarding category captainship misconduct, e.g. tortillas, cranberries, and carbonated soft drinks (Greenberger 2003).

Motivated by these conflicting observations, our research aims to better understand the impact of category captainship initiatives and to develop recommendations for its imple-
mentation. To this end, we consider a two-stage supply chain where asymmetric competing manufacturers each sell one product to consumers through a common shelfspace-constrained retailer; the products are substitutes. The scope of category management is pricing and assortment. We analyze retail category management, where the retailer makes category decisions, and category captainship, where the retailer assigns a category captain, specifies a target category profit and relies on the category captain’s decisions. We compare these two scenarios to investigate the impact of switching to category captainship on all parties, including the non-captain manufacturer and consumers. In addition, we compare different forms (price vs. price and assortment) and structures (leading manufacturer versus other manufacturer) of category captainship to investigate the impact of the scope and structure of category captainship. Based on these results, we conclude with recommendations about category captainship implementations.

The rest of the paper is organized as follows. In §2, we review the related literature and position our research. In §3, we describe the model and discuss our assumptions. Then, in §4, we analyze the retail category management and the category captainship scenarios with respect to pricing and investigate the impact of category captainship on all the involved supply chain partners. Building on this analysis, §5 asks how the retailer can best implement category captainship. In particular, we provide design recommendations as to the scope and structure of category captainship implementations. In §6, we discuss the robustness of our results to the modelling assumptions. §7 concludes with various implications of our analysis.

2 Literature Review

In this section, we discuss the literature in operations management, marketing and economics that is related to our research, and outline our contributions to each stream.

The supply chain collaboration literature mainly focuses on collaborations in single manufacturer, single retailer supply chains. For example, the retailer might share the parameters of its inventory holding policy (e.g., Gavirneni et al. 1999, Lee et al. 2000) or the supply chain partners may collaborate on forecasting the market demand for their product (e.g., Aviv 2001, Kurtuluş and Toktay 2004). The focus of this literature is to show that supply
chain collaboration can be beneficial to both partners and to characterize conditions under which collaboration is most beneficial or likely. In this paper, we extend the framework beyond a single manufacturer and a single retailer, and investigate manufacturer-retailer collaboration in a broader context. In particular, we investigate the impact of supply chain collaboration on the remaining supply chain members: the non-captain manufacturer and the consumers.

One of the category management decisions is assortment selection. Van Ryzin and Mahajan (1999) study the relationship between inventory costs and variety benefits in retail assortment. This paper determines the optimal assortment and provides insights on how various factors affect the optimal level of assortment variety. Various extensions to the model by van Ryzin and Mahajan have been considered. Hopp and Xu (2003) extend the model by assuming a risk-averse decision maker. Aydin and Hausman (2003) extend the model by studying the supply chain coordination problem in assortment planning. Cachon et al. (2002) study retail assortment in the presence of consumer search. Cachon and Kök (2003) study assortment planning with multiple categories and consider the interaction between the categories. All these papers focus on the retailer’s optimal assortment problem taking the retail prices as given. In contrast, we investigate how retail assortment under category captainship may differ from that under retail category management in a shelfspace-constrained setting with downward sloping demand curves.

Third, there is an emerging marketing literature that particularly focuses on the transition from retail category management to category captainship (Niraj and Narasimhan 2003 and Wang et al. 2003). Niraj and Narasimhan (2003) define category management as an information sharing alliance between the retailer and all manufacturers in the category. The information shared is a signal about the uncertain intercept of the linear demand function. Category captainship is defined as an exclusive alliance between the retailer and only one manufacturer. The paper determines the conditions under which category captainship emerges in equilibrium. In Wang et al. (2003), the retailer and the category captain act as an integrated firm. The authors investigate whether it is profitable for the retailer to delegate pricing authority to the category captain. The main result is that using a category captain for category management is profitable for both the retailer and the category
captain. We focus on the broader impact of category captainship by investigating its impact on the non-captain manufacturer and consumers. In addition to pricing, we consider how the choice of assortment changes under category captainship. Competitive exclusion is shown to arise under category captainship; this effect is more pronounced when shelfspace is constrained.

Finally, some economists have raised antitrust concerns related to category captainship (Steiner 2001, Desrochers et al. 2003). These articles hypothesize that category captainship may lead to competitive exclusion, which refers to situations where the category captain takes advantage of its position to disadvantage other manufacturers. Our model contributes to the ongoing debate about whether category captainship can lead to anticompetitive practices, and offers some theoretical support regarding the competitive exclusion hypothesis.

3 The Model

We consider a two-stage supply chain model with two manufacturers that each produce one product in a given category and sell them to consumers through a common retailer. Below, we discuss our main modelling assumptions. §6 discusses the robustness of our results to some of these assumptions.

**Assumption 1** The demand for each product at the retailer is given by the following linear demand functions:

\[
q_1 = a_1 - p_1 + \theta(p_2 - p_1) \quad q_2 = a_2 - p_2 + \theta(p_1 - p_2)
\]

where \(p_1\) and \(p_2\) are the retail prices of the two products and \(\theta \in [0, 1]\).

The parameters in the demand system have the following interpretation. If the retail prices for both products are the same, the relative demand for each product is determined by the parameters \(a_1\) and \(a_2\). Therefore, we interpret \(a_1\) and \(a_2\) as the relative brand strength of each product. The parameter \(\theta\) is the cross-price sensitivity parameter that shows by how much the demand for product \(j\) increases as a function of a unit price increase in product \(i\). The assumption \(\theta \in [0, 1]\) implies that the products are substitutable. As \(\theta\) increases, the demand for product \(i\), \(q_i\), becomes more sensitive to price changes of product \(j\), \(p_j\).
Therefore, we interpret the parameter $\theta$ as being the *degree of product differentiation*; the higher the parameter $\theta$ the less differentiated the products are.

This type of linear demand system that is consistent with Shubik and Levitan (1980) is widely used in marketing (McGuire and Staelin 1983, Choi 1991, Wang et al. 2003) and economics (Vives 2000, and references therein). The demand functions can be justified on the basis of an underlying consumer utility model: They are derived by assuming that consumers maximize the utility they obtain from consuming quantities $q_1$ and $q_2$ at prices $p_1$ and $p_2$, respectively. The underlying utility model and the corresponding demand function derivation are given in Appendix A. The utility representation is useful as it allows us to investigate how consumers are influenced by different pricing policies and different product assortments via a calculation of the consumer surplus.

**Assumption 2** The manufacturers are in wholesale price competition. This competition is impacted by the limited shelfspace at the retailer.

Let $c_i$ and $w_i$ denote the production cost and wholesale price of manufacturer $i$, $i = 1, 2$. Under retail category management, manufacturers play a simultaneous-move wholesale price game. Under category captainship, the non-captain manufacturer moves first and the category captain determines retail prices and its own wholesale price accordingly. Demand and cost parameters are common knowledge.

As discussed in the introduction, manufacturer competition has intensified due to the proliferation of products in conjunction with the relative scarcity of retail shelfspace. Since retailers operate on very thin margins, every unit of space allocation to manufacturers is scrutinized for profitability. In our model, we do not take into account operational level costs that may play a role in determining the profitability per unit shelfspace allocated to each product. Rather, we capture the increased competition between manufacturers that arises from retailer resource constraints by incorporating a shelfspace constraint $S$ and imposing $q_1 + q_2 \leq S$. This model admits two interpretations. In the first interpretation, $q_1$ and $q_2$ can be viewed as demand rates for each product per replenishment period; the retailer prices the products so that the total demand rate does not exceed the shelfspace availability. In the second interpretation, $q_1$ and $q_2$ can be viewed as the long-term volumes.
to be purchased and sold subject to a total volume target for the category. We assume that $S$ is given.

**Assumption 3** There is a fixed cost for undertaking category management activities and the category captain is more effective than the retailer in undertaking these activities.

This cost can be interpreted as the cost of collecting and analyzing data that is necessary to estimate the demand parameters, for example. One of the reasons that retailers prefer manufacturer involvement in category management is their own lack of resources, and the leading manufacturers’ typically superior knowledge about the retail categories in which they compete. To capture this differential, we assume that the manufacturer’s fixed cost of managing the category is lower than the retailer’s cost $F$, and represent it by $\gamma F$, where $\gamma < 1$.

**Assumption 4** The retailer adopts the category captain’s recommendations.

In practice, some retailers implement their category captain’s recommendations as they are, whereas other retailers implement the recommendations only after filtering and selecting the appropriate recommendations. We focus on category captainship implementations where the retailer adopts the recommendations as they are and discuss (in §4.3) how relaxing this assumption would affect our results.

### 4 The Impact of Category Captainship

In this section, we assume that the retailer has already decided which two products will be offered to the consumers (i.e. which manufacturers he will work with); the scope of category management is pricing. We number the manufacturers such that $a_1 > a_2$, and refer to the first manufacturer as the ‘leading’ or the ‘stronger brand’ manufacturer. We consider the following two scenarios that differ in who manages the category, i.e., who determines retail prices. In the first scenario, Retail Category Management (RCM), we assume that the retailer is responsible for category management and sets the retail price for each product to maximize the category profit subject to the shelfspace constraint. The manufacturers set their wholesale prices strategically in expectation of the quantity demanded of their
own product. In the second scenario, Category Captainship (CC), the retailer delegates the pricing decisions to one of the manufacturers in return for a target category profit. In this section, we assume that the retailer assigns the first manufacturer as category captain. In other words, the retailer assigns the manufacturer with the stronger brand as category captain. The second manufacturer sets its wholesale price strategically in expectation of the quantity demanded of its product. In §4.1 and §4.2, we analyze these two scenarios and in §4.3, we compare them to identify the impact of category captainship on all stakeholders.

4.1 Retail Category Management (RCM)

In the retail category management scenario, the manufacturers play a simultaneous-move Nash game in the wholesale prices. Then the retailer determines the retail prices for both products. Figure 1 illustrates the sequence of events for the RCM model.

![Figure 1: Sequence of Events for the RCM scenario](image)

We solve the problem by backward induction. First, for given wholesale prices \( w_1 \) and \( w_2 \), the retailer solves the following constrained profit maximization problem:

\[
\max_{p_1, p_2} \quad (p_1 - w_1)q_1(p_1, p_2) + (p_2 - w_2)q_2(p_1, p_2) - F \\
\text{s.t.} \quad q_1(p_1, p_2) + q_2(p_1, p_2) \leq S \\
\quad q_1(p_1, p_2) \geq 0, \quad q_2(p_1, p_2) \geq 0
\]

where \( F \) is the fixed cost of managing the category. Let \( \hat{q}_1(w_1, w_2) \) and \( \hat{q}_2(w_1, w_2) \) denote the optimal quantities determined in the above optimization problem for given wholesale prices \( (w_1, w_2) \). Appendix B.1 fully characterizes \( \hat{q}_1(w_1, w_2) \) and \( \hat{q}_2(w_1, w_2) \) for all possible wholesale price combinations.
Manufacturer $i$’s profit is

$$\Pi_i(w_i, w_j) = (w_i - c_i)\hat{q}_i(w_i, w_j) \quad \text{for } i, j = 1, 2 \text{ and } i \neq j.$$ 

In the first stage, anticipating the retailer’s response functions $\hat{q}_i(w_1, w_2)$ and $\hat{q}_2(w_1, w_2)$, the manufacturers play a simultaneous move wholesale price game. The resulting Nash equilibria are characterized in Appendix B.2 for $c_1 \leq \frac{a_1(1+\theta)+a_2(1+\theta)-2S\theta}{1+2\theta}$ and $c_2 \leq \frac{a_1\theta+a_2(1+\theta)-2S\theta}{1+2\theta}$ and are summarized in Figure 5. With these cost assumptions, we discard cases where one of the products is so expensive that it is excluded from the category by the retailer even though there is ample shelfspace.

Define $S_1 = \frac{(1+2\theta)(a_1+a_2-c_1-c_2)}{2(3+2\theta)}$ and $S_2 = \frac{(1+\theta)(a_1+a_2-c_1-c_2)}{2(2+\theta)}$. Then $S_1 < S_2 \ \forall \theta \in [0,1]$.

Let $q_1^R$ and $q_2^R$ denote the equilibrium sales volumes in the retail category management scenario.

**Lemma 1** If $S < S_1$, then there exists a unique equilibrium in the wholesale prices leading to $q_1^R + q_2^R = S$. If $S \in [S_1, S_2]$, then there exist multiple equilibria in the wholesale prices leading to $q_1^R + q_2^R = S$. If $S > S_2$, then there exists a unique wholesale price equilibrium leading to $q_1^R + q_2^R < S$.

**Proof** The proof is in Appendix B.3. $\blacksquare$

When $S < S_1$, the shadow price for the shelfspace constraint, denoted by $\lambda$, is strictly positive, implying that the retailer would benefit from an increase in the shelfspace. When $S \in [S_1, S_2]$, there is a possibility that there exist multiple equilibria in the wholesale price game. In this region, the shadow price for the shelfspace constraint, $\lambda$, is 0, implying that the retailer does not benefit from an increase in the shelfspace. When $S > S_2$, there is ample shelfspace: $q_1^R + q_2^R < S$. In this case, the shadow price for the shelfspace constraint $\lambda = 0$. In the remainder of the analysis, we focus on $S < S_1$.

Let $A = a_1 - a_2$ and $C = c_1 - c_2$. The parameters $A$ and $C$ capture the brand differential and the cost differential between the products. Since $a_1 \geq a_2$, $A \geq 0$. The higher the brand differential $A$ and the lower the cost differential $C$, the more advantage the first manufacturer has over the second manufacturer.
Lemma 2 If $S < S_1$, then $q_1^R + q_2^R = S$ and equilibrium sales volumes are given by

$$(q_1^R, q_2^R) = \begin{cases} 
\left( \frac{S}{2} + \frac{A - (1+2\theta)C}{12}, \frac{S}{2} - \frac{A - (1+2\theta)C}{12} \right) & \text{if } 6S > A - (1 + 2\theta)C, \, 6S > (1 + 2\theta)C - A \\
(S, 0) & \text{if } 6S \leq A - (1 + 2\theta)C \\
(0, S) & \text{if } 6S \leq (1 + 2\theta)C - A 
\end{cases}$$

Proof The proof is in Appendix B.3. ■

The retailer either offers both products or one of the products to the consumers. This depends on the relative profitability that each manufacturer can provide to the retailer. When both products are symmetric ($a_1 = a_2$ and $c_1 = c_2$), the retailer sells an equal volume of each product. However, as the difference (both in terms of brand differential and/or cost differential) between the leading manufacturer and the non-leading manufacturer becomes bigger, the non-leading manufacturer can no longer compete with the leading manufacturer and as a result cannot provide the retailer with the margin that is necessary to ensure a positive sales volume.

We assume that $C < \bar{C} = \frac{6S + A}{1 + 2\theta}$ to ensure that the leading manufacturer's product is always offered to the consumers. For $S < S_1$, let $\Omega^1_{RCM}(S) = \{(A, C, \theta)|q_1^R > 0 \text{ and } q_2^R > 0\}$ be the set of parameters where both products are offered to the consumers and $\Omega^2_{RCM}(S) = \{(A, C, \theta)|q_1^R = S \text{ and } q_2^R = 0\}$ be the set of parameters where the first product is offered and the second product is not offered to the consumers, respectively.

4.2 Category Captainship (CC)

In the category captainship scenario, we assume that the retailer delegates the pricing authority to the first manufacturer in return for a fixed target category profit denoted by $K$. Recall that the retailer implements the recommendations exactly as they are made by the category captain. The sequence of events in the category captainship scenario is illustrated in Figure 2.

In stage 1, the retailer offers a take-it-or-leave-it contract to the category captain (i.e., the retailer asks for a target category profit of $K$). The contract is accepted if the category captain can profit at least as much as it can under the RCM scenario; it is rejected otherwise. In stage 2, the second manufacturer offers a wholesale price for its product to the retailer.
This manufacturer is aware that the other manufacturer is the category captain. In stage 3, the category captain determines the retail prices for both products as well as the wholesale price to be used for its own product.

We solve the problem by backward induction. First, for given \( \{w_2, S, K\} \), we solve the following optimization problem faced by the category captain:

\[
\text{CC: } \max_{p_1, p_2, w_1} (w_1 - c_1)q_1(p_1, p_2) - \gamma F
\]

s.t. \( (p_1 - w_1)q_1(p_1, p_2) + (p_2 - w_2)q_2(p_1, p_2) = K \)

\( q_1(p_1, p_2) + q_2(p_1, p_2) \leq S \)

\( q_1(p_1, p_2) \geq 0, \quad q_2(p_1, p_2) \geq 0 \)

where the parameter \( \gamma \leq 1 \) captures the category captain’s relative effectiveness in developing category management recommendations. The optimization problem given in (2) has the following interpretation, which is further discussed at the beginning of Appendix C.1. The retailer and the category captain form an alliance and the category captain maximizes the total alliance profit by setting the retail prices for both products. Then, the alliance profit is shared between the retailer and the category captain through the wholesale price of the first product, \( w_1 \). The retailer sets its target profit \( K \) such that the category captain’s wholesale price choice \( w_1 \) leaves the category captain with a profit equal to what he would have received under retail category management.

Let \( \hat{q}_1(w_2) \) and \( \hat{q}_2(w_2) \) denote the optimal quantities determined in the above optimization problem for a given \( w_2 \). Appendix C.1 fully characterizes \( \hat{q}_1(w_2) \) and \( \hat{q}_2(w_2) \) for all possible \( w_2 \). In the second stage, given the category captain’s response functions \( \hat{q}_1(w_2) \) and \( \hat{q}_2(w_2) \), the second manufacturer sets its wholesale price \( w_2 \). The resulting equilibrium

![Figure 2: Sequence of events in the CC scenario](image-url)
wholesale prices \( w_1^C \) and \( w_2^C \) and equilibrium sales volumes \( q_1^C \) and \( q_2^C \) are characterized in Appendix C.2 for \( c_1 \leq \frac{a_1(1+\theta)+a_2\theta-2S\theta}{1+2\theta} \) and \( c_2 \leq \frac{a_1\theta+a_2(1+\theta)\theta-2S\theta}{1+2\theta} \).

Finally, in the first stage, the retailer sets the target category profit level \( K \) to leave the category captain indifferent between accepting and rejecting the contract. The retailer sets \( K \) such that
\[
(w_1^C(K) - c_1)q_1^C - \gamma F = \Pi_R^1,
\]
where \( \Pi_R^1 \) is the first manufacturer’s profit under retail category management. Also note that the retailer outsources category management to the category captain only if \( K \geq \Pi_R^R \) where \( \Pi_R^R \) is retailer’s profit under retail category management.

Let us define the shelfspace threshold levels
\[
S_{C1}^C = \frac{(3+4\theta)a_1+(1+4\theta)a_2-3(1+2\theta)c_1-(1+2\theta)c_2}{2(3+4\theta)}
\]
and
\[
S_{C2}^C = \frac{2(1+\theta)a_1+(1+2\theta)a_2-(3+2\theta)c_1-(1+\theta)c_2}{4(1+\theta)}.
\]

**Lemma 3** If \( S < S_{C2}^C \), then there exists a unique equilibrium in the wholesale prices leading to \( q_1^C + q_2^C = S \). If \( S > S_{C2}^C \), then there exists a unique wholesale price equilibrium leading to \( q_1^C + q_2^C < S \).

**Proof** The proof is in Appendix C.3.

If \( S < S_{C1}^C \), the shadow price for the shelfspace constraint, denoted by \( \lambda \), is strictly positive implying that the retailer would benefit from an increase in the shelfspace. When \( S \in [S_{C1}^C, S_{C2}^C] \), the shadow price for the shelfspace constraint \( \lambda = 0 \), implying that the retailer does not benefit from an increase in the shelfspace. Finally, when \( S > S_{C2}^C \), there is ample shelfspace. As in the RCM scenario, we focus on \( S < S_{C1}^C \) in the remainder of the analysis.

**Lemma 4** If \( S < S_{C1}^C \), then \( q_1^C + q_2^C = S \) and equilibrium sales volumes are given by
\[
(q_1^C, q_2^C) = \begin{cases} 
\left( \frac{3S}{4} + \frac{A-(1+2\theta)C}{8}, \frac{S}{4} - \frac{A-(1+2\theta)C}{8} \right) & \text{if } 2S > A - (1+2\theta)C, \ 6S > (1+2\theta)C - A \\
(S, 0) & \text{if } 2S \leq A - (1+2\theta)C \\
(0, S) & \text{if } 6S \leq (1+2\theta)C - A
\end{cases}
\]

**Proof** The proof is in Appendix C.3.

As in the RCM scenario, either both products are offered to the consumers or one of the products is offered to the consumers. We assume that \( C < \bar{C} = \frac{6S+A}{1+2\theta} \) to ensure that the first manufacturer’s product is always offered to the consumers. For \( S < S_{C1}^C \), let
\[
\Omega_{CC}^1(S) = \{ (A, C, \theta) | q_1^C > 0 \ \text{and} \ q_2^C > 0 \}
\]
are offered to the consumers and $\Omega_{CC}^2(S) = \{(A, C, \theta) | q_1^C = S$ and $q_2^C = 0\}$ be the set of parameters where only the first manufacturer’s product is offered to the consumers.

4.3 The Impact of Category Captainship on the Stakeholders

In this section, we compare the retail category management and category captainship scenarios and investigate the effect of the transition to category captainship on all stakeholders. In this discussion, we focus only on cases where both scenarios are constrained by shelf-space availability. In particular, we focus on the cases where the shelfspace parameter $S < S_1 \leq S_C$.

**Proposition 1** For $S < S_1$ and $C < \frac{A}{1+2\theta}$, (i) $q_1^C > q_1^R > 0$ and $0 < q_2^C < q_2^R$ in $\Omega_{CC}^1(S)$; (ii) $q_2^R > 0$ and $q_2^C = 0$ in $\Omega_{RCM}^1(S) \setminus \Omega_{CC}^1(S)$.

**Proof** The proof is in Appendix D. □

If both products are offered to the consumers in both scenarios (in region $\Omega_{CC}^1(S)$ with $S < S_1$), the category captain takes advantage of its position and allocates a higher sales volume to its own product and disadvantages the competing manufacturer by allocating him a lower sales volume. Furthermore, there are cases where this result takes an extreme form: There always exists a nonempty region $(\Omega_{RCM}^1(S) \setminus \Omega_{CC}^1(S)$ with $S < S_1$) in the parameter space where the retailer would have offered the non-captain brand to the consumers, but where the category captain does not do so. Figure 3 illustrates the results in Proposition 1.

![Figure 3](image)

Figure 3: There is a region where the second product is excluded from the category under category captainship but not under retail category management.
Competitive exclusion refers to the phenomenon where the category captain takes advantage of its position to advantage its own brand and disadvantage competitors’ products. Our results suggest that in some cases, the category captain would indeed prefer to go so far as to exclude the non-captain manufacturer’s brand from the category. The UST vs. Conwood case is a good example of a high level of competitive exclusion. In practice, competitive exclusion may take many different forms, most of them less extreme than completely excluding competitors. For example, displaying the non-captain manufacturers’ brands at the bottom of the shelf, or promoting the non-captain manufacturers’ brands at a less desirable time would be some less obvious forms of competitive exclusion.

Examining the region \( \Omega_{RCM}^1(S) \setminus \Omega_{CC}^1(S) \), we identify a number of factors that would play a role on the likelihood of exclusion. First, as the brand differential between the captain brand and another brand in the category increases, it is more likely that the captain manufacturer excludes that brand from the category. Second, if in addition to the brand differential, the captain manufacturer has a cost advantage over the non-captain brand, then exclusion is much easier. Finally, as the level of product differentiation between captain and non-captain brands increases, exclusion is more likely.

A natural question to ask is: What measures can the retailer take to avoid competitive exclusion? One obvious solution would be for the retailer to mandate that the category captain not exclude any of the brands in the category. However, as we mentioned already, exclusion may take many different and non-obvious forms, which may make it difficult for the retailer to monitor the exclusion of the non-captain brands from the category. A second measure is for the retailer to filter the category captain’s recommendations before implementing them. This would avoid the more blatant forms of exclusion. Of course, for the same reason as before, it may not be easy for the retailer to detect biased recommendations when they are subtle. In §5, we explore a structural approach to avoid competitive exclusion: the choice of category captain.

Let us denote the retailer’s and the manufacturers’ equilibrium profits in the retail category management scenario as \( \Pi_{R}^{R}, \Pi_{C}^{R}, \) and \( \Pi_{C}^{R} \) and equilibrium profits in the category captainship scenario as \( \Pi_{R}^{C}, \Pi_{C}^{C}, \) and \( \Pi_{C}^{C} \). Also let \( CS^{R} \) and \( CS^{C} \) denote the consumer surplus in the RCM and CC scenarios. The following proposition compares the profits and
the consumer surplus in both scenarios.

**Proposition 2** In region $\Omega_{CC}^1(S)$ with $S < S_1$ and $C$ low enough

(i) The consumers are better off under category captainship ($CS^C > CS^R$).

(ii) The retailer is better off under category captainship ($\Pi_R^C > \Pi_R^R$).

(iii) The category captain is indifferent between the two scenarios ($\Pi_1^C = \Pi_1^R$).

(iv) The second manufacturer is worse off under category captainship ($\Pi_2^C < \Pi_2^R$).

**Proof** The proof is in Appendix D.

Category captainship provides partial coordination by mitigating the double marginalization between the retailer and the category captain. The benefits of eliminating the double marginalization are shared between the retailer, the category captain, and the consumers. As a result, the consumers are better off under category captainship. The retailer is better off under category captainship both because of the mitigation of double marginalization and because the retailer benefits from the reduction $(1 - \gamma)F$ in the fixed cost of category management. The category captain, on the other hand, is indifferent between the RCM and the CC scenarios because of our assumption that the retailer is the powerful party and offers a take-it-or-leave-it target profit contract to the category captain. It would be possible to extend the model where the benefits of category captainship are shared via bargaining between the retailer and the category captain, in which case the category captain would be better off. Finally, the non-captain manufacturer is always worse off under category captainship because the competitive pressure on this manufacturer increases under category captainship: In the RCM scenario, the non-captain manufacturer competes with the leader manufacturer and the retailer separately whereas in the CC scenario, the non-captain manufacturer competes with the alliance formed by the category captain and the retailer and is forced to reduce its wholesale price.

Therefore, we conclude that category captainship can benefit some of the involved parties, the retailer, the category captain, and the consumers, however, that benefit comes at the expense of the non-captain manufacturer.

Our result about the increase in consumer surplus under category captainship also holds in the region $(\Omega_{RCM}^1(S) \setminus \Omega_{CC}^1(S))$ where competitive exclusion takes place. One might have
expected that consumer surplus would decrease under category captainship because the non-captain brand is excluded and the consumers have less choice. This result is due to the fact that the underlying consumer utility model does not specifically account for the utility that consumers may derive from having access to a variety of products. If on the other hand, consumers do value having access to a number of brands, then category captainship may result in a decrease in consumer surplus. In addition, competitive exclusion may lead to effective monopolization of the category by the category captain (as in the UST example) and can provide the captain manufacturer with significant power over the retailer, which, over time, could lead to increases in wholesale, and consequently, retail prices, again hurting the consumers. This point is further discussed in §6.

5 Design Implications: Scope and Structure of Category Captainship

In our analysis, we have assumed that the retailer has delegated only pricing authority to its category captain and the category captain choice is driven by brand strength only. In this section, we extend our model by relaxing each of these assumptions. In §5.1, we analyze the case where the retailer, in addition to pricing, also delegates the retail assortment decision to the category captain. We compare the retailer’s versus the category captain’s assortment selection. Then, in §5.2, we assume that the assortment decision has been taken by the retailer and analyze the retailer’s category captain selection problem.

5.1 Delegating Assortment Selection

In practice, in addition to pricing, many retailers rely on their category captains for recommendations on assortment planning. However, the retailer and the category captain may have conflicting goals as a result of which their category assortment selections may differ. In this section, we investigate whether delegating the assortment decisions to the category captain is the best choice for the retailer.

Suppose that the retailer assigns a manufacturer with brand strength $a$ and production cost $c$ as category captain. We assume that there are $k$ potential candidates for becoming
the second manufacturer in the category, and each manufacturer is characterized by its
brand strength $a_i$, cross price sensitivity $\theta_i$ with the captain’s product, and production cost
c. We assume that $a > a_i$, $i = 1, \ldots, k$, that is, the captain has the strongest brand. Let
$A_i = a - a_i$. We assume that all $(A_i, \theta_i) \in \Omega_{CC}^1(S)$ and $S < S_1$ so that both products are
offered to the consumers.

**Proposition 3** If $\theta_i = \theta \forall i$, both the retailer and the category captain select manufactur-
er $j$ such that $a_j = \min\{a_1, \ldots, a_k\}$. If $\theta_j \neq \theta_i$, then the retailer’s and the category captain’s
selection of a second brand may be different, with the category captain preferring a highly
differentiated assortment and the retailer preferring a less differentiated assortment when
$C$ is low enough.

**Proof** The proof is in Appendix D. ■

The retailer’s and the category captain’s choice of a second product are in line if the
cross-price sensitivity of all products is the same; in both cases, the assortment that leads
to the highest brand differential is chosen. The reason is that the profit of the alliance
formed by the retailer and the category captain increases as $A_i$ increases, benefiting both
parties. The higher the brand strength of the category captain, $a$, the more are the benefits
of eliminating the double marginalization because a higher fraction of the total volume sold
to the consumers is sold through the alliance.

However, the choice of the second product may be different when the cross price sens-
sitivity of the products is different. The reason is that the category captain prefers to
offer highly differentiated products in the category as opposed to the retailer’s preferences
that are in favor of less differentiated products. The intuition behind this result is that
the retailer benefits from higher competition between manufacturers and is able to extract
more profit when the products in the category are less differentiated, whereas the captain
manufacturer benefits from competing with a brand that is clearly differentiated from its
own brand. As a result, consumers are better off under category captainship because they
get higher utility from having access to differentiated products.

The results about the selection of a second brand have some implications regarding
the implementation of category captainship practices. In practice, retailers rely on their
category captains for recommendations on retail assortment, pricing and shelfspace management. Surprisingly, there are cases where the retailer’s and category captain’s choices of a second product coincide, but it is much more likely that the choices of a second product diverge. Therefore, it may not be best option for the retailer to delegate the assortment decisions to the category captain because of the category captain’s and the retailer’s conflicting goals.

5.2 Selecting a Category Captain

So far, we have assumed that the manufacturer with the strongest brand is assigned as category captain. We demonstrated that in this case, category captainship may lead to competitive exclusion as a result of which consumers are offered less variety. However, consumers may value the flexibility of having access to a number of products in which case the retailer may want to avoid competitive exclusion for competitive reasons. In this section, we explore the possibility of avoiding competitive exclusion by design choices and investigate whether manufacturer selection can be used to this end.

We assume that the retailer has decided the products to be offered in the category. Let $\gamma_1$ and $\gamma_2$ denote the first and second manufacturers’ relative effectiveness in managing the category with $0 < \gamma_1, \gamma_2 \leq 1$. We again take $a_1 > a_2$. If the retailer assigns manufacturer $i$ as category captain, the retailer requires a target category profit of $K^i_R$. In either case, the retailer is able to extract the entire surplus and leave the category captain indifferent between accepting and rejecting the contract.

**Proposition 4** For $S < S_1$ and $(A, C, \theta) \in \Omega^1_{RCM}(S) \setminus \Omega^1_{CC}(S)$, the retailer can avoid competitive exclusion by assigning the non-captain manufacturer as category captain. Furthermore, if

$$\gamma_1 - \gamma_2 > \frac{S(A - (1 + 2\theta)C)}{12F(1 + 2\theta)} > 0,$$

the retailer is strictly better off under the category captainship of the second manufacturer ($K^2_R > K^1_R$).

**Proof** The proof is in Appendix D. ■
When the manufacturer with the stronger brand is assigned as category captain, he can meet the target profit level set by the retailer without including the other manufacturer in the category, which in turn leads to competitive exclusion. However, when a non-leader manufacturer is assigned as category captain, the non-leader manufacturer may not be able to meet the target profit level set by the retailer without including the leader manufacturer, which in turn prevents competitive exclusion. However, the retailer has an incentive to assign the non-leader manufacturer as category captain only if doing so is more profitable. We show that if $\gamma_1 - \gamma_2$ is higher than some positive threshold, the retailer is also better off under the category captainship of the non-leader manufacturer.

In practice, retailers typically simply assign their leading manufacturers as category captains, however, our results suggest that this may lead to competitive exclusion and may not be the most profitable choice. Our result suggests that the retailers should also consider assigning non-leader manufacturers as category captains in case these manufacturers are more effective in category management.

6 Robustness of the Results to Modelling Assumptions

We have made a number of assumptions that simplify our analysis. In this section, we discuss the robustness of our results to these assumptions.

Complementary Products: Our analysis assumed that the products are substitutes ($\theta > 0$). Product categories can consist of products that consumers perceive to be substitutable and/or interrelated. For example, the products in the soft drinks category are substitutes, whereas in the oral care category, retailers offer complementary products such as toothbrushes and toothpaste. With complementary products ($\theta < 0$), the sales of the second product have a positive influence on the sales of the category captain’s own product. Therefore, competitive exclusion is less likely compared to the case where the products in the category are substitutable. On the assortment side, if we allow negative cross-price sensitivity, it is even more likely that the retailer’s and the category captain’s assortment choices are different. Therefore, we conclude that the retailers should be less concerned about competitive exclusion and more concerned about assortment recommendations when
implementing category captainship in categories where they offer complementary products as compared to categories consisting of substitutable products.

**Inclusion of own-price sensitivity:** The demand functions can be generalized to $q_1 = a_1 - b_1p_1 + \theta(p_2 - p_1)$ and $q_2 = a_2 - b_2p_2 + \theta(p_1 - p_2)$, where the parameters $b_1$ and $b_2$ capture own-price sensitivity. Suppose that $c_1 = c_2$. If $b_1 = b_2$, then the condition for competitive exclusion under category captainship is $2S \leq a_1 - a_2$, which is the same as that in Lemma 4 for $C = 0$. If $b_2 < b_1$, it is more difficult to exclude the non-leader brand (the exclusion region shrinks). The parameter $b$ can be seen as a proxy for brand loyalty by consumers, where $b_2 < b_1$ implies that the non-captain manufacturer’s customers are more loyal. The more loyal consumers the non-leader brand has, the less likely its exclusion from the category. On the assortment side, the category captain prefers the second manufacturer to have high own-price sensitivity, whereas the retailer prefers it to have low own-price sensitivity. This makes it more likely that the retailer’s and the category captain’s assortment choices differ when own-price sensitivity is taken into account.

**The Demand Model:** We considered a linear demand model that is based on an underlying representative consumer utility model. How robust are our results to the choice of the demand model? To investigate this, we considered Hotelling’s linear city model of differentiated products. In this model, consumers are uniformly distributed on the line segment $[0, 1]$ and the two products are located at both ends of this segment. By consuming product $i$, a consumer obtains net utility of the form $U = R_i - p_i - tx_i$, where $R_i$ is the utility that the consumer obtains from consuming product $i$, $p_i$ is the retail price of product $i$, $t$ is the unit travelling cost and $x_i$ is the consumer’s distance to product $i$. Unlike the representative consumer model, with this model, the total customer base segments into (at most three) distinct segments: those who buy product 1, those who buy product 2 and those who buy no product. Nevertheless, when we analyzed the basic RCM and CC scenarios under the linear city model, we found the same qualitative results. In particular, we identified an equivalence between the linear city model and the representative consumer model: The parameter $R_i$ in the linear city model behaves like the parameter $a_i$ in the representative consumer model, and the unit travelling cost $t$ in the linear city model behaves like the inverse of the cross-price sensitivity parameter $\theta$ in the representative consumer model. Since our analysis
leads to the same insights when carried out with the linear city model, we conclude that our results about the impact of category captainship on the stakeholders are robust to the choice of demand model.

**Information Structure:** We assumed a common information structure throughout our analysis. In practice, one reason for outsourcing retail category management may be the manufacturers’ superior knowledge about demand. The inclusion of private information about the demand parameters would considerably complicate our analysis and most likely not allow us to obtain closed form solutions for wholesale prices, sales volumes and profits. However, even without analysis, we can assert that our results about the existence of competitive exclusion would be further enhanced by the inclusion of private information because it would make it more difficult for the retailer to detect biased recommendations by the category captain. In addition, if the category captain has private information about the demand, the retailer would not be able to extract the entire benefit of category captainship and would have to pay an information rent to the category captain. In this sense, our results can be seen as a best-case bound for retailer and non-captain manufacturer profits.

**Variety-Seeking Consumers:** Our model does not specifically take into account the fact that consumers might value the option of having access to a number of different products. There is a marketing literature (e.g., Broniarczyk et al. 1998, Hoch et al. 1999, Kahn and Wansink 2004) that investigates the influence of (perceived) variety on consumers. This literature argues that variety almost always exerts a positive influence on consumers. A paper that is particularly relevant is Kim et al. (2002). This paper proposes a model to compute the monetary equivalent of the consumer’s loss in utility from the removal of a variant from the assortment. We expect that such an inclusion of variety seeking behavior on the consumer’s part can reverse our results about the impact of category captainship on consumers. In particular, if consumers value the option of having access to a variety of products, there would be cases where consumers are worse off under category captainship because competitive exclusion decreases the number of variants offered.
7 Conclusions

We consider a retailer who delegates category pricing and/or assortment decisions to one of the manufacturers in the category, a practice known as category captainship. We investigate the impact of category captainship on the retailer, the manufacturers and the consumers.

We demonstrate that category captainship benefits the collaborating partners at the expense of the non-captain manufacturer. In particular, if the retailer assigns the stronger brand manufacturer as category captain, weaker brands may be excluded from the category. Category captainship may increase consumer surplus and offer more differentiated products in the short-run, increasing customer satisfaction. However, if consumers value variety, category captainship has the potential of harming consumers through competitive exclusion. Therefore, retailers should be more vigilant about competitive exclusion in categories where consumers value high variety, and in cases where the leading brand is very powerful.

Our results have implications for retailers about the preferred scope and structure of category captainship. First, retailers should be aware that what is in the best interest of the category captain may not be the best for them. In particular, if the assortment decision is left to the category captain, the level of differentiation in the category may increase, undercutting the retailer’s power over the manufacturers, and leading to lower margins. Therefore, including assortment planning within the scope of category captainship may not be the best approach for the retailer. Second, simply choosing the stronger brand manufacturer to serve as category captain may not be the best choice. For example, we find that competitive exclusion can be avoided if the retailer assigns the non-leader manufacturer as category captain. This choice is also more profitable for the retailer if the non-leader manufacturer is more efficient in managing the category.

A point worth considering is the long-term impact of depending on the manufacturer for category management. Traditionally, manufacturers such as P&G and Unilever were the main players in the fast-moving consumer goods industry and retailers were just a means of reaching consumers. The early nineties saw an increase in the number of high quality new product introductions and the emergence of other strong manufacturers, which led to higher competition for shelfspace. This, combined with the retailers’ awareness of the
importance of being in contact with end consumers, provided the basis for a shift in power from manufacturers to retailers. Many retailers such as Wal-Mart, Carrefour, and Metro owe their rapid growth to these developments. According to Corstjens and Corstjens (1995), ‘... the giant retailers, now, stand as an obstacle between the manufacturers and the end consumers, about as welcome as a row of high-rise hotels between the manufacturer’s villa and the beach.’ It is therefore no surprise that manufacturers would advocate any initiative that can increase their influence over retail decisions, and category captainship is such a practice. By outsourcing category management to their leading manufacturers, retailers may lose their capabilities in managing product categories and their knowledge about consumers. This loss of capability may prepare the basis for a shift of power back from the retailers to the manufacturers. Therefore, retailers should adopt a strategic perspective in evaluating category captainship type practices, and trade off the short-run benefits against the long-term potential disadvantages.

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**References**


Appendix A

The demand system in equation (1) is derived from the so called ‘representative consumer’ model introduced in Shubik and Levitan (1980). The representative consumer model assumes that there is a single consumer in the end market, whose behavior, when magnified sufficiently, will reflect that of the market. By consuming quantities \((q_1, q_2)\) the representative consumer obtains utility

\[
U(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - \frac{1}{2} \left( \beta (q_1^2 + q_2^2) + 2\delta q_1 q_2 \right),
\]

where \(\beta > \delta\) to ensure strict concavity (Shubik and Levitan 1980). When these products are purchased at prices \(p_1\) and \(p_2\), respectively, the consumer surplus is

\[
CS(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - \frac{1}{2} \left( \beta (q_1^2 + q_2^2) + 2\delta q_1 q_2 \right) - p_1 q_1 - p_2 q_2.
\]

The representative consumer solves \(\max_{q_1, q_2} CS(q_1, q_2)\), which yields \(q_1 = a_1 - p_1 + \theta(p_2 - p_1)\) and \(q_2 = a_2 - p_2 + \theta(p_1 - p_2)\). Here, \(a_1 = (\alpha_1 \beta - \alpha_2 \delta) / (\beta^2 - \delta^2)\), \(a_2 = (\alpha_2 \beta - \alpha_1 \delta) / (\beta^2 - \delta^2)\), \(1 + \theta = \beta / (\beta^2 - \delta^2)\), and \(\theta = \delta / (\beta^2 - \delta^2)\). Rewriting the consumer surplus in terms of parameters \(a_1, a_2,\) and \(\theta\), we obtain

\[
CS(q_1, q_2) = \frac{q_1 \left( a_1 + \theta (a_1 + a_2) \right)}{1 + 2\theta} + \frac{q_2 \left( a_2 + \theta (a_1 + a_2) \right)}{1 + 2\theta} - \frac{2q_1 q_2 \theta + (q_1^2 + q_2^2) (1 + \theta)}{2(1 + 2\theta)} - p_1 q_1 - p_2 q_2.
\]

Appendix B.1

The Lagrangian of the optimization problem RCM is given by

\[
\mathcal{L}(p_1, p_2, \lambda, \mu_1, \mu_2) = (p_1 - w_1)q_1(p_1, p_2) + (p_2 - w_2)q_2(p_1, p_2) - \lambda[q_1 + q_2 - S] + \mu_1 q_1 + \mu_2 q_2
\]
The Kuhn-Tucker conditions are

\[
\frac{\partial \mathcal{L}(p_1, p_2, \lambda, \mu_1, \mu_2)}{\partial p_1} = (2p_2 - 2p_1 + w_1 - w_2 - \mu_1 + \mu_2)\theta + a_1 - 2p_1 + w_1 + \lambda - \mu_1 = 0
\]

\[
\frac{\partial \mathcal{L}(p_1, p_2, \lambda, \mu_1, \mu_2)}{\partial p_2} = (2p_1 - 2p_2 + w_2 - w_1 + \mu_1 - \mu_2)\theta + a_2 - 2p_2 + w_2 + \lambda - \mu_2 = 0
\]

\[
\lambda \geq 0, \mu_1 \geq 0, \mu_2 \geq 0, q_1 + q_2 \leq S, q_1 \geq 0, q_2 \geq 0
\]

\[
\lambda(q_1 + q_2 - S) = 0, \mu_1 q_1 = 0, \mu_2 q_2 = 0
\]

**Case (I):** \(q_1 + q_2 < S, q_1 > 0, \) and \(q_2 > 0.\) Then \(\lambda = 0, \mu_1 = 0, \mu_2 = 0.\) Solving the first order conditions for \(p_1\) and \(p_2\) with these multiplier values, we get

\[
\hat{p}_1 = \frac{w_1 + 2\theta w_1 + (1 + \theta) a_1 + \theta a_2}{2 + 4\theta} \quad \hat{p}_2 = \frac{w_2 + 2\theta w_2 + \theta a_1 + (1 + \theta) a_2}{2 + 4\theta},
\]

which yields

\[
\hat{q}_1(w_1, w_2) = \frac{\theta w_2 - (1 + \theta) w_1 + a_1}{2} \quad \hat{q}_2(w_1, w_2) = \frac{\theta w_1 - (1 + \theta) w_2 + a_2}{2}.
\]

We now need to check whether \(\hat{q}_1 + \hat{q}_2 < S, \hat{q}_1 > 0, \) and \(\hat{q}_2 > 0\) hold. Substituting and simplifying, we find that \(\hat{q}_1 + \hat{q}_2 < S\) holds if \(S > (a_1 + a_2 - w_1 - w_2)/2.\) For nonnegativity of the demands, it must be that \((1 + \theta)w_1 - \theta w_2 < a_1\) and \((1 + \theta)w_2 - \theta w_1 < a_2.\)

Manufacturers’ profits are

\[
\pi_i^M = (w_1 - c_1)\frac{\theta w_2 - (1 + \theta) w_1 + a_1}{2} \quad \pi_i^M = (w_2 - c_2)\frac{\theta w_1 - (1 + \theta) w_2 + a_2}{2}
\]

**Case (II):** \(q_1 + q_2 < S, q_1 = 0, \) and \(q_2 > 0.\) Then \(\lambda = 0, \mu_1 \geq 0, \mu_2 = 0.\) Solving the first order conditions for \(p_1\) and \(p_2\) with these multiplier values, we get

\[
\hat{p}_1 = \frac{w_1 + 2\theta w_1 + (1 + \theta) a_1 + \theta a_2 - \mu_1(1 + 2\theta)}{2 + 4\theta} \quad \hat{p}_2 = \frac{w_2 + 2\theta w_2 + \theta a_1 + (1 + \theta) a_2}{2 + 4\theta},
\]

which yields

\[
\hat{q}_1(w_1, w_2) = \frac{\theta w_2 - (1 + \theta) w_1 + a_1 + (1 + \theta)\mu_1}{2} \quad \hat{q}_2(w_1, w_2) = \frac{\theta w_1 - (1 + \theta) w_2 + a_2 - \mu_1\theta}{2}.
\]

The condition \(\hat{q}_1 = 0\) gives \(\mu_1 = \frac{\theta(w_1 - w_2) - a_1 + w_1}{1 + \theta}.\) Substituting in the expression for \(\hat{q}_2,\) we obtain \(\hat{q}_2 = \frac{(1 + \theta)a_2 + a_1\theta - w_2(1 + 2\theta)}{2 + 2\theta}.\) We also need to check for \(\frac{\theta(w_1 - w_2) - a_1 + w_1}{1 + \theta} \geq 0.\) The remaining conditions impose \(0 < \frac{(1 + \theta)a_2 + a_1\theta - w_2(1 + 2\theta)}{2 + 2\theta} < S.\)
Manufacturers’ profits are given by $\pi_{M1}^I = 0$ and $\pi_{M2}^I = (w_2 - c_2)^{(1+\theta)α_2 + α_1θ - w_2(1+2θ)}_{2+2θ}$.

M2’s profit is independent of $w_1$.

**Case (III):** $q_1 + q_2 < S$, $q_1 > 0$, and $q_2 = 0$. Then $\lambda = 0, \mu_1 = 0, \mu_2 \geq 0$. This is symmetric to case (II). With the same analysis, we obtain $\mu_2 = \frac{θ(w_2 - w_1) - a_2 + w_2}{1+θ}$ and $\hat{q}_1 = \frac{(1+θ)α_1 + α_2θ - w_1(1+2θ)}{2+2θ}$ under the conditions $\frac{θ(w_2 - w_1) - a_2 + w_2}{1+θ} \geq 0$ and $0 < \frac{(1+θ)α_1 + α_2θ - w_1(1+2θ)}{2+2θ} < S$.

Manufacturers’ profits are $\pi_{M1}^I = (w_1 - c_1)^{(1+θ)α_1 + α_2θ - w_1(1+2θ)}_{2+2θ}$ and $\pi_{M2}^I = 0$. M1’s profit is independent of $w_2$.

**Case (IV):** $q_1 + q_2 < S$, $q_1 = 0$, and $q_2 = 0$. Then $\lambda = 0, \mu_1 \geq 0, \mu_2 \geq 0$. Proceeding in the same way, we obtain

$$\mu_1 = \frac{w_1(1+2θ) - a_1(1+θ) - a_2θ}{1+2θ} \quad \text{and} \quad \mu_2 = \frac{w_2(1+2θ) - a_2(1+θ) - a_1θ}{1+2θ}.$$  

Thus, the conditions for this case to hold are $w_1(1+2θ) - a_1(1+θ) - a_2θ \geq 0$ and $w_2(1+2θ) - a_2(1+θ) - a_1θ \geq 0$. Manufacturers’ profits in case IV are given by $\pi_{M1}^I = 0$ and $\pi_{M2}^I = 0$.

**Case (V):** $q_1 + q_2 = S$, $q_1 > 0$, and $q_2 > 0$. Then $\lambda \geq 0, \mu_1 = 0, \mu_2 = 0$. Solving the first order conditions for $p_1$ and $p_2$ with these multiplier values, we get

$$\hat{p}_1 = \frac{w_1 + 2θw_1 + (1+θ)α_1 + α_2θ + λ(1+2θ)}{2+4θ} \quad \hat{p}_2 = \frac{w_2 + 2θw_2 + θα_1 + (1+θ)α_2 + λ(1+2θ)}{2+4θ}$$

which yields

$$\hat{q}_1(w_1, w_2) = \frac{θw_2 - (1+θ)w_1 + a_1 - λ}{2} \quad \hat{q}_2(w_1, w_2) = \frac{θw_1 - (1+θ)w_2 + a_2 - λ}{2}.$$  

The condition $\hat{q}_1 + \hat{q}_2 = S$ yields $λ = \frac{a_1 + a_2 - w_1 - w_2 - 2S}{2}$. Substituting, we find

$$\hat{q}_1(w_1, w_2) = \frac{2S + (1+2θ)(w_2 - w_1) + a_1 - a_2}{4} \quad \hat{q}_2(w_1, w_2) = \frac{2S + (1+2θ)(w_1 - w_2) - a_1 + a_2}{4}$$

This case holds under the conditions

$$S ≤ (a_1 + a_2 - w_1 - w_2)/2 \quad w_1 - w_2 < \frac{2S + a_1 - a_2}{1+2θ} \quad w_2 - w_1 < \frac{2S - a_1 + a_2}{1+2θ}.$$  

Manufacturers’ profits are

$$\pi_{M1}^V = (w_1 - c_1)^{(2S + (1+2θ)(w_2 - w_1) + a_1 - a_2)}_{4}.$$
\[ \pi_{V}^{M_2} = (w_2 - c_2) \frac{2S + (1 + 2\theta)(w_1 - w_2) - a_1 + a_2}{4} \]

**Case (VI):** \( q_1 + q_2 = S, q_1 = 0, \) and \( q_2 = S. \) Then \( \lambda \geq 0, \mu_1 \geq 0, \mu_2 = 0. \) Solving the first order conditions for \( p_1 \) and \( p_2 \) with these multiplier values, we get

\[
\hat{p}_1 = \frac{w_1(1 + 2\theta) + (1 + \theta) a_1 + \theta a_2 + (\lambda - \mu_1)(1 + 2\theta)}{2 + 4\theta} \quad \hat{p}_2 = \frac{w_2(1 + 2\theta) + \theta a_1 + (1 + \theta) a_2 + \lambda(1 + 2\theta)}{2 + 4\theta}
\]

which yields

\[
\hat{q}_1(w_1, w_2) = \frac{\theta w_2 - (1 + \theta) w_1 + a_1 - \lambda + \mu_1(1 + \theta)}{2} \quad \hat{q}_2(w_1, w_2) = \frac{\theta w_1 - (1 + \theta) w_2 + a_2 - \mu_1 \theta - \lambda}{2}.
\]

The conditions \( \hat{q}_1 = 0, \hat{q}_2 = S \) yield \( \lambda = \frac{a_1 + a_2(1 + \theta) - w_2(1 + 2\theta) - 2S(1 + \theta)}{1 + 2\theta} \) and \( \mu_1 = \frac{(1 + 2\theta)(w_1 - w_2) - (a_1 - a_2) - 2S}{1 + 2\theta}. \)

Substituting, we verify \( \hat{q}_1 = 0 \) and \( \hat{q}_2 = S. \) The prices in this case are given by

\[
p_1 = \frac{a_1(1 + \theta) + a_2 \theta - S \theta}{1 + 2\theta} \quad p_2 = \frac{a_1 + a_2(1 + \theta) - S (1 + \theta) w_2}{1 + 2\theta}
\]

This case holds under the conditions \( a_1 \theta + a_2(1 + \theta) - w_2(1 + 2\theta) - 2S(1 + \theta) \geq 0 \) and \( (1 + 2\theta)(w_1 - w_2) - (a_1 - a_2) - 2S \geq 0. \)

Manufacturers’ profits in this case are given by \( \pi_{V}^{M_1} = 0 \) and \( \pi_{V}^{M_2} = (w_2 - c_2)S. \)

**Case (VII):** \( q_1 + q_2 = S, q_1 = S, \) and \( q_2 = 0. \) Then \( \lambda \geq 0, \mu_1 = 0, \mu_2 \geq 0. \) This is symmetric to case (vi). We find that \( \lambda = -\left( \frac{2S(1+\theta)+(1+2\theta)(w_1-a_1(1+\theta)-\theta a_2)}{1+2\theta} \right) \) and \( \mu_2 = -\left( \frac{2S+(1+2\theta)(w_1-(1+2\theta)w_2-a_1+a_2)}{1+2\theta} \right). \) Solving it in the same way, we obtain that this case holds under the conditions \( a_2 \theta + a_1(1 + \theta) - w_1(1 + 2\theta) - 2S(1 + \theta) \geq 0 \) and \( (1 + 2\theta)(w_2 - w_1) - (a_2 - a_1) - 2S \geq 0. \) The retail prices in this case are given by

\[
p_1 = \frac{(1 + \theta) a_1 + \theta a_2 - S (1 + \theta)}{1 + 2\theta} \quad p_2 = \frac{\theta a_1 + (1 + \theta) a_2 - S \theta}{1 + 2\theta}
\]

Manufacturers’ profits in this case are given by \( \pi_{V}^{M_1} = (w_1 - c_1)S \) and \( \pi_{V}^{M_2} = 0. \)

**Case (VIII):** \( q_1 + q_2 = S, q_1 = 0, \) and \( q_2 = 0. \) This is infeasible.

We have derived \( \hat{q}_1(w_1, w_2) \) and \( \hat{q}_2(w_1, w_2) \) and the resulting manufacturer profits in seven mutually exclusive and collectively exhaustive regions of \((w_1, w_2)\) space. The regions are illustrated in Figure 4.

**Appendix B.2 (Wholesale Price Game in RCM)**

Given \( \hat{q}_1(w_1, w_2) \) and \( \hat{q}_2(w_1, w_2) \) for any \((w_1, w_2)\), we focus on the wholesale price game that takes place between the manufacturers in the first stage. The best response of manufacturer
Figure 4: Illustration of the possible regions in the wholesale price game.

\( i \) is the solution to

\[
\max_{w_i} (w_i - c_i)\hat{q}_i(w_i, w_j) \quad \text{s.t. } w_i \geq c_i
\]

for \( i = 1, 2 \) and \( i \neq j \), from which we can derive wholesale price equilibria under different parameter combinations, as well as the resulting sales volumes. To simplify the analysis, we assume that \( c_1 \leq \frac{a_1(1+\theta) + a_2 \theta - 2S\theta}{1+2\theta} \) and \( c_2 \leq \frac{a_1\theta + a_2(1+\theta) - 2S\theta}{1+2\theta} \), which eliminates equilibria in Regions II and III or at their boundary with Region I. This means that we discard cases where one of the products is so expensive that it is excluded from the category by the retailer even though there is ample shelfspace. Our results are summarized in Figure 5 and are derived below.

**Case (I):** Suppose that \( w_1 + w_2 > a_1 + a_2 - 2S \), \( (1+\theta)w_1 - \theta w_2 < a_1 \), and \( (1+\theta)w_2 - \theta w_1 < a_2 \) so that the wholesale prices are in region (I). Then the manufacturers expect

\[
\hat{q}_1 = \frac{\theta w_2 - (1 + \theta) w_1 + a_1}{2} \quad \hat{q}_2 = \frac{\theta w_1 - (1 + \theta) w_2 + a_2}{2}.
\]

The best response functions in this region are given by

\[
w_1(w_2) = \frac{\theta w_2 + a_1 + (1 + \theta) c_1}{2 + 2 \theta} \quad w_2(w_1) = \frac{\theta w_1 + a_2 + (1 + \theta) c_2}{2 + 2 \theta}.
\]

Figure 6 illustrates the best response functions when the wholesale price equilibrium is in region (I). If an equilibrium exists in this region, the equilibrium wholesale prices are given
Figure 5: Summary of the wholesale price game at time $t = 1$.

<table>
<thead>
<tr>
<th>Eq. Sales Volume</th>
<th>Conditions</th>
<th>Eq. Wholesale Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^R_1 &gt; 0$, $q^R_2 &gt; 0$  $q^R_1 + q^R_2 &lt; S$</td>
<td>$S &gt; (1 + \theta) (a_1 + a_2 - c_1 - c_2) = S_2$  $2 (1 + \theta) a_1 + \theta a_2 - (2 + \theta (4 + \theta)) c_1 + \theta (1 + \theta) c_2 &gt; 0$  $\theta a_1 + (1 + \theta) (2 a_2 + \theta c_1) - (2 + \theta (4 + \theta)) c_2 &gt; 0$</td>
<td>$w^R_1 = \frac{\theta a_2 + (1 + \theta) (2 a_1 + \theta c_2 + 2 (1 + \theta) c_1)}{4 + 8 \theta + 3 \theta^2}$  $w^R_2 = \frac{\theta a_1 + (1 + \theta) (2 a_2 + \theta c_1 + 2 (1 + \theta) c_2)}{4 + 8 \theta + 3 \theta^2}$</td>
</tr>
<tr>
<td>$q^R_1 &gt; 0$, $q^R_2 &gt; 0$  $q^R_1 + q^R_2 = S$</td>
<td>$S &lt; (1 + 2 \theta) (a_1 + a_2 - c_1 - c_2) = S_1$  $6 s + a_1 - a_2 - (1 + 2 \theta) (c_1 - c_2) &gt; 0$  $6 s - a_1 + a_2 + (1 + 2 \theta) (c_1 - c_2) &gt; 0$</td>
<td>$w^R_1 = \frac{6 S + a_1 - a_2 + 2 c_1 + c_2}{3 (1 + 2 \theta)}$  $w^R_2 = \frac{6 S - a_1 + a_2 + c_1 + 2 c_2}{3 (1 + 2 \theta)}$</td>
</tr>
<tr>
<td>$q^R_1 &gt; 0$, $q^R_2 &gt; 0$  $q^R_1 + q^R_2 = S$</td>
<td>$S &lt; S_2$  $w^<em>_2 - w^</em>_1 &lt; \frac{2 S - a_1 + a_2}{1 + 2 \theta}$  $w^<em>_1 - w^</em>_2 &lt; \frac{2 S + a_1 - a_2}{1 + 2 \theta}$</td>
<td>Multiple equilibria: $(w^R_1, w^R_2)$ such that $w^R_1 + w^R_2 = a_1 + a_2 - 2 S$</td>
</tr>
<tr>
<td>$q^R_1 = 0$, $q^R_2 = S$  $q^R_1 + q^R_2 = S$</td>
<td>$6 s + a_1 - a_2 - (1 + 2 \theta) (c_1 - c_2) \leq 0$</td>
<td>$w^R_1 = c_1$  $w^R_2 = c_1 - \frac{2 S + a_1 - a_2}{1 + 2 \theta}$</td>
</tr>
<tr>
<td>$q^R_1 = s$, $q^R_2 = 0$  $q^R_1 + q^R_2 = S$</td>
<td>$S &lt; S_2$  $w^R_1 = c_2$  $w^R_2 = \frac{2 S - a_1 + a_2}{1 + 2 \theta}$</td>
<td></td>
</tr>
</tbody>
</table>

by

$$w^R_1 = \frac{\theta a_2 + (1 + \theta) (2 a_1 + \theta c_2 + 2 (1 + \theta) c_1)}{4 + 8 \theta + 3 \theta^2}$$

$$w^R_2 = \frac{\theta a_1 + (1 + \theta) (2 a_2 + \theta c_1 + 2 (1 + \theta) c_2)}{4 + 8 \theta + 3 \theta^2}$$

For this equilibrium to exist, it must be that

$$w^R_1 + w^R_2 = \frac{a_1 + a_2 + (1 + \theta) (c_1 + c_2)}{2 + \theta} > a_1 + a_2 - 2 S$$

Rearranging the terms we get

$$S > S_2 \equiv \frac{(1 + \theta) (a_1 + a_2 - c_1 - c_2)}{2 (2 + \theta)}$$

Second it must be that $q_1^R > 0$, which holds if

$$\frac{(1 + \theta) (2 (1 + \theta) a_1 + \theta a_2 - (2 + \theta (4 + \theta)) c_1 + \theta (1 + \theta) c_2)}{(2 + \theta) (2 + 3 \theta)} > 0$$
and $q^R_2 > 0$ which holds if
\[
\frac{(1 + \theta) \left( \theta a_1 + (1 + \theta) (2 a_2 + \theta c_1) - (2 + \theta (4 + \theta)) c_2 \right)}{(2 + \theta) (2 + 3 \theta)} > 0
\]
The conditions for $w^R_1 > c_1$ and $w^R_2 > c_2$ are the same as the conditions for $q^R_1 > 0$ and $q^R_2 > 0$. This is because a positive demand guarantees the manufacturer a strictly positive margin. The retail prices in this case are given by
\[
\begin{align*}
p^R_1 &= \frac{(1 + \theta) \left( (1 + 2 \theta) (c_2 \theta + 2 c_1 (1 + \theta)) + 3 (2 + \theta (4 + \theta)) a_1 \right) + \theta \left( 5 + \theta (10 + 3 \theta) \right) a_2}{2 (2 + \theta) (1 + 2 \theta) (2 + 3 \theta)} \\
p^R_2 &= \frac{\theta \left( 5 + 10 \theta + 3 \theta^2 \right) a_1 + (1 + \theta) \left( (1 + 2 \theta) (c_1 \theta + 2 c_2 (1 + \theta)) + 3 (2 + 4 \theta + \theta^2) a_2 \right)}{2 (2 + \theta) (1 + 2 \theta) (2 + 3 \theta)}
\end{align*}
\]

Figure 6: Best response functions when the equilibrium is in region (I) for $c_1 = c_2 = 0$.

**Case (II):** For $w_1$ large enough, the best response of the second manufacturer falls in the interior of Region II and is equal to $w_2(w_1) = \frac{a_2 + a_1 (1 + \theta) + c_2 (1 + 2 \theta)}{2 (1 + 2 \theta)}$, independently of $w_1$, which gives $q^R_1 = 0$ and $q^R_2 > 0$. However, under our assumption about the cost of production of the first manufacturer, $c_1 \leq \frac{a_1 (1 + \theta) + a_2 (1 + \theta) - 2 S_0}{1 + 2 \theta}$, such an equilibrium is not possible as the first manufacturer can reduce its price enough to capture a positive volume.

**Case (III):** This is symmetric to Case II. Under our assumption about the cost of production of the second manufacturer, $c_2 \leq \frac{a_1 (1 + \theta) + a_2 (1 + \theta) - 2 S_0}{1 + 2 \theta}$, no equilibrium exists in Region III.

**Case (IV):** Since $\hat{q}_1 = 0$ and $\hat{q}_2 = 0$ in Region IV, the best response of neither manufacturer falls in this region, as a result of which no equilibrium exists in this region.
**Case (V):** Suppose that \( w_1 + w_2 \leq a_1 + a_2 - 2S \), \( w_1 - w_2 < \frac{2S + a_1 - a_2}{1 + 2\theta} \), and \( w_2 - w_1 < \frac{2S - a_1 + a_2}{1 + 2\theta} \). We consider the following two subcases: (i) \( w_1 + w_2 < a_1 + a_2 - 2S \) and (ii) \( w_1 + w_2 = a_1 + a_2 - 2S \) separately.

**Subcase (i)** \( w_1 + w_2 < a_1 + a_2 - 2S \), \( w_1 - w_2 < \frac{2S + a_1 - a_2}{1 + 2\theta} \), and \( w_2 - w_1 < \frac{2S - a_1 + a_2}{1 + 2\theta} \). In this case, the manufacturers expect

\[
\hat{q}_1 = \frac{2S + (1 + 2\theta)(w_2 - w_1) + a_1 - a_2}{4}
\]

\[
\hat{q}_2 = \frac{2S + (1 + 2\theta)(w_1 - w_2) - a_1 + a_2}{4}
\]

The best response functions are given by

\[
w_1(w_2) = \frac{2S + (1 + 2\theta)w_2 + a_1 - a_2 + (1 + 2\theta)c_1}{2 + 4\theta}
\]

\[
w_2(w_1) = \frac{2S + (1 + 2\theta)w_1 - a_1 + a_2 + (1 + 2\theta)c_2}{2 + 4\theta}
\]

Figure 7 illustrates the best response functions when the wholesale price equilibrium is in region (V,i). The equilibrium wholesale prices are given by

\[
w_1^R = \frac{6S + a_1 - a_2}{3(1 + 2\theta)} + \frac{2c_1}{3} + \frac{c_2}{3}
\]

\[
w_2^R = \frac{6S - a_1 + a_2}{3(1 + 2\theta)} + \frac{c_1}{3} + \frac{2c_2}{3}
\]

For \((w_1^R, w_2^R)\) to satisfy the conditions that define this region, it must be that

\[
w_1^R + w_2^R = \frac{4S}{1 + 2\theta} + c_1 + c_2 < a_1 + a_2 - 2S
\]

Rearranging the terms we get

\[
S < S_1 = \frac{(1 + 2\theta)(a_1 + a_2 - c_1 - c_2)}{2(3 + 2\theta)}
\]

Moreover, it can be shown that \( S_1 < S_2 \) by showing that \( S_2 - S_1 > 0 \) when \( w_1 + w_2 \leq a_1 + a_2 - 2S \).

Second, it must be that \( w_1^R - w_2^R < \frac{2S + a_1 - a_2}{1 + 2\theta} (q_1^R > 0) \). Rearranging the terms we get \( 6S + a_1 - a_2 - (1 + 2\theta)(c_1 - c_2) > 0 \). Third, it must be that \( w_2^R - w_1^R < \frac{2S - a_1 + a_2}{1 + 2\theta} (q_2^R > 0) \), rearranging the terms we get \( 6S - a_1 + a_2 + (1 + 2\theta)(c_1 - c_2) > 0 \). Finally we check that \( w_1^R > c_1 \) and \( w_2^R > c_2 \). The conditions for \( w_1^R > c_1 \) and \( w_2^R > c_2 \) are the same as the conditions for \( q_1^R > 0 \) and \( q_2^R > 0 \). This is because a positive demand guarantees the manufacturer a strictly positive margin.
The best response functions are given by

\[
\begin{align*}
p_1^R &= \frac{(11 + 12 \theta) a_1 + (1 + 12 \theta) a_2}{12(1 + 2 \theta)} - \frac{1}{12} (6 S - c_1 + c_2) \\
p_2^R &= \frac{(1 + 12 \theta) a_1 + (11 + 12 \theta) a_2}{12(1 + 2 \theta)} - \frac{1}{12} (6 S + c_1 - c_2)
\end{align*}
\]

and the equilibrium sales volumes are given by

\[
q_1^R = \frac{1}{12} (6 S + a_1 - a_2 + (1 + 2 \theta) (c_2 - c_1)) \quad q_2^R = \frac{1}{12} (6 S - a_1 + a_2 + (1 + 2 \theta) (c_1 - c_2))
\]

**Subcase (ii)** $w_1 + w_2 = a_1 + a_2 - 2S$, $w_1 - w_2 < \frac{2S-a_1+a_2}{1+2\theta}$, and $w_2 - w_1 < \frac{2S-a_1+a_2}{1+2\theta}$. The best response functions are given by $w_1(w_2) = -w_2 + a_1 + a_2 - 2S$ for

\[
w_2^B = \frac{a_1(1 + 4\theta) + a_2(3 + 4\theta) - 2S(3 + 4\theta) - c_1(1 + 2\theta)}{3(1 + 2\theta)} < w_2 \leq w_2^A = \frac{a_1(1 + 2\theta) + 2a_2(1 + \theta) - 4S(1 + \theta) - c_1(1 + \theta)}{2 + 3\theta}
\]

and $w_2(w_1) = -w_1 + a_1 + a_2 - 2S$ for

\[
w_2^C = \frac{a_2(1 + 4\theta) + a_1(3 + 4\theta) - 2S(3 + 4\theta) - c_2(1 + 2\theta)}{3(1 + 2\theta)} < w_1 \leq w_2^D = \frac{a_2(1 + 2\theta) + 2a_1(1 + \theta) - 4S(1 + \theta) - c_2(1 + \theta)}{2 + 3\theta}
\]

Therefore, there is the possibility that there exist multiple equilibria. Figure 8 illustrates the best response functions when the wholesale price equilibrium is in region (V.ii).

Let $w_1^* \equiv \max(w_1^A, w_1^C, \bar{w}_1)$ and $w_2^f \equiv \min(w_1^B, w_1^D, \bar{w}_1)$ where $\bar{w}_1 = \frac{(1+\theta)a_1+\theta a_2-2S(1+\theta)}{1+2\theta}$ and $\bar{w}_1 = \frac{(1+\theta)a_1-\theta a_2-2S\theta}{1+2\theta}$. All the points from $w_1^*$ to $w_2^f$ on the line $w_1 + w_2 = a_1 + a_2 - 2S$ that satisfy the conditions $w_1^f - w_1^* \geq 0$, $w_2^f - w_1^* < \frac{2S-a_1+a_2}{1+2\theta}$, and $w_1^f - w_2^f < \frac{2S+a_1-a_2}{1+2\theta}$ are equilibria of the wholesale price game in Region V.ii. The first condition is equivalent to $S_1 \leq S \leq S_2$. 

Figure 7: Best response functions when the equilibrium is in region (V.i) for $c_1 = c_2 = 0$. 

![Figure 7: Best response functions when the equilibrium is in region (V.i) for $c_1 = c_2 = 0$.](image-url)
Figure 8: Best response functions when the equilibrium is in region (V.ii) for $c_1 = c_2 = 0$.

**Case (VI):** Suppose that $w_2 \leq \frac{a_1 \theta + a_2 (1+\theta) - 2S(1+\theta)}{1+2\theta}$ and $(w_1 - w_2) \geq \frac{2S+a_1-a_2}{1+2\theta}$. In this case, $\hat{q}_1 = 0$ and $\hat{q}_2 = S$. The second manufacturer maximizes its profit $(w_2 - c_2)S$. Since this profit is increasing in $w_2$, the equilibrium, if it exists in this region, can only be on the boundary with region (V) or with region (II). The assumption $c_1 \leq \frac{a_1 (1+\theta) + a_2 \theta - 2S \theta}{1+2\theta}$ eliminates the possibility of an equilibrium on the boundary with region (II), and allows us to focus on a possible equilibrium on the boundary with (V), defined by the line $(w_1 - w_2) = \frac{2S+a_1-a_2}{1+2\theta}$. The second manufacturer’s profit is maximized at the boundary for all $w_1$ such that $\frac{6S-a_1+a_2}{3(1+2\theta)} + c_1 + \frac{2c_2}{3} \leq w_1 \geq \frac{2S+a_1-a_2}{1+2\theta}$, which is equivalent to $w_1 \geq \frac{12S+2a_1-2a_2+(1+2\theta)(c_1+2c_2)}{3(1+2\theta)}$. For this equilibrium to exist, it must be that $\frac{12S+2a_1-2a_2+(1+2\theta)(c_1+2c_2)}{3(1+2\theta)} \leq c_1$ which is $6S + a_1 - a_2 - (1 + 2\theta)(c_1 - c_2) \leq 0$. The wholesale price equilibrium in this case is given by $w_1^R = c_1$ and $w_2^R = c_1 - \frac{2S+a_1-a_2}{1+2\theta}$.

**Case (VII):** Suppose that $w_1 \leq \frac{a_2 \theta + a_1 (1+\theta) - 2S(1+\theta)}{1+2\theta}$ and $(w_2 - w_1) \geq \frac{2S-a_1+a_2}{1+2\theta}$. In this case, $\hat{q}_1 = S$ and $\hat{q}_2 = 0$. The first manufacturer maximizes its profit $(w_1 - c_1)S$. Our assumption $c_2 \leq \frac{a_1 \theta + a_2 (1+\theta) - 2S \theta}{1+2\theta}$ allows us to focus on only the equilibria on the boundary between regions (V) and (VII). The first manufacturer’s profit is maximized at the boundary for all $w_2$ such that $\frac{6S+a_1-a_2}{3(1+2\theta)} + \frac{2c_1}{3} + \frac{c_2}{3} \leq w_2 \geq \frac{2S-a_1+a_2}{1+2\theta}$, which is equivalent to $w_2 \geq \frac{12S-2a_1+2a_2+(1+2\theta)(2c_1+c_2)}{3(1+2\theta)}$. For this equilibrium to exist it must be that $\frac{12S-2a_1+2a_2+(1+2\theta)(2c_1+c_2)}{3(1+2\theta)} \leq c_2$ which is $6S - a_1 + a_2 + (1 + 2\theta)(c_1 - c_2) \leq 0$. The only possible wholesale price equilibrium is on the boundary and is given by $w_1^R = c_2 - \frac{2S-a_1+a_2}{1+2\theta}$.
and \( w_2^R = c_2 \).

**Appendix B.3. Proof of the Lemmas in Section 4.1**

**Proof of Lemma 1:** As shown in Appendix B.2, if \( S < S_1 \), there are three possible cases. The wholesale prices are in one of the regions V.i, VI, or VII. Therefore, there are three possible equilibria: (i) \( q_1^R > 0 \) and \( q_2^R > 0 \) corresponding to region V.i; (ii) \( q_1^R = 0 \) and \( q_2^R = S \) corresponding to region VI; and (iii) \( q_1^R = S \) and \( q_2^R = 0 \) corresponding to region VII. In all cases the equilibrium is unique and the shelfspace constraint is binding \((q_1^R + q_2^R = S)\) with \( \lambda > 0 \).

As shown in Appendix B.2, \( S \in [S_1, S_2] \) implies that the wholesale price equilibrium is on the line segment \( w_1^R + w_2^R = a_1 + a_2 - 2S \). There are three possible cases: (i) multiple wholesale price equilibria leading to multiple possible sales volumes such that \( q_1^R > 0 \) and \( q_2^R > 0 \) corresponding to the interior of the line segment V.ii; (ii) \( q_1^R = 0 \) and \( q_2^R = S \) corresponding to the endpoint bordering region VI; and (iii) \( q_1^R = S \) and \( q_2^R = 0 \) corresponding to the endpoint bordering region VII. In all cases, the shelfspace constraint is binding \((q_1^R + q_2^R = S)\) with \( \lambda = 0 \).

Finally, if \( S > S_2 \), then \( q_1^R + q_2^R < S \) and \( \lambda = 0 \). In this case, the only relevant region is region (I) where \( q_1^R > 0 \) and \( q_2^R > 0 \) and the equilibrium is unique.

**Proof of Lemma 2:** If \( S < S_1 \), there are three possible cases. The wholesale prices are in one of the regions V.i, VI, or VII. The wholesale price equilibrium in region V.i is given by

\[
w_1^R = \frac{6S + a_1 - a_2}{3(1 + 2\theta)} + \frac{2c_1}{3} + \frac{c_2}{3} \quad w_2^R = \frac{6S - a_1 + a_2}{3(1 + 2\theta)} + \frac{c_1}{3} + \frac{2c_2}{3}
\]

which results in the following equilibrium sales volumes

\[
q_1^R = \frac{S}{2} + \frac{A - (1 + 2\theta)C}{12} \quad q_2^R = \frac{S}{2} - \frac{A - (1 + 2\theta)C}{12}
\]

The wholesale price equilibrium in region VI is given by \( w_1^R = c_1 \) and \( w_2^R = c_1 - \frac{2S - a_1 + a_2}{1 + 2\theta} \) resulting in \( q_1^R = 0 \) and \( q_2^R = S \). Finally, the wholesale price equilibrium in region VII is given by \( w_1^R = c_2 - \frac{2S - a_1 + a_2}{1 + 2\theta} \) and \( w_2^R = c_2 \) resulting in \( q_1^R = S \) and \( q_2^R = 0 \). The conditions defining each case can be found in the table in Figure 5.
Appendix C.1

Recall the category captain’s optimization problem given in (2)

\[
\begin{align*}
\max_{p_1, p_2, w_1} & \quad (w_1 - c_1)q_1 - \gamma F \\
\text{s.t.} & \quad (p_1 - w_1)q_1 + (p_2 - w_2)q_2 = K \\
& \quad q_1 + q_2 \leq S, \quad q_1 \geq 0, \quad q_2 \geq 0
\end{align*}
\]

Equivalently, this optimization problem can be written as

\[
\begin{align*}
\max_{p_1, p_2, w_1} & \quad (p_1 - c_1)q_1 + (p_2 - w_2)q_2 - \gamma F \\
\text{s.t.} & \quad (p_1 - w_1)q_1 + (p_2 - w_2)q_2 = K \\
& \quad q_1 + q_2 \leq S, \quad q_1 \geq 0, \quad q_2 \geq 0
\end{align*}
\]

In this formulation, the category captain maximizes the alliance profit \( \Pi_A = (p_1 - c_1)q_1 + (p_2 - w_2)q_2 - \gamma F \) by setting the retail prices for both products. The equivalence can be shown by realizing that the alliance profit can be rewritten as \( \Pi_A = (p_1 - w_1)q_1 + (w_1 - c_1)q_1 + (p_2 - w_2)q_2 - \gamma F = (w_1 - c_1)q_1 + K - \gamma F \). The alliance profit is shared between the retailer and the category captain through the category captain’s wholesale price, \( w_1 \): With the appropriate choice of \( K \left( (w_1(K) - c_1)q_1 - \gamma F = \Pi_{1}^{R} \right) \), the category captain is left with a profit equal to what he would get under retail category management.

The Lagrangian of the optimization problem in (2) is given by

\[
L^C(p_1, p_2, w_1, \lambda, \beta, \mu_1, \mu_2) = (w_1 - c_1)q_1 - \lambda(q_1 + q_2 - S) - \beta[K - (p_1 - w_1)q_1 - (p_2 - w_2)q_2] + \mu_1 q_1 + \mu_2 q_2
\]

The Kuhn-Tucker conditions are

\[
\begin{align*}
\frac{\partial L^C}{\partial p_1} &= -2p_1 \beta - (2p_1 - w_2) \beta \theta - w_1 (1 - \beta) (1 + \theta) + \lambda + \beta a_1 + (1 + \theta) c_1 - (1 + \theta) \mu_1 + \theta \mu_2 = 0 \\
\frac{\partial L^C}{\partial p_2} &= w_1 \theta + 2p_2 \beta (1 + \theta) - \beta (w_2 + (w_1 + w_2) \theta) + \lambda - \beta a_2 - \mu_2 - \theta (c_1 - \mu_1 + \mu_2) = 0 \\
\frac{\partial L^C}{\partial w_1} &= (-1 + \beta) (p_1 + p_1 \theta - p_2 \theta - a_1) = 0 \\
\frac{\partial L^C}{\partial \beta} &= (p_1 - w_1)q_1 + (p_2 - w_2)q_2 - K = 0 \\
\lambda &\geq 0, \mu_1 \geq 0, \mu_2 \geq 0, q_1 + q_2 \leq S, q_1 \geq 0, q_2 \geq 0 \\
\lambda(q_1 + q_2 - S) &= 0, \mu_1 q_1 = 0, \mu_2 q_2 = 0
\end{align*}
\]
Solving for $p_1$, $p_2$, and $w_1$ we get

$$
\hat{p}_1 = \frac{2\theta \lambda + \theta (a_1 + a_2) + \lambda - \mu + c_1 + a_1 + 2\theta c_1 + \lambda - 2\mu_1}{2(1+2\theta)}
$$

$$
\hat{p}_2 = \frac{2\theta \lambda + \theta (a_1 + a_2) + \lambda + a_2 + w_2 - \mu_2 + 2 w_2 \theta - 2\mu_2}{2(1+2\theta)}
$$

$$
\hat{w}_1 = \frac{(1+\theta) a_1^2 - 2 (1+2\theta) a_2 w_2 + 2\theta a_1 a_2 + (1+\theta) a_1^2}{2(1+2\theta) (w_2 \theta - \lambda + a_1 - (1+\theta) c_1 + (1+\theta) \mu_1 - \theta \mu_1)}
$$

and $\beta = 1$.

**Case (I):** $\lambda = 0, (q_1 + q_2 < S), \mu_1 = 0 (q_1 > 0), \mu_2 = 0 (q_2 > 0)$.

$$
\hat{p}_1 = \frac{(a_1 + a_2) \theta + c_1 + a_1 + 2\theta c_1}{2(1+2\theta)} \quad \hat{p}_2 = \frac{(a_1 + a_2) \theta + a_2 + w_2 + 2 w_2 \theta}{2(1+2\theta)}
$$

The sales volumes are given by

$$
\hat{q}_1 = \frac{a_1 + \theta w_2 - (1+\theta) c_1}{2} \quad \hat{q}_2 = \frac{a_2 - (1+\theta) w_2 + \theta c_1}{2}
$$

The conditions for this case are $a_1 + a_2 - c_1 - w_2 < 2S$, $a_1 + \theta w_2 - (1+\theta)c_1 > 0$ and $a_2 - (1+\theta)w_2 + \theta c_1 > 0$.

**Case (II):** $\lambda = 0, \mu_1 \geq 0, \mu_2 = 0$. The retail prices are

$$
\hat{p}_1 = \frac{(a_1 + a_2) \theta + \mu_1 + c_1 + a_1 + 2\theta c_1 - 2\mu_1}{2(1+2\theta)} \quad \hat{p}_2 = \frac{(a_1 + a_2) \theta + a_2 + w_2 + 2 w_2 \theta}{2(1+2\theta)}
$$

The sales volumes are

$$
\hat{q}_1 = \frac{a_1 + \theta w_2 - (1+\theta) c_1 + (1+\theta) \mu_1}{2} \quad \hat{q}_2 = \frac{a_2 - (1+\theta) w_2 + \theta c_1 - \theta \mu_1}{2}
$$

The condition $\hat{q}_1 = 0$ gives $\mu_1 = \frac{-w_2 \theta - a_1 + c_1}{1+\theta}$. Substituting in the expression for $\hat{q}_2$, we obtain $\hat{q}_2 = \frac{\theta a_1 + (1+\theta) a_2 - w_2 (1+2\theta)}{2(1+\theta)}$. The conditions for this case are $\frac{-(a_2 \theta) - a_1 + c_1}{1+\theta} \geq 0$ and $0 < \frac{\theta a_1 + (1+\theta) a_2 - w_2 (1+2\theta)}{2(1+\theta)} < S$.

**Case (III):** $\lambda = 0, \mu_1 = 0, \mu_2 \geq 0$. The retail prices are

$$
\hat{p}_1 = \frac{(a_1 + a_2) \theta + c_1 + a_1 + 2\theta c_1}{2(1+2\theta)} \quad \hat{p}_2 = \frac{(a_1 + a_2) \theta + a_2 + w_2 - \mu_2 + 2 w_2 \theta - 2\mu_2}{2(1+2\theta)}
$$

The condition $\hat{q}_2 = 0$ gives $\mu_2 = \frac{w_2 + w_2 \theta - a_2 - \theta c_1}{1+\theta}$. Substituting in the expression for $\hat{q}_1$, we obtain $\hat{q}_1 = \frac{(1+\theta) a_1 + \theta a_2 - (1+2\theta) c_1}{2(1+\theta)}$. The conditions for this case are $\frac{w_2 + w_2 \theta - a_2 - \theta c_1}{1+\theta} \geq 0$ and $0 < \frac{(1+\theta) a_1 + \theta a_2 - (1+2\theta) c_1}{2(1+\theta)} < S$. 

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Case (IV): $\lambda = 0$, $\mu_1 \geq 0$, $\mu_2 \geq 0$. The retail prices are
\[
\hat{p}_1 = \frac{(a_1 + a_2) \theta - \mu_1 + c_1 + a_1 + 2 \theta c_1 - 2 \mu_1 \theta}{2(1 + 2 \theta)}
\]
\[
\hat{p}_2 = \frac{(a_1 + a_2) \theta + a_2 + w_2 - \mu_2 + 2 w_2 \theta - 2 \mu_2 \theta}{2(1 + 2 \theta)}
\]
The corresponding sales volumes are
\[
\hat{q}_1 = \frac{w_2 \theta + a_1 - (1 + \theta) c_1 + (1 + \theta) \mu_1 - \theta \mu_2}{2}
\]
\[
\hat{q}_2 = \frac{-(w_2 (1 + \theta)) + a_2 + \mu_2 + \theta (c_1 - \mu_1 + \mu_2)}{2}
\]
We find $\mu_1 = -\frac{(w_2 \theta - a_1 + c_1 + \theta c_1 + \theta \mu_2)}{1 + \theta}$ and $\mu_2 = \frac{w_2 + w_2 \theta - a_2 - \theta c_1 + \theta \mu_1}{1 + \theta}$.

Case (V): $\lambda \geq 0$, $\mu_1 = 0$, $\mu_2 = 0$. The retail prices are
\[
\hat{p}_1 = \frac{2 \theta \lambda + (a_1 + a_2) \theta + c_1 + a_1 + 2 \theta \lambda c_1 + \lambda}{2(1 + 2 \theta)}
\]
\[
\hat{p}_2 = \frac{2 \theta \lambda + (a_1 + a_2) \theta + \lambda + a_2 + w_2 + 2 w_2 \theta}{2(1 + 2 \theta)}
\]
the corresponding sales volumes are
\[
\hat{q}_1 = \frac{2 S + w_2 (1 + 2 \theta) + a_1 - a_2 - (1 + 2 \theta) c_1}{4}
\]
\[
\hat{q}_2 = \frac{2 S - w_2 (1 + 2 \theta) - a_1 + a_2 + (1 + 2 \theta) c_1}{4}
\]
The condition $q_1 + q_2 = S$ yields $\lambda = -\frac{2 S - w_2 + a_1 + a_2 - c_1}{2}$. Substituting $\lambda$, we find
\[
\hat{q}_1 = \frac{2 S + w_2 (1 + 2 \theta) + a_1 - a_2 - (1 + 2 \theta) c_1}{4}
\]
\[
\hat{q}_2 = \frac{2 S - w_2 (1 + 2 \theta) - a_1 + a_2 + (1 + 2 \theta) c_1}{4}
\]
The conditions for this case are $2 S \leq a_1 + a_2 - c_1 - w_2$, $c_1 - w_2 < \frac{2 S + a_1 - a_2}{1 + 2 \theta}$, and $w_2 - c_1 < \frac{2 S - a_1 + a_2}{1 + 2 \theta}$.

Case (VI): $\lambda \geq 0$, $\mu_1 \geq 0$, $\mu_2 = 0$. The retail prices are
\[
\hat{p}_1 = \frac{2 \theta \lambda + (a_1 + a_2) \theta - \mu_1 + c_1 + a_1 + 2 \theta c_1 - 2 \mu_1 \theta}{2(1 + 2 \theta)}
\]
\[
\hat{p}_2 = \frac{2 \theta \lambda + (a_1 + a_2) \theta + \lambda + a_2 + w_2 + 2 w_2 \theta}{2(1 + 2 \theta)}
\]
We can find $\lambda = -\left(\frac{2 S + w_2 + 2 w_2 \theta - a_1 + a_2 - a_2^2}{1 + 2 \theta}\right)$, and $\mu_1 = -\left(\frac{2 S + w_2 + 2 w_2 \theta + a_1 - a_2 - c_1 - 2 \theta c_1}{1 + 2 \theta}\right)$.

The corresponding sales volumes are given by $\hat{q}_1 = 0$ and $\hat{q}_2 = S$.

The conditions are $a_1 \theta + (1 + \theta) a_2 - 2 S - w_2 - 2 S \theta - 2 w_2 \theta \geq 0$ and $2 S + w_2 + 2 w_2 \theta + a_1 - a_2 - c_1 - 2 \theta c_1 \leq 0$.

Case (VII): $\lambda \geq 0$, $\mu_1 = 0$, $\mu_2 \geq 0$. As in case (VI), we find $\lambda = -\left(\frac{2 S + 2 S \theta - a_1 - a_2 + c_1 + 2 \theta c_1}{1 + 2 \theta}\right)$, and $\mu_2 = -\left(\frac{2 S - w_2 + 2 w_2 \theta - a_1 + a_2 + c_1 + 2 \theta c_1}{1 + 2 \theta}\right)$.
The conditions are $-(2S + 2S\theta - a_1 -\theta a_1 - \theta a_2 + c_1 + 2\theta c_1) \geq 0$ and $-(2S - w_2 - 2w_2\theta - a_1 + a_2 + c_1 + 2\theta c_1) \geq 0$.

**Case (VIII):** $q_1 + q_2 = S$, $q_1 = 0$, $q_2 = 0$. This is infeasible. We have derived $\hat{q}_1(w_2)$ and $\hat{q}_2(w_2)$ for each possible value of $w_2$.

**Appendix C.2 (Second Manufacturer’s Wholesale Price in CC)**

Given $\hat{q}_1(w_2)$ and $\hat{q}_2(w_2)$ for any $w_2$, we focus on the second manufacturer’s wholesale price decision that takes place at time $t = 2$ in the RCM scenario. The second manufacturer maximizes its profit $(w_2 - c_2)\hat{q}_2(w_2)$ in each region. To simplify the analysis we assume that $c_1 \leq \frac{a_1(1+\theta)+a_2\theta-2S\theta}{1+2\theta}$ and $c_2 \leq \frac{a_1\theta+a_2(1+\theta)-2S\theta}{1+2\theta}$. Our results are summarized in Figure 9 and are derived below.

<table>
<thead>
<tr>
<th>Eq. Sales Volume</th>
<th>Conditions</th>
<th>Second Manufacturer’s Eq. Wholesale Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{q}_1^c &gt; 0$, $\hat{q}_2^c &gt; 0$</td>
<td>$S &gt; \frac{2(1+\theta)a_1 + (1+2\theta)w_2 - (3+2\theta)c_1 - (1+\theta)c_2}{4(1+\theta)}$</td>
<td>$w_2^c = \frac{a_2 + c_1\theta + c_2(1+\theta)}{2(1+\theta)}$</td>
</tr>
<tr>
<td>$\hat{q}_1^c + \hat{q}_2^c &lt; S$</td>
<td>$2(1+\theta)a_1 + \theta a_2 - c_1(\theta^2 + 4\theta + 2) + c_2(1+\theta) &gt; 0$</td>
<td></td>
</tr>
<tr>
<td>$\hat{q}_1^c &gt; 0$, $\hat{q}_2^c &gt; 0$</td>
<td>$S &lt; \frac{(3+4\theta)c_1 + (1+4\theta)a_2 - 3(1+2\theta)c_1 - (1+2\theta)c_2}{2(3+4\theta)}$</td>
<td>$w_2^c = \frac{2S - a_1 + a_2 + (c_1 + c_2)(1+2\theta)}{2+4\theta}$</td>
</tr>
<tr>
<td>$\hat{q}_1^c + \hat{q}_2^c = S$</td>
<td>$6S + a_1 - a_2 - (c_1 - c_2)(1+2\theta) &gt; 0$</td>
<td></td>
</tr>
<tr>
<td>$\hat{q}_1^c &gt; 0$, $\hat{q}_2^c &gt; 0$</td>
<td>$a_1(1+\theta) + a_2\theta - c_1(1+2\theta) - 2S\theta &gt; 0$</td>
<td>$w_2^c = \frac{a_1 + a_2 - c_1 - 2S}{2}$</td>
</tr>
<tr>
<td>$\hat{q}_1^c + \hat{q}_2^c = S$</td>
<td>$2S(1+\theta) - a_1(1+\theta) - a_2\theta + c_1(1+2\theta) &gt; 0$</td>
<td></td>
</tr>
<tr>
<td>$\hat{q}_1^c = 0$, $\hat{q}_2^c = S$</td>
<td>$6S + a_1 - a_2 - (c_1 - c_2)(1+2\theta) \leq 0$</td>
<td>$w_2^c = c_1 - \frac{2S + a_1 - a_2}{1+2\theta}$</td>
</tr>
</tbody>
</table>

Figure 9: Summary of the wholesale prices set by the second manufacturer at time $t = 2$.

**Case (I):** Suppose that $a_1 + a_2 - c_1 - w_2 < 2S$, $a_1 + \theta w_2 - (1+\theta)c_1 > 0$ and $a_2 - (1+\theta)w_2 + \theta c_1 > 0$. In this case, $\hat{q}_2 = \frac{(a_2 - (1+\theta)w_2 + \theta c_1)}{2}$. The second manufacturer maximizes its profit function $(w_2 - c_2)(a_2 - (1+\theta)w_2 + \theta c_1)/2$ by choosing

$$w_2^c = \frac{a_2 + c_1\theta + c_2(1+\theta)}{2(1+\theta)}.$$
The corresponding sales volumes are

\[ q_1^C = \frac{2(1 + \theta) a_1 + \theta a_2 - c_1(\theta^2 + 4\theta + 2) + c_2\theta(1 + \theta)}{4(1 + \theta)} \]
\[ q_2^C = \frac{a_2 + c_1\theta - c_2(1 + \theta)}{4} \]

The conditions for this case are

\[ S \geq S_2^C = \frac{2(1 + \theta) a_1 + (1 + 2\theta) a_2 - (3 + 2\theta)c_1 - (1 + \theta)c_2}{4(1 + \theta)} \]

\[ 2(1 + \theta) a_1 + \theta a_2 - c_1(\theta^2 + 4\theta + 2) + c_2\theta(1 + \theta) > 0 \text{ and } a_2 + c_1\theta - c_2(1 + \theta) > 0. \]

**Case (II):** Suppose that \( \frac{-(w_2\theta - a_1 + c_1 + \theta c_1)}{1 + \theta} \geq 0 \) and \( 0 < \frac{\theta a_1 + (1 + \theta) a_2 - w_2(1 + 2\theta)}{2(1 + \theta)} < S \). Our assumption of \( c_1 \leq \frac{a_1\theta + a_2(1 + \theta) - 2S\theta}{1 + 2\theta} \) eliminates the possibility of an equilibrium in this region.

**Case (III):** In these case, \( \hat{q}_2 = 0 \). Under our assumption \( c_2 \leq \frac{a_1\theta + a_2(1 + \theta) - 2S\theta}{1 + 2\theta} \), no equilibrium exists in this region.

**Case (IV):** Since \( \hat{q}_1 = 0 \) and \( \hat{q}_2 = 0 \), the best response of neither manufacturer falls in this region as a result of which no equilibrium exists in this region.

**Case (V):** Suppose that \( w_2 \leq a_1 + a_2 - c_1 - 2S \), \( c_1 - w_2 < \frac{2S + a_1 - a_2}{1 + 2\theta} \), and \( w_2 - c_1 < \frac{2S - a_1 + a_2}{1 + 2\theta} \).

We consider the two cases (i) \( w_2 < a_1 + a_2 - c_1 - 2S \) and (ii) \( w_2 = a_1 + a_2 - c_1 - 2S \) separately.

**Subcase (i):** Suppose that \( w_2 < a_1 + a_2 - c_1 - 2S \). In this case, \( \hat{q}_2 = \frac{2S - w_2(1 + 2\theta) - a_1 + a_2 + (1 + 2\theta)c_1}{4} \).

The second manufacturer’s wholesale price is given by

\[ w_2^C = \frac{2S - a_1 + a_2 + (c_1 + c_2)(1 + 2\theta)}{2 + 4\theta} \]

The retail prices are

\[ p_1^C = \frac{(7 + 8\theta) a_1 + (1 + 8\theta) a_2 + (c_1 - c_2)(1 + 2\theta) - 2S(3 + 4\theta)}{8(1 + 2\theta)} \]
\[ p_2^C = \frac{(1 + 8\theta) a_1 + (7 + 8\theta) a_2 + (c_2 - c_1)(1 + 2\theta) - 2S(1 + 4\theta)}{8(1 + 2\theta)} \]

The corresponding sales volumes are

\[ q_1^C = \frac{1}{8}(6S + a_1 - a_2 - (c_1 - c_2)(1 + 2\theta)) \quad q_2^C = \frac{1}{8}(2S - a_1 + a_2 + (c_1 - c_2)(1 + 2\theta)) \]

The conditions for this case are

\[ S \leq S_1^C = \frac{(3 + 4\theta)a_1 + (1 + 4\theta)a_2 - 3(1 + 2\theta)c_1 - (1 + 2\theta)c_2}{2(3 + 4\theta)} \]
6S + a_1 - a_2 - (c_1 - c_2)(1 + 2\theta) > 0 \text{ and } 2S - a_1 + a_2 + (c_1 - c_2)(1 + 2\theta) > 0.

Also for a given target level $K$, the category captain’s wholesale price is given by

$$w^C_1 = \frac{a_1 - a_2 - 10S}{1 + 2\theta} + 3c_1 + c_2 - \frac{32(K - S^2(1 - \theta) + 2K\theta - S((1 + \theta)a_1 + \theta a_2) + S(1 + 2\theta)c_1)}{(1 + 2\theta)(6S + a_1 - a_2 - (1 + 2\theta)c_1 + c_2 + 2\theta c_2)}$$

**Subcase (ii):** Suppose that $w^C_2 = a_1 + a_2 - c_1 - 2S \geq c_2$. The equilibrium sales volumes are

$$q^C_1 = \frac{1}{2}(a_1(1 + \theta) + a_2\theta - c_1(1 + 2\theta) - 2S\theta)$$

$$q^C_2 = \frac{1}{2}(2S(1 + \theta) - a_1(1 + \theta) - a_2\theta + c_1(1 + 2\theta))$$

First it must be that $a_1 + a_2 - c_1 - 2S \leq \frac{2S - a_1 + a_2 + (c_1 + c_2)(1 + 2\theta)}{2 + 4\theta}$ which translates into $S \geq S^C_1$.

Second it must be that $a_1 + a_2 - c_1 - 2S \geq \frac{a_2 + c_1\theta + c_2(1 + \theta)}{2(1 + \theta)}$ which translates into $S \leq S^C_2$.

The conditions for this case are $S \in [S^C_1, S^C_2]$, $a_1(1 + \theta) + a_2\theta - c_1(1 + 2\theta) - 2S\theta > 0$, $2S(1 + \theta) - a_1(1 + \theta) - a_2\theta + c_1(1 + 2\theta) > 0$, and $a_1 + a_2 - c_1 - 2S \geq 0$.

**Case (VI):** Suppose that $a_1\theta + (1 + \theta)a_2 - 2S - w_2 - 2S\theta - 2w_2\theta \geq 0$ and $2S + w_2 + 2w_2\theta + a_1 - a_2 - c_1 - 2\theta c_1 \leq 0$. In this case $\hat{q}_1 = 0$ and $\hat{q}_2 = S$. The second manufacturer maximizes $(w_2 - c_2)S$. Our assumption $c_1 \leq \frac{a_1(1 + \theta) + a_2\theta - 2S\theta}{1 + 2\theta}$ allows us to focus on cases where $w_2 \leq \frac{c_1 - 2S + a_1 - a_2}{1 + 2\theta}$. The second manufacturer’s profit is maximized at $w^C_2 = c_1 - \frac{2S + a_1 - a_2}{1 + 2\theta}$. The condition for this equilibrium is $\frac{2S - a_1 + a_2 + (c_1 + c_2)(1 + 2\theta)}{2 + 4\theta} \leq c_1 - \frac{2S + a_1 - a_2}{1 + 2\theta}$ which is equivalent to $6S + a_1 - a_2 - (c_1 - c_2)(1 + 2\theta) \leq 0$.

**Case (VII):** Suppose that $-(2S + 2S\theta - a_1 - \theta a_1 - a_2 + c_1 + 2\theta c_1) \geq 0$ and $-(2S - w_2 - 2w_2\theta - a_1 + a_2 + c_1 + 2\theta c_1) \geq 0$. In this case, $\hat{q}_1 = S$ and $\hat{q}_2 = 0$. Therefore, the second manufacturer’s wholesale price decision is not relevant in this case.

**Retailer’s Target Profit Setting**

Now, we look at the retailer’s target profit level setting problem at time $t = 1$. The retailer sets the target level parameter $K$ such that the category captain is indifferent between accepting and rejecting the category captainship contract. Retailer sets $K$ such that $(w^C_1(K) - c_1)q^C_1 - \gamma F = \Pi^R_1$. If $S < S_1$, then the retailer sets the target level to $K^1_R$, and
where

\[ K_R^1 = \frac{A^2 - 36S^2 (15 + 8\theta) + C^2(1 + 2\theta)^2 - 156C S (1 + 2\theta)}{288 (1 + 2\theta)} + \frac{-2A(C(1 + 2\theta) - 6S(13 + 24\theta))}{288 (1 + 2\theta)} + S(a_2 - c_2) - \gamma F. \]  

(3)

Appendix C.3. Proof of the Lemmas in Section 4.2

Proof of Lemma 3: As shown in Appendix C.2, if \( S < S^C_1 \), there are three possible cases: (1) \( q^C_1 > 0 \) and \( q^C_2 > 0 \) corresponding to region V.i; (ii) \( q^C_1 = 0 \) and \( q^C_2 = S \) corresponding to region VI; and (iii) \( q^C_1 = S \) and \( q^C_2 = 0 \) corresponding to region VII. In all cases the equilibrium is unique and the shelfspace constraint is binding \((q^C_1 + q^C_2 = S)\) with \( \lambda > 0 \).

If \( S \in [S^C_1, S^C_2] \), there are three possible cases: (1) \( q^C_1 > 0 \) and \( q^C_2 > 0 \) corresponding to the interior of the line segment V.ii; (ii) \( q^C_1 = 0 \) and \( q^C_2 = S \) corresponding to the endpoint bordering region VI; and (iii) \( q^C_1 = S \) and \( q^C_2 = 0 \) corresponding to the endpoint bordering region VII. In all cases the equilibrium is unique and the shelfspace constraint is binding \((q^C_1 + q^C_2 = S)\) with \( \lambda = 0 \).

If \( S > S^C_2 \), then \( q^C_1 + q^C_2 < S \). In this case, the only relevant region is region (I) where \( q^C_1 > 0 \) and \( q^C_2 > 0 \) with \( \lambda = 0 \), and the equilibrium is unique.

Proof of Lemma 4: If \( S < S^C_1 \), then the relevant regions are V.i, VI, and VII. The equilibrium sales volumes in region V.i are

\[ q^C_1 = \frac{3S}{4} + \frac{A - (1 + 2\theta)C}{8} \]
\[ q^C_2 = \frac{S}{4} - \frac{A - (1 + 2\theta)C}{8} \]

In region VI, the equilibrium sales volumes are \( q^C_1 = 0 \) and \( q^C_2 = S \). Finally, in region VII, the equilibrium sales volumes are \( q^C_1 = S \) and \( q^C_2 = 0 \). The conditions defining these cases can be found in Figure 9.

Appendix D

Proof of Proposition 1: Recall

\[ \Omega_{RCM}^1(S) = \{(A, C, \theta) | 6S > A - (1 + 2\theta)C \text{ and } 6S > (1 + 2\theta)C - A\} \]
\[ \Omega_{CC}^1(S) = \{(A, C, \theta) | 2S > A - (1 + 2\theta)C \text{ and } 6S > (1 + 2\theta)C - A\} \]
By definition, $\Omega_{CC}^1(S) \subset \Omega_{RCM}^1(S)$ for a given $S$.

We can show that $S_1 < S^C_1$ if $C < \frac{A}{1+2\theta}$. To show this, we define

$$S_1^C - S_1 = \frac{2a_2 \theta + a_1 (3 + 4 \theta) + (1 + 2 \theta) (c_2 \theta - c_1 (3 + \theta))}{(3 + 2 \theta) (3 + 4 \theta)} = \frac{A (3 + 4 \theta) - (1 + 2 \theta) (-3a_2 + 3c_2 + C (3 + \theta))}{9 + 18 \theta + 8 \theta^2} = \frac{3(1 + 2\theta)(a_2 - c_2) + (3 + 4\theta)A - (1 + 2\theta)(3 + \theta)C}{9 + 18 \theta + 8 \theta^2}$$

Assuming $a_2 > c_2$, we need to show that $(3 + 4\theta)A - (1 + 2\theta)(3 + \theta)C > 0$. Note

$$\frac{3 + 4\theta}{3 + \theta} \geq 1 \text{ for } \theta \in [0, 1]. \text{ Thus, } S_1 < S^C_1 \text{ if } C < \frac{A}{1+2\theta}.$$ Therefore, we can conclude that if $C < \frac{A}{1+2\theta}$, then $q_1^C, q_1^R, q_2^C, q_2^R$ are all positive in $\Omega_{RCM}^1(S) \cap \Omega_{CC}^1(S) = \Omega_{CC}^1(S)$ for $S < S_1$.

Note that $q_1^C - q_1^R = S/4 + (A - (1 + 2\theta)C)/24 > 0$, so $q_1^C > q_1^R$. Similarly we can show that $q_2^C < q_2^R$. By definition, in the region $\Omega_{RCM}^1(S) \setminus \Omega_{CC}^1(S)$ with $S < S_1$, $q_2^R > 0$ and $q_2^C = 0$.

**Proof of Proposition 2:** For $C < \frac{A}{1+2\theta}$,

Part (i): The consumer surplus in the RCM scenario for $S < S_1$ is given by

$$CS^R = \begin{cases} \frac{S^2}{4} + \frac{(A - (1 + 2\theta)C)^2}{144(1+2\theta)} & \text{in } \Omega_{RCM}^1(S), \\ \frac{S^2(1+\theta)}{2(1+2\theta)} & \text{in } \Omega_{RCM}^2(S). \end{cases}$$

Similarly, the consumer surplus in the CC scenario for $S < S^C_1$ is given by

$$CS^C = \begin{cases} \frac{4S^2(5+8\theta)+(A-(1+2\theta)C)(4S^2+A-(1+2\theta)C)}{64(1+2\theta)} & \text{in } \Omega_{CC}^1(S), \\ \frac{S^2(1+\theta)}{2(1+2\theta)} & \text{in } \Omega_{CC}^2(S), \end{cases}$$

We show that $CS^C > CS^R$ in $\Omega_{CC}^1(S)$ by showing that

$$CS^C - CS^R = \frac{(5A + 6S - 5C (1 + 2\theta)) (A + 6S - C (1 + 2\theta))}{576 (1 + 2\theta)} > 0$$

since $5A + 6S - 5C (1 + 2\theta) > 0$ and $A + 6S - C (1 + 2\theta) > 0$. It can also be shown that the same result holds in $\Omega_{RCM}^1(S) \setminus \Omega_{CC}^1(S)$ where the second product is excluded under category captainship.

Part (ii): We show that the retailer is better off under the category captainship by showing that

$$\Pi^C_R - \Pi^R_R = \frac{60 S^2 - A^2 - C^2(1 + 2\theta)^2 + 4S(A - (1 + 2\theta)C) + 2AC (1 + 2\theta)}{96 (1 + 2\theta)} + (1 - \gamma) F > 0$$
since $60 S^2 - A^2 - C^2 (1 + 2 \theta)^2 > 0$, $4S(A - (1 + 2\theta)C) > 0$, and $2AC (1 + 2 \theta) > 0$.

Part (iii): Since the retailer sets $K$ such that the category captain is indifferent between retail category management and category captainship, we have that $\Pi_1^R = \Pi_1^C$ by assumption.

Part (iv): We show that the second manufacturer is worse off under the category captainship by showing that

$$\Pi_2^R - \Pi_2^C = \frac{(6S + A - (1 + 2 \theta)C)}{144 (1 + 2 \theta)} (18S - 5A + 5(1 + 2 \theta) C) > 0$$

since both $6S + A - (1 + 2 \theta)C > 0$ and $18S - 5A + 5(1 + 2 \theta) C > 0$.

**Proof of Proposition 3:** We assume that $S < S_1$ and all $A_i, \theta_i \in \Omega_{CC}^1$ so that both products are offered to the consumers. The category captain prefers manufacturer $j$ to manufacturer $i$ if $\Pi_1^C(A_j, \theta_j, S) \geq \Pi_1^C(A_i, \theta_i, S)$ and $\Pi_1^C(A_i, \theta_i, S) = \Pi_1^R = (w_i^R - c_i)q_i^R = \frac{(6S + A_i)^2}{36(1 + 2 \theta_i)}$.

Substituting and simplifying, we find that the category captain prefers manufacturer $j$ to $i$ if $A_j \geq \bar{A}_j^C(S, \theta_j, \theta_i, A_i)$ where the threshold is given by

$$\bar{A}_j^C(S, \theta_j, \theta_i, A_i) = -6 S + \frac{\sqrt{(6S + A_i)^2 (1 + 2 \theta_i) (1 + 2 \theta_j)}}{1 + 2 \theta_i}.$$  

The retailer, on the other hand, prefers manufacturer $j$ to manufacturer $i$ if $K_1^R(S, A_j, \theta_j) \geq K_1^R(S, A_i, \theta_i)$ where $K_1^R(S, A_i, \theta_i)$ is found from equation (3) by substituting $C = 0$:

$$K_1^R(S, A_i, \theta_i) = \frac{A_i^2 + 12 A_i S (13 + 24 \theta_i) - 36S(8c + 15S + 8(2c + S) \theta_i)}{288 (1 + 2 \theta_i)} + a_i S - \gamma F.$$  

Substituting and simplifying, we find that the retailer prefers manufacturer $j$ to $i$ if $A_j \geq \bar{A}_j^R(S, \theta_j, \theta_i, A_i)$, where

$$\bar{A}_j^R(S, \theta_j, \theta_i, A_i) = -6 S (13 + 24 \theta_j) + \frac{\sqrt{W}}{1 + 2 \theta_i}$$  

with

$$W = (1 + 2 \theta_i) \left( A_i^2 (1 + 2 \theta_j) + 12 A_i S (13 + 24 \theta_i) (1 + 2 \theta_j) + 36 S^2 (1 + 2 \theta_i) (13 + 24 \theta_j)^2 \right)$$

$$+ (1 + 2 \theta_i) (72 S (11 S (\theta_i - \theta_j) + 4 a_i (1 + 2 \theta_i) (1 + 2 \theta_j) - 4 a_j (1 + 2 \theta_i) (1 + 2 \theta_j)))).$$

If $\theta_i = \theta$ for all $i$, then $\bar{A}_j^C(S, \theta_j, \theta_i, A_i) = \bar{A}_j^R(S, \theta_j, \theta_i, A_i) = A_i$, and both the retailer and the category captain’s choices are in line with each other: Both prefer manufacturer $j$
such that $a_j = \min\{a_1, \ldots, a_k\}$, the manufacturer with the lowest brand strength. However, when $\theta_j \neq \theta_i$, then $\bar{A}_j^C(S, \theta_j, \theta_i, A_i) \neq \bar{A}_j^R(S, \theta_j, \theta_i, A_i)$, implying that the retailer’s and the category captain’s choice of a second manufacturer may not be the same.

In particular, if $C < \frac{A}{1 + 2\theta}$, the category captain prefers a highly differentiated assortment (low $\theta$) because

$$\frac{\partial \Pi^C}{\partial \theta_i} = \frac{(6S + A + (1 + 2\theta)C)(-6S - A + (1 + 2\theta)C)}{18(1 + 2\theta)^2} < 0.$$ 

If $C < \frac{A}{5(1 + 2\theta)}$, the retailer prefers a less differentiated assortment (high $\theta$) because

$$\frac{\partial K^1_R}{\partial \theta_i} = \frac{-A^2 - 12AS + 396S^2 + C^2(1 + 2\theta)^2}{144(1 + 2\theta)^2}\frac{(33S + A + (1 + 2\theta)C)(12S - A + (1 + 2\theta)C) + 9S(A - 5(1 + 2\theta)C)}{144(1 + 2\theta)^2} > 0,$$

since $33S + A + (1 + 2\theta)C > 0$ and $12S - A + (1 + 2\theta)C > 0$. Therefore, everything else being equal, the category captain prefers a highly differentiated product assortment whereas the retailer prefers a less differentiated assortment if $C < \frac{A}{5(1 + 2\theta)}$.

**Proof of Proposition 4:** Suppose that the retailer assigns the first manufacturer as category captain. Further suppose that $S < S_1$ and $(A, C, \theta) \in \Omega^1_{RCM}(S) \setminus \Omega^1_{CC}(S)$. In this case, only the category captain’s product is offered to the consumers.

Second, suppose that the retailer assigns the second manufacturer as category captain. Assuming $S < S_1$ and $(A, C, \theta) \in \Omega^1_{RCM}(S) \setminus \Omega^1_{CC}(S)$, the second manufacturer allocates $\frac{1}{8}(2S + A - (1 + 2\theta)C) > 0$ to the first product and $\frac{1}{8}(6S - A + (1 + 2\theta)C) > 0$ to the second product since $2S \leq A - (1 + 2\theta)C \leq 6S$ when $(A, C, \theta) \in \Omega^1_{RCM} \setminus \Omega^1_{CC}$. Therefore, for $S < S_1$, if the parameters $(A, C, \theta)$ are such that the second product is excluded under the category captainship of the first manufacturer, then the first product is not excluded under the category captainship of the second manufacturer.

If the retailer assigns the first manufacturer as category captain, the retailer requires a target category profit of $K^1_R$, where (by Eq. 3)

$$K^1_R = \frac{A^2 - 36S^2(15 + 8\theta) + (C^2 - 156CS)(1 + 2\theta)^2}{288(1 + 2\theta)} - 2A(C + 2C\theta - 6S(13 + 24\theta)) + S(a_2 - c_2) - \gamma_1F.$$

If, on the other hand, the retailer assigns the second manufacturer as category captain, he requires a target category profit of $K^2_R$,

$$K^2_R = \frac{A^2 + (C^2 - 132CS)(1 + 2\theta)^2 - 2A(C + 2C\theta - 6S(11 + 24\theta)) - 36S^2(15 + 8\theta)}{288(1 + 2\theta)} + S(a_2 - c_2) - \gamma_2F.$$
The retailer benefits from selecting the second manufacturer as category captain if $K_R^2 \geq K_R^1$, or

$$SA > (1 + 2\theta)(12F(\gamma_1 - \gamma_2) + SC)$$

or

$$\gamma_1 - \gamma_2 > \frac{S(A - (1 + 2\theta)C)}{12F(1 + 2\theta)}.$$