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Abstract

The profitability of remanufacturing systems for different cost, technology and logistics structures has been extensively investigated in the literature. We provide an alternative and somewhat complementary approach that considers demand-related issues, such as the existence of green segments, OEM competition and product life cycle effects. The profitability of a remanufacturing system strongly depends on these issues as well as on their interactions. For a monopolist, we show that there exist thresholds on the remanufacturing cost savings, the green segment size, market growth rate and consumer valuations for the remanufactured products, above which remanufacturing is profitable. More importantly, we show that under competition remanufacturing becomes an effective marketing strategy, which allows the manufacturer to defend its market-share via price discrimination.

1 Introduction

Remanufacturing recovers value from used products by replacing components or reprocessing used parts to bring the product to like-new condition. Since it reduces both the natural resources needed and the waste produced, remanufacturing helps reduce the environmental burden. Because remanufactured products are kept out of the waste stream longer, landfill space is preserved and air pollution is reduced from products that would have had to be re-smelted or otherwise reprocessed (EPA1 2005, Remanufacturing Central 2005).

Examples from industry show that there is a big market for remanufactured products. According to Remanufacturing Central (2005), in 1997, the estimated total annual sales of 73,000 remanufacturing firms in the US was 53 billion dollars. As the remanufacturing literature (Guide and Van Wassenhove 2003, Geyer et.al. 2006, Guide et.al. 2006) points out, successful examples from industry, such as those of Kodak (Geyer et.al. 2006), BMW, IBM, DEC and Xerox (Ayres et.al. 1997) show that remanufacturing can be profitable.

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The decision to remanufacture is difficult since managers have little guidance and industry practice is very diverse. Some manufacturers fear cannibalization from remanufactured products. They either do not remanufacture at all, or they sell remanufactured products to invisible/secondary channels to avoid cannibalization. Other manufacturers sell remanufactured products through direct channels (see e.g., Bosch Tools, Gateway and Sun). The central question manufacturers seem to face is "When do benefits from remanufacturing outweigh losses from cannibalization?" For example, Bosch Tools of USA does not know how exactly remanufactured products affect primary product sales, so they use simple heuristics to decide on remanufacturing. Bosch generally remanufactures products only if their market share is small and remanufacturing leads to sufficiently high cost savings.[1] Management acknowledges that it needs more sophisticated tools for making effective and differentiated remanufacturing decisions.

The primary goal of this paper is to provide manufacturers with guidelines for remanufacturing decisions. We identify profitability conditions for remanufacturing by considering the following important characteristics of a remanufactured product:

1. The remanufactured product is typically a natural **low cost** alternative to the new product.
2. Remanufactured products usually have **lower valuation** from regular consumer segments.
3. Remanufacturing has a **green image**, as it reduces waste generated and reuses old material. As such, it provides high value to a relatively small (albeit growing) green consumer segment.
4. A remanufactured product usually has the same functionality as a new product. Since it is a low-price alternative, manufacturers often believe it **cannibalizes** new product sales.
5. The remanufacturing supply is bounded from above by the number of returns from previous sales. Thus, remanufacturable products face **supply constraints**.

In particular, we focus on the demand side aspects of the problem and identify three important drivers of remanufacturing profitability: (i) direct competition between OEMs, (ii) the existence of a green segment and (iii) properties of the product life cycle, i.e. the market growth rate.

OEM Competition: The market share concern of Bosch suggests that *OEM competition* (as an indicator of lower market share) may have a significant impact on the profitability of remanufacturing. In other words, remanufacturing may be a better strategy under competition, as the remanufactured product potentially cannibalizes the competitor's new product sales. This issue has been largely neglected in the literature. Our analysis shows that the degree of competition is a significant driver of remanufacturing profitability.

[1]Based on the authors' personal interview with Randy Valenta, product service director at Robert Bosch Tool Corporation.

Green Segments: For certain products, the environmental burden can be very high. Government legislation (such as the WEEE and ELV Directives of the EU) or "green" consumer initiatives (NPOs) create important incentives for companies to seriously consider remanufacturing. For instance, ToxicDude (2005) targets companies like Dell and Apple for sustainable production and forces them to take responsibility for the reuse or recycling of their products. EPA (2005b) advises consumers to buy "green" products, i.e. products designed with environmental attributes and recycled inputs. Thus, besides the direct benefits of cost reduction and value added recovery, remanufacturing may provide firms with side benefits such as a "green image" which, in turn, extends the consumer base and improves consumer relations. In other words, the existence of a *green consumer segment* represents an important marketing opportunity for remanufacturers.[2]

Product Life Cycle and Market Growth: It is well-documented in the marketing literature (see e.g. Bass (1969)) that products undergo a product life cycle, the stages of which can be characterized by the speed of growth (diffusion) of the market. As market growth rate determines the likely market size next period, it clearly impacts the remanufacturing decisions of the firm, although *a priori* it is not clear how. This suggests that the *market growth rate* can be a driver of profitable remanufacturing. For example, the firm may wonder whether to delay the introduction of the remanufactured product under fast product diffusion to benefit from more new product sales or, instead, speed it up to benefit from higher subsequent return rates.

In this paper, we explore the potential of remanufacturing as a strategic marketing tool with a major impact on the firm's competitive advantage, rather than thinking of it as a cost saving device or as compliance with legal requirements. Our results confirm that the three factors mentioned above, competition, product life cycle and size of the green segment, have a significant direct impact on the remanufacturing decision. Furthermore, no single factor among the three dominates the others. Instead, these effects are intimately linked and exhibit strong interactions that can nevertheless be summarized in a framework that readily speaks to practice.

The next section positions our research in the remanufacturing literature. Section 3 presents a static monopolist model setup that is used as a benchmark. This analysis compares the remanufacturing scenario to one where no remanufacturing is considered. In Section 4, we extend our monopolistic results to a setting that considers the impact of the product life cycle and identify the existence of an optimal market growth rate for the introduction of remanufactured products. Section 5 extends the benchmark monopoly case to a competitive setting to show that remanufacturing may be a better strategy under OEM competition. Here, we also consider the case where the manufacturer does not remanufacture

[2]For instance, EIA-CEI (EIA 2005), a manufacturer alliance from the US electronics industry publishes the list of companies that participate in the reuse programs and informs consumers about participating companies. Xerox's "Green Line" products can be considered as a response to this green consumer initiative (Ayres et.al. 1997, Geyer et.al. 2006).

but a local remanufacturer competes with the OEM's product. Section 6 combines the previous two extensions and considers the impact of OEM competition as well as product life cycle effects. Section 7 concludes, highlights limitations and investigates possible avenues for future research. To improve readability most mathematical details are relegated to an Appendix.

2 Relevant Literature

This paper is related to two main streams of research in the operations literature: market segmentation and remanufacturing. Several papers address the issue of market segmentation for remanufactured products. Majumder and Groenevelt (2001) consider the pricing/remanufacturing decisions of an OEM facing competition from a local remanufacturer and derive conditions on cost/price relations for different reverse logistics settings. Ferrer and Swaminathan (2005) study the joint pricing of new and remanufactured products for a monopolist in a multi-period setting. They characterize the Nash equilibrium outcome and discuss the impact of various system parameters when the manufacturer competes with a local remanufacturer. Debo et al. (2005) investigate joint technology selection and pricing decisions for new and remanufactured products. They derive the manufacturer's optimal remanufacturing decisions as well as conditions on the viability of remanufacturing. They also extend their results to the case of competing remanufacturers. Ferguson and Toktay (2006) consider the pricing and remanufacturing/collection decisions when facing a competing local remanufacturer, similar to Majumder and Groenevelt (2001). They derive conditions on costs, under which remanufacturing or collection is profitable for a monopoly or under competition, in addition to strategies that deter remanufacturer entry.

Interestingly, all these articles mainly consider competition against local remanufacturers that use an original equipment manufacturer's (OEM's) product returns for remanufacturing. Heese et al. (2005) appears to be the only reference analyzing the profitability of remanufacturing under direct OEM competition. The authors use a Stackelberg duopoly model to show that remanufacturing can be a profitable strategy for the first-moving firm, if the underlying cost structure and market share are appropriate.

Although most articles consider a dynamic multi-period setting, they ignore product diffusion over time. It is well known in marketing (see Bass (1969)) that product sales follow a bell-shaped pattern over the product life cycle. Debo et al. (2006) investigate the impact of diffusion on remanufacturing decisions in the context of a monopoly. They find that optimal remanufacturability levels are higher for faster diffusing products.

We contribute to this literature by bringing a marketing perspective to the remanufacturing problem through a focus on factors related to the demand faced by the firm. We combine three aspects of the demand that are typically examined separately by the literature. First, we consider direct OEM competition. Second, we observe the existence of a secondary (green) market segment, which consists of consumers who do not discount the value of the remanufactured product. Finally, we consider the impact of market growth, which turns out to have very significant effects on the profitability of remanufacturing. We also show that remanufacturing decisions may depend on the existence of local remanufacturers. To the best of our knowledge, this is the first paper that simultaneously combines these three demand-side factors in a remanufacturing context.

3 Benchmark: Static Monopoly

We start our analysis with a benchmark scenario that focuses on the impact of *the green segment.* Our target is to identify the demand conditions under which remanufacturing is profitable.

Consider a *monopolist* with *unconstrained remanufacturable product supply* throughout the product life cycle. Assume that, at a certain stage of the product life cycle, there are Δ consumers in the market. Consumers are heterogenous with respect to their willingness to pay θ, assumed to be uniformly distributed between 0 and 1. There are two types of consumers: primary consumers and green consumers. The green consumers' proportion in the market is $\beta < 1$. When a primary consumer values the new product at θ, she values the remanufactured product lower, i.e. $\delta\theta$, where $\delta < 1$. The green consumers on the other hand value the remanufactured and new products the same. With this representation, the green segment not only represents consumers who are environmentally conscious but also consumers who care only for the functionality of the product rather than its newness.[3] The consumers in the green segment are the types where cannibalization is a real issue since they will buy remanufactured product when these are offered at a lower price.

Given the assumptions and the definitions in Table 1, the consumer utilities can be obtained as:

- primary consumer utility from the new product: $U_n^P = \theta - p_n$,
- primary consumer utility from remanufactured product $U_r^P = \delta\theta - p_r$,
- green consumer utility from the new product $U_n^G = \theta - p_n$,
- green consumer utility from the remanufactured product $U_r^G = \theta - p_r$.

Thus, the primary consumers purchase the new product if $U_n^P > 0$ and $U_n^P > U_r^P$. Otherwise the primary consumer purchases the remanufactured product if $U_r^P > 0$. Similarly, the green consumers

[3] In fact, there could even be consumers that are valuing the remanufactured product more because of its environmental friendliness. Although we will not consider this in our models, we should note that the existence of such consumers would reinforce our results as it would make remanufacturing a more profitable option.

Parameter	Definition
β	Ratio of green consumers in the market
δ	Primary consumer value discount for the remanufactured product
Δ	Total number of consumers in the market
p_n	Sales price for the new product
q_n	New product sales
c_n	Manufacturing cost of the new product
p_r	Sales price for the remanufactured product
q_r	Remanufactured product sales
c_r	Collection and processing cost of the remanufactured product

Table 1: Monopoly model parameters

purchase the new product if $U_n^G > 0$ and $U_n^G > U_r^G$. Otherwise, the green consumers purchase the remanufactured product if $U_r^G > 0$. Note that under these assumptions the manufacturer will always price the remanufactured product lower than the new product, i.e. $p_n > p_r$. This feature is also the relevant case for practice since in real-life remanufactured products are always priced lower.

3.1 Demand

When the remanufactured product is not offered, the market can be represented via a single consumer type, since green consumers cannot be differentiated. In this case the market consists of Δ consumers, whose valuations (θ) are uniformly distributed over $[0, 1]$. It is easy to see that the demand can be written as $q_n = 1 - p_n$.

Now, assume that both new and remanufactured products are sold. Due to the existence of two different consumer segments in the market, the manufacturer has two options :

1. Keep the price low ($p_r \leq \delta p_n$) to sell remanufactured products to both segments. When $p_r \leq \delta p_n$, the new product's demand only comes from primary consumers with high willingness to pay:

$$q_n^P = -\Delta \left(\frac{(-1+\beta)\,(-1+\delta+p_n-p_r)}{-1+\delta} \right). \tag{1}$$

The price-sensitive primary consumers will buy the remanufactured product:

$$q_r^P = \Delta \frac{(-1+\beta)\,(\delta\, p_n - p_r)}{(-1+\delta)\,\delta}. \tag{2}$$

Green consumers will not buy the new product: $q_n^G = 0$ since $p_n > p_r$, but they will buy the remanufactured product:

$$q_r^G = \Delta(1-p_r)(\beta). \tag{3}$$

Then, $q_n = q_n^P$ and $q_r = q_r^P + q_r^G$.

2. Keep the price high ($p_r > \delta p_n$) to maximize profits from the green segment only. When $p_r > \delta p_n$ primary consumers do not buy the remanufactured product and the demand is formed as follows:

$$q_n = \Delta(1-\beta)(1-p_n). \tag{4}$$

$$q_r = \Delta\beta(1-p_r). \tag{5}$$

As such, the overall demand is kinked with two possible demand regimes. The manufacturer maximizes profits by solving the following problem:

$$\max_{p_n, p_r} \Pi_R = (p_n - c_n)q_n + (p_r - c_r)q_r.$$

3.2 No Remanufacturing Scenario

Assume first that remanufacturing is not an option, i.e. the manufacturer maximizes profits by offering only the new product. The profit function is $\Pi_{NR} = (p_n - c_n)q_n$, which is maximized by $p_n = (1+c_n)/2$. Sales are $q_n = \Delta(1-c_n)/2$ and the profit is given by $\Pi_{NR} = \Delta\frac{(1-c_n)^2}{4}$.

3.3 Static Remanufacturing Scenario

When the monopolist remanufactures, the remanufactured and new products are priced simultaneously to maximize profits. Assume that the costs are such that both new and remanufactured products are offered (see Corollaries 6 and 7 in Appendix). In this case, the manufacturer uses the pricing strategy described in the following proposition.

Proposition 1 *There exists a β^* (detailed in Appendix) such that when $\beta \leq \beta^*$, the solution to the monopolist's problem is given by* $q_n^* = \frac{(1-\beta)\,(1-c_n+c_r-\delta)\,\Delta}{2\,(1-\delta)}$, $q_r^* = \frac{(\delta\,(c_n-\beta\,(-1+c_n+\delta))+c_r\,(-1+\beta\,(1+(-1+\delta)\,\delta)))\,\Delta}{2\,(1-\delta)\,\delta}$, $p_n^* = \frac{-\left(-1+\beta-c_n+\beta\,c_n-2\,\beta\,\delta-\beta\,c_n\,\delta+\beta\,\delta^2\right)}{2\,(1-\beta+\beta\,\delta)}$ *and* $p_r^* = \frac{-(-c_r+\beta\,c_r-\delta-\beta\,c_r\,\delta)}{2\,(1-\beta+\beta\,\delta)}$. *Otherwise the solution to the monopolist's problem is given by* $q_n^* = \Delta(1-\beta)\frac{1-c_n}{2}$, $q_r^* = \Delta\beta\frac{1-c_r}{2}$, $p_n^* = \frac{1+c_n}{2}$ *and* $p_r^* = \frac{1+c_r}{2}$.

Proposition 1 shows that the manufacturer should use two different pricing regimes depending on the green segment size. When the green segment is small ($\beta \leq \beta^*$), the manufacturer prices the remanufactured product low to price discriminate and also capture the price sensitive customers in the primary market. However, when the green segment is large, the manufacturer uses the high pricing regime to get the maximum profit out of the green segment. This way, the losses from cannibalization are compensated by the high price charged to the green segment. In practice however, the green segment is expected to be small and we would typically observe a low pricing strategy by manufacturers.

Having identified the optimal pricing strategy under remanufacturing, we would like to know whether remanufacturing is profitable or not. We can compare the no remanufacturing (NR) and remanufacturing (R) scenarios to answer this question. Corollary 1 states that the only condition under which a monopolist would remanufacture is when the remanufacturing costs are sufficiently low.[4]

Corollary 1 *When $\beta > 0$, remanufacturing is profitable for a monopolist if and only if $c_r < c_n$. Otherwise, remanufacturing is profitable for a monopolist if $c_r < \delta c_n$.*

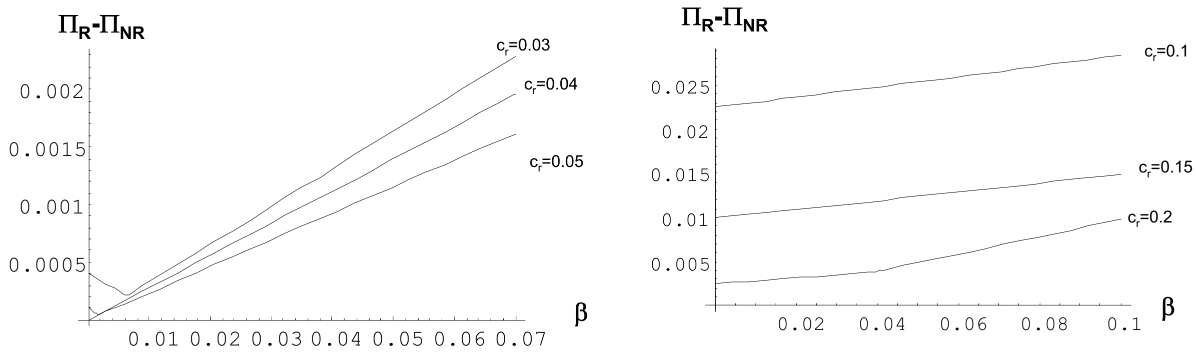
Corollary 2 *When $\beta \leq \beta^*$, q_r is increasing in δ and decreasing in β. q_n is decreasing in δ and β. The profit is convex in β. When $\beta > \beta^*$, q_r and Π_R are increasing in β.*

We would also like to understand what really drives cannibalization and to what extent cannibalization impacts profitability. Obviously, there are two types of cannibalization in this system: (i) cannibalization from the primary segment (price sensitive primary consumers purchase the remanufactured product instead of a new one) and (ii) cannibalization from the green segment (green consumers purchase the cheaper remanufactured product instead of the new one). According to Corollary 2 cannibalization does not necessarily lead to lower profit. There are two main drivers of the cannibalization impact: (i) green segment size and (ii) primary consumer discount rate (δ).

The impact of the green segment changes depending on the size of this segment. When the green segment is large ($\beta \geq \beta^*$), there is a strong niche in the market that can be captured via the remanufactured product. If remanufacturing is cheaper than new production, the profit increases in β. When the green segment is small ($\beta \leq \beta^*$), the profit is convex in β, which means that the profit can be decreasing in the green segment size. To facilitate the interpretation of these results, the additional profit from remanufacturing is explored in Figures 1 and 2.

Figure 1 represents the extra profit from remanufacturing, (i.e. the profit difference between selling vs. not selling remanufactured products) for different green segment rates and remanufacturing costs. Figure 1a assumes low cost savings from remanufacturing and shows that cannibalization is a concern when the green segment is small. Specifically, when $\beta < \beta^*$, the manufacturer uses the low pricing regime and sells both to primary and green segments. In this case, profits from remanufacturing are decreasing in β This is due to the fact that the green consumers who could have bought the new product if the remanufactured product were not sold are going for the remanufactured product since it is cheaper. However, when β exceeds β^*, profits are increasing in the green segment size because the manufacturer charges a high price for the green segment. Figure 1b illustrates the case when the cost savings from remanufacturing are high. With sufficiently high cost savings (in absolute terms), benefits from remanufacturing always overcome the cannibalization effect.

[4] The reader should note that this result is valid under a zero fixed cost assumption. Remanufacturing is profitable only if extra profit obtained from remanufacturing compensates for any fixed costs involved.



(a) Low cost savings: $c_n = 0.1$, $0.05 \leq c_n - c_r \leq 0.07$

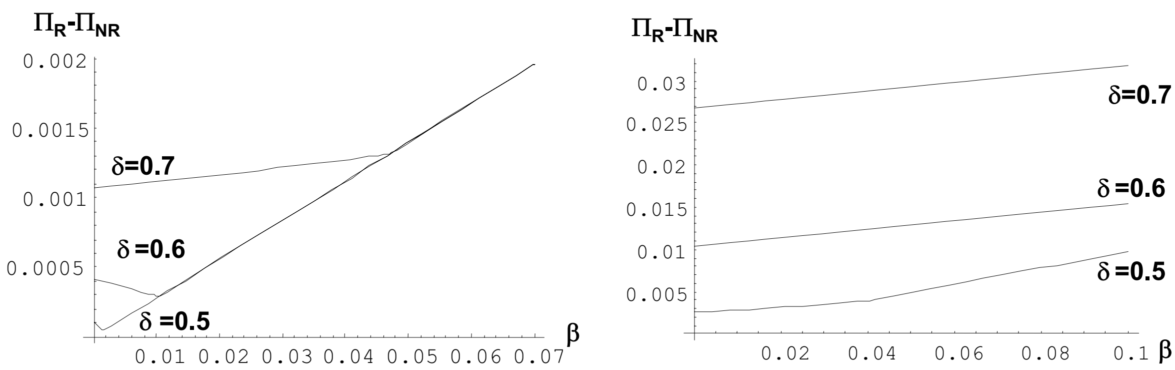
(b) High cost savings: $c_n = 0.5$, $0.3 \leq c_n - c_r \leq 0.4$

Figure 1: Extra profits from remanufacturing when $\delta = 0.5$, $\Delta = 1$ and $c_r \leq \delta c_n$

Primary consumer valuations for the remanufactured product is also a significant driver of the cannibalization impact. Figure 2 explores the profitability of remanufacturing under different levels of primary consumer discount rate (δ) for the remanufactured product. When δ is high, the negative cannibalization impact of the green segment is lost. This is due to the fact that with high δ, the primary consumers can be charged a higher price for the remanufactured product. With this high price, the green consumers are also paying more for the remanufactured product.

4 Monopolist Facing Product Life Cycle

So far, we have ignored the origins of remanufacturable product supply. Remanufacturable products are reusable returns from earlier sales. To take this into consideration, we assume a two-period monopolistic model. In the first period the market size is normalized to 1. We denote the sales of new products as q_1 and the price as p_1. Total market demand in period 1 is given by $q_1 = 1 - p_1$. In the second period the market size expands (shrinks) to $\Delta > 1$ (< 1), depending on the product's position in the life cycle. Figure 3 illustrates the two situations captured by our model: when Δ is larger (smaller) than 1 our model applies to early (late) stages of the product life cycle with a growing (shrinking) market. In the case of a growing market, Δ can be considered as the product's diffusion rate (Bass 1969). The novelty of this approach lies in representing the *product life cycle effects* in a two-period model. Using a multi-period model would not add many insights but tremendously complicate the analysis. It would also make it impossible to address the strategic (i.e. competitive) role of remanufacturing.



(a) Low cost savings: $c_n = 0.1$, $c_r = 0.04$

(b) High cost savings: $c_n = 0.5$, $c_r = 0.2$

Figure 2: Extra profits from remanufacturing when $\Delta = 1$ and $c_r \leq \delta c_n$

4.1 No Remanufacturing Scenario

When the manufacturer maximizes profits by offering only the new product, the return flows have no impact on the manufacturer's decision. The profit function is given by $\Pi_{NR} = (p_n - c_n)q_n + (p_1 - c_n)q_1$, which is maximized by $p_n = p_1 = (1 + c_n)/2$. Sales are $q_n = \Delta(1 - c_n)/2$ and $q_1 = (1 - c_n)/2$. The profit is given by $\Pi_{NR} = (\Delta + 1)\frac{(1-c_n)^2}{4}$.

4.2 Remanufacturing Scenario

In general, only a proportion ρ of used products from the first period is available and remanufacturable in the second period (see Debo et al. (2005) and Geyer et al. (2006) for a general discussion.) We assume the collection cost is included in the c_r parameter.[5] With these elements the manufacturer's two-period objective can be written as:

$$\max_{p_n, p_r, p_1} \Pi_R = (p_1 - c_n)q_1 + (p_n - c_n)q_n + (p_r - c_r)q_r \tag{6}$$

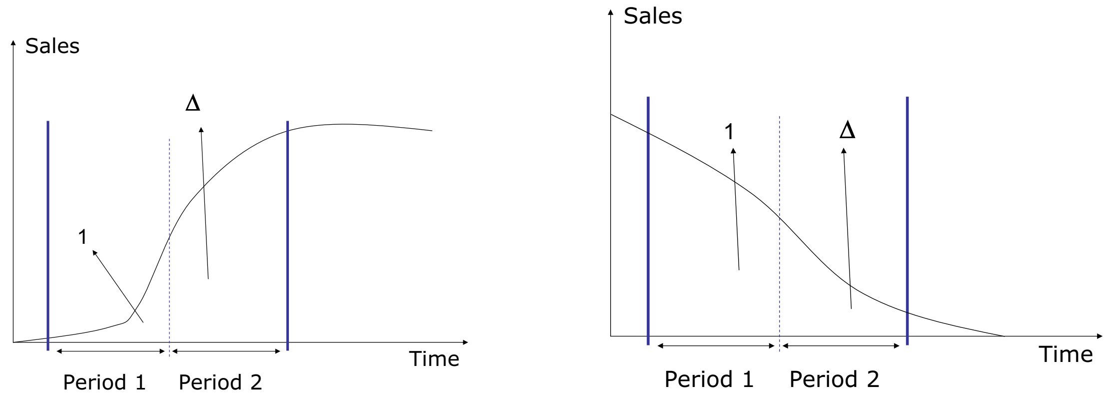
$$s.t. \quad q_r \leq q_1 \rho, \tag{7}$$

Given the assumptions, the first period demand will be defined by the equation $q_1 = 1 - p_1$, while the second period demand is given by (1), (2), (3), (4) and (5).

Lemma 1 *Constraint (7) is binding if $\Delta > \bar{\Delta}$ or equivalently if $\rho < \bar{\rho}$ where*

$$\begin{cases} \bar{\Delta} = \frac{\rho\,(1-c_n)\,(-1+\delta)\,\delta}{\delta\,(-c_n+\beta\,(-1+c_n+\delta))-c_r\,(-1+\beta\,(1+(-1+\delta)\,\delta))}, \bar{\rho} = \frac{(\delta\,(c_n-\beta\,(-1+c_n+\delta))+c_r\,(-1+\beta\,(1+(-1+\delta)\,\delta)))\,\Delta}{(-1+c_n)\,(-1+\delta)\,\delta} & \text{when } \beta < \beta^* \\ \bar{\Delta} = \frac{-\rho+c\,c_n}{\beta\,(-1+c_r)}, \ \bar{\rho} =\leq \frac{-(\beta\,\Delta)+\beta\,c_r\,\Delta}{-1+c_n} & \text{otherwise.} \end{cases}$$

[5]We assume that collection cost is linear in the quantity collected. See Ferguson and Toktay (2006) for a discussion on the impact of non-linear collection costs.



(a) Growth phase ($\Delta > 1$)

(b) Decline phase ($\Delta < 1$)

Figure 3: Market growth model

Lemma 1 shows that there exists a market growth (diffusion) level below which remanufacturing is unconstrained. In that case, Proposition 1 applies. However, when the remanufacturable supply is constrained, the pricing rules in Proposition 1 do not apply. Similar to the benchmark scenario, we assume that costs are such that both new and remanufactured products are offered (see Corollaries 8 and 9 in Appendix for details). The following proposition identifies the manufacturer's optimal decision.

Proposition 2 *Assume $\Delta > \bar{\Delta}$, where $\bar{\Delta}$ is defined by Lemma 1. Then, there exists a β' (detailed in Appendix) such that when $\beta < \beta'$, the monopolist's optimal prices are given by:*
$p_r^* = \frac{\rho\,(-1+\delta)\,\delta\,(-1+c_n+\beta\,(-1+c_n+\rho\,c_r)\,(-1+\delta)+\rho\,(c_r+\delta))+\delta\left(-1-c_n+\beta\left(-(c_n\,(-2+\delta))+\delta^2+\beta\,(-1+\delta)\,(-1+c_n+\delta)\right)\right)\Delta}{2\,(1+\beta\,(-1+\delta))\,((-1+\beta)\,\Delta+(-1+\delta)\,\delta\,(\rho^2+\beta\,\Delta))}$, $p_1^* = \frac{2\,\rho^2\,(-1+\delta)\,\delta+(1+c_n)\,(-1+\beta\,(1+(-1+\delta)\,\delta))\,\Delta+\rho\,(\delta\,(c_n-\beta\,(-1+c_n+\delta))+c_r\,(-1+\beta\,(1+(-1+\delta)\,\delta)))\,\Delta}{2\,(-1+\beta)\,\Delta+2\,(-1+\delta)\,\delta\,(\rho^2+\beta\,\Delta)}$ *and* $p_n^* = \frac{1+c_n+\beta\,(1+c_n-\delta)\,(-1+\delta)}{2+2\,\beta\,(-1+\delta)}$. *Otherwise the optimal prices of the monopolist are given by* $p_1^* = \frac{-\left(-2\,\rho^2-\beta\,\Delta+\beta\,\rho\,\Delta-\beta\,c_n\,\Delta-\beta\,\rho\,c_r\,\Delta\right)}{2\,(\rho^2+\beta\,\Delta)}$, $p_n^* = \frac{1+c_n}{2}$ *and* $p_r^* = \frac{-\left(\rho-\rho^2-\rho\,c_n-\rho^2\,c_r-2\,\beta\,\Delta\right)}{2\,(\rho^2+\beta\,\Delta)}$.

Proposition 2 has the same structure as Proposition 1. The monopolist uses a two-level pricing regime (depending on the green segment size) to maximize profits. Similar to the benchmark scenario, to overcome the negative impact of cannibalization, the remanufacturing cost should be sufficiently low. However, there is an important difference when the product life cycle is considered. Comparing the no remanufacturing (NR) and remanufacturing (R) scenarios we obtain:

Corollary 3 *When $\beta > 0$, the condition $c_r < c_n$ is not sufficient for profitable remanufacturing when $\Delta > \bar{\Delta}$. There exists a $\tau = \frac{(1-c_n)\,(1-c_n+2\,\rho\,(1-c_r))}{(-1+c_r)^2}$ such that remanufacturing is profitable if $\Delta\beta \leq \tau$ when $\beta > \beta^*$. τ is decreasing in c_n and increasing in c_r.*

In the benchmark case, we have found (Corollary 1) that the only condition under which a firm would remanufacture is if the cost of remanufacturing is sufficiently low, i.e. $c_n > c_r$. However, when there is limited supply, an additional condition is required, i.e. $\Delta\beta \leq \tau$. When the remanufacturable supply is constrained, either the market growth rate or the green segment size should be sufficiently low for profitable remanufacturing. Moreover, the profitability depends on the cost margins involved as well as the green segment size and market growth rate. Basically, supply is not sufficient to match demand to compensate the losses from cannibalization if the demand for the remanufacturable products is high and profit margins are low. Therefore, a monopolist facing the product life cycle should consider the market growth rate when making remanufacturing decisions. *In particular, under a fast market growth and high demand for remanufactured products, remanufacturing should be avoided.*

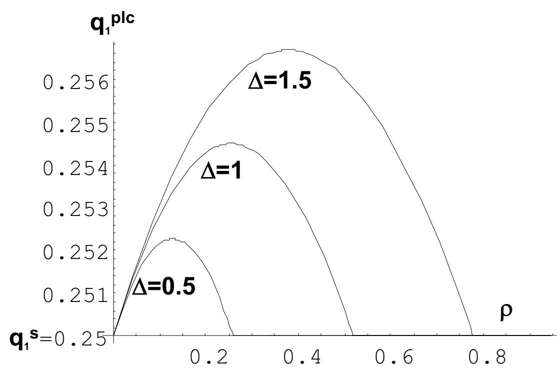
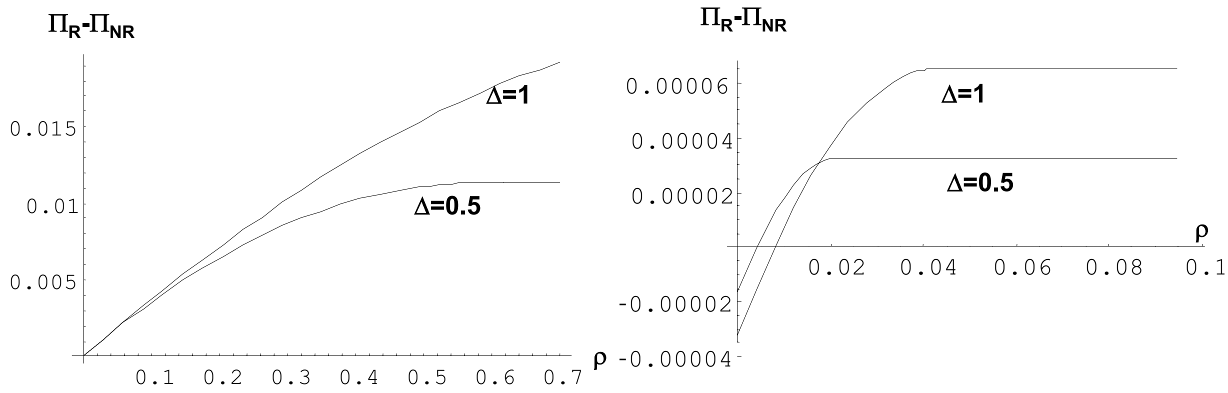


Figure 4: Optimal first period sales for different market growth and remanufacturability rates when $c_n = 0.5$, $c_r = 0, 2$, $\delta = 0.5$, $\beta = 0.1$

Another difference with the static monopoly case is that the manufacturer is forward-looking when facing a product life cycle. Figure 4 illustrates the optimal new product sales quantities in period 1 for different market growth and remanufacturability levels. The optimal sales quantity under product life cycle, say q_1^{plc}, is always larger than in the unconstrained case where the static optimal new product sales in the first period, q_1^s, would be $\frac{1-0.5}{2} = 0.25$. This is because the first-period sales determine the remanufacturable product availability in the second period. When the reusability rate is low, the new product prices will be lower to increase higher new product sales and assure higher availability of remanufacturable products. The optimal first-period new product sales quantities are increasing up to a certain accessibility threshold and then decrease down to the unconstrained case. This accessibility threshold is increasing in the market growth rate (Δ) since faster growth requires higher remanufacturable product availability. Therefore, manufacturers should consider the return accessibility/reusability rates as well as the rate of market growth when making pricing decisions, even before they introduce remanufactured products. Examples show that these rates can vary depending on the industry and product specifications. Toktay (2003) and Guide et al. (2006) report that accessibility rates are between 5 and 35 per cent and reusability rates range between 40 to 93 per cent.



(a) $c_n = 0.5$, $c_r = 0, 1$, $\beta = 0.004$

(b) $c_n = 0.1$, $c_r = 0.04$, $\beta = 0.001$

Figure 5: Extra profit from remanufacturing when $\delta = 0.5$

Figure 5 illustrates the impact of market growth and remanufacturability rates on the profitability of remanufacturing. Remanufacturable product availability increases extra profit from remanufacturing and this effect is stronger for higher market growth levels and higher remanufacturing cost savings, which is quite intuitive. In Figure 5a we observe that extra profit from remanufacturing increases in the return availability when the absolute cost savings from remanufacturing are high. Figure 5b on the other hand illustrates that the remanufacturability rate is not that significant when remanufacturing cost savings are low. Figure 5b leads to the interesting observation that there is an interaction between the market growth level and remanufacturability rate determining the profitability of remanufacturing.

From Corollary 3, we also know that there is an interaction between the green segment size and the market growth rate. For example, when we expect large green segments, low market growth rate is required for profit making from remanufacturing. These observations suggest that there is an optimal market growth rate to introduce remanufactured products. This optimal market growth rate is very easy to determine numerically. Figure 6 illustrates the impact of market growth on the profitability of remanufacturing. The optimal remanufactured product market introduction requires lower market growth rates for lower remanufacturability rates.

It is important to note that we have assumed constant remanufacturability rate ρ throughout the life cycle. This assumption is reasonable for simple products with short return lead times such as printer cartridges. However, it may not necessarily hold for other product categories. For instance,

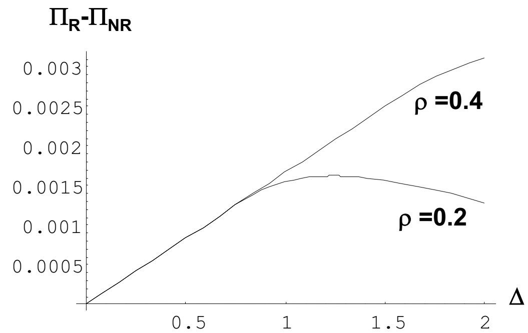


Figure 6: Impact of market growth level on extra profits from remanufacturing at $\delta = 0.7$, $\beta = 0.1$, $c_n = 0.1$ and $c_r = 0.04$

the remanufacturability rates of consumer electronics differ throughout the life cycle. Early on, most product returns are commercial or warranty returns for which return rates are low but reusability rates are high. Later in the life cycle, one would expect end-of-use type of returns, for which return rates can be high but reusability rates are usually low. In general, one would expect higher remanufacturability rates earlier in the life cycle, i.e. when the market growth rate is high, which means more profitable remanufacturing according to our analytical results.

5 Static Competition

In this section we revert to the static model but introduce a competitor. To improve readability we call our original monopolist the Manufacturer. The OEM offering an alternative new product is called the Competitor. All consumers value the competitor's product with $\alpha\theta$, where α can be thought of as the brand image of the competitor when compared to the manufacturer. We limit our analysis to the relevant case where $\alpha \leq 1$.[6] Our goal is to show that remanufacturing can be used as a competitive strategy. To this end, we consider a worst-case scenario for remanufacturing where $\delta \leq \alpha$. This assumption biases our analysis against the remanufactured product since the primary consumers consider the competitor's product as a better alternative than the remanufactured product. (In case $\delta > \alpha$, remanufacturing would always be a better alternative.)

The manufacturer prices the remanufactured product as the cheap alternative in the market, which results in the relevant case where $p_n > p_c > p_r$ (see Table 2). Similar to the benchmark scenario, the primary consumers purchasing the competitor's product can be given by the set $\{\theta | U_c^P(\theta) > 0,\ U_c^P(\theta) >$

[6] In case of $\alpha > 1$, the outcome would be similar to the monopoly case since the manufacturer's remanufactured product would mostly compete with his own new product.

$U_r^P(\theta),\ U_c^P(\theta) > U_n^P(\theta)\}$. Green consumers go for the remanufactured product as it is the cheapest alternative.

Parameter	Definition
p_c	Sales price for the new product of (C)
q_c	Competitor's (C) new product sales
c_c	Manufacturing cost of the new product of (C)
α	Consumer valuation ratio for the new product of (C)

Table 2: Additional model parameters under OEM competition

5.1 Demand

First we consider the case without remanufactured products. Consumers with high willingness-to-pay buy the manufacturer's new product:

$$q_n = \frac{\Delta\ (-1+\alpha-p_c+p_n)}{-1+\alpha}. \tag{8}$$

The price sensitive consumers buy the competitor's product:

$$q_c = \frac{\Delta\ (p_c-\alpha p_n)}{(-1+\alpha)\ \alpha}. \tag{9}$$

Now, assume that the manufacturer offers a remanufactured product. Similar to the benchmark scenario, the manufacturer can choose one of two pricing regimes.

1. If the manufacturer keeps the price low to sell the remanufactured product to both primary and green segments,i.e., $p_c/\alpha \geq p_r/\delta$, the manufacturer's new product demand will come from the primary consumers with high willingness to pay:

   $$q_n = \frac{\Delta(1-\beta)\ (-1+\alpha-p_c+p_n)}{-1+\alpha}. \tag{10}$$

   Some primary consumers with lower willingness-to-pay will go for the competitor's new product:

   $$q_c = (1-\beta)\ \Delta\left(\frac{p_c-p_n}{-1+\alpha}+\frac{p_c-p_r}{-\alpha+\delta}\right). \tag{11}$$

   The price sensitive primary consumers will buy the remanufactured product:

   $$q_r^P = \frac{(1-\beta)\ \Delta\ (\delta\, p_c-\alpha\, p_r)}{(\alpha-\delta)\ \delta}. \tag{12}$$

   Green consumers will buy the remanufactured product:

   $$q_r^G = \beta\,\Delta\ (1-p_r)\,. \tag{13}$$

2. If the manufacturer keeps the price high, i.e. $p_c/\alpha < p_r/\delta$, he aims at selling the remanufactured product to the green segment only. In this case, the primary consumers do not buy the remanufactured product and the demand is formed as follows. Primary consumers with high willingness-to-pay buy the manufacturer's new product:

$$q_n = \frac{\Delta(1-\beta)\ (-1+\alpha-p_c+p_n)}{-1+\alpha}. \tag{14}$$

The price-sensitive primary consumers buy the competitor's product:

$$q_c = \frac{\Delta(1-\beta)\ (p_c-\alpha p_n)}{(-1+\alpha)\ \alpha}. \tag{15}$$

The green consumers buy the remanufactured product:

$$q_r = \Delta\beta(1-p_r). \tag{16}$$

Again, the overall demand is kinked with the two demand regimes identified. The manufacturer maximizes profits by solving the problem: $\max_{p_n,p_r} \Pi_R = (p_n-c_n)q_n + (p_r-c_r)q_r$, and the competitor uses $\max_{p_c} \Pi_C = (p_c-c_c)q_c$ in the competitive game.

5.2 No Remanufacturing

Without remanufacturing, the manufacturer maximizes profits via the objective $\max_{p_n} \Pi_{NR} = (p_n - c_n)q_n$ and the competitor similarly uses $\max_{p_c} \Pi_C = (p_c-c_c)q_c$ in the game. The resulting optimal prices are $p_n^* = \frac{2-2\,\alpha+2\,c_n+c_c}{4-\alpha}$ and $p_c^* = \frac{\alpha-\alpha^2+2\,c_c+\alpha\,c_n}{4-\alpha}$ and the optimal sales quantities are $q_n^* = \frac{(2+c_c+\alpha\,(-2+c_n)-2\,c_n)\,\Delta}{4-5\,\alpha+\alpha^2}$ and $q_c^* = \frac{(-2\,c_c+\alpha\,(1-\alpha+c_c+c_n))\,\Delta}{(-4+\alpha)\,(-1+\alpha)\,\alpha}$. The optimal profit of the manufacturer equals: $\Pi_{NR} = \frac{(2+c_c+\alpha\,(-2+c_n)-2\,c_n)^2\,\Delta}{(-4+\alpha)^2\,(1-\alpha)}$.

5.3 Static Remanufacturing under Competition from an OEM

Under the remanufacturing option, the Nash Equilibrium of the competitive game between the manufacturer and the competitor can be characterized as follows.

Proposition 3 *There is a unique Nash equilibrium under competition in which there exists a β'' such that when $\beta \le \beta''$ the equilibrium prices are set at:*

$$p_n^* = \frac{\alpha\ \left((-1+\beta)\ (4+2\,c_c+4\,c_n+c_r) - (-2-2\,c_c-3\,c_n+\beta\ (7+4\,c_c 7\,c_n+c_r))\ \delta + \beta\ (-1+2\,c_c+4\,c_n-c_r)\ \delta^2 + 4\,\beta\,\delta^3\right)}{2\ (\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta))}$$

$$+\frac{\alpha^2\ (4+c_r+\beta\ (4+c_r-4\,\delta)\ (-1+\delta)-3\,\delta) + \delta\ \left(1+c_n+\beta\ \left(-1-c_n+(5+2\,c_c+4\,c_n+c_r)\ \delta - 2\ (2+c_c+2\,c_n)\ \delta^2\right)\right)}{2\ (\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta))}, \tag{17}$$

$$p_r^* = \frac{\alpha\ (\delta\ (-1-c_n-\delta+\beta\ (-3+c_n+3\,\delta)) + c_r\ (-4+3\,\delta+\beta\ (4+\delta\ (-7+2\,\delta))))}{2\ (\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta))}$$

$$+\frac{\delta\ (2\,c_c\ (-1+\beta+\delta-\beta\,\delta) + \delta\ (1+c_n-\beta\ (-3+c_n-4\,c_r+3\ (1+c_r)\ \delta))) + \alpha^2\ (c_r\ (1+\beta\ (-1+\delta)) + \delta) +}{2\ (\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta))}, \tag{18}$$

$$p_c^* = \frac{\alpha \left(1 + \alpha^2 \beta + 2 c_c + c_n - \beta (2 + 4 c_c + c_n + c_r) + \alpha (-1 + \beta (1 - c_n + c_r))\right) \delta}{\alpha^2 (1 + \beta (-1 + \delta)) + 2 \alpha (1 + \beta (-1 + \delta)) (-2 + \delta) + \delta + \beta \delta (-1 + (4 - 3 \delta) \delta)}$$

$$+ \frac{-(\alpha (-1 + \beta) (-2 c_c - c_r + \alpha (-1 + \alpha - c_n + c_r))) + \beta (1 + 2 c_c + \alpha (1 - 2 \alpha + 2 c_c + 2 c_n - c_r) + c_r) \delta^2 + \beta (-1 + \alpha - 2 c_c - c_n) \delta^3}{\alpha^2 (1 + \beta (-1 + \delta)) + 2 \alpha (1 + \beta (-1 + \delta)) (-2 + \delta) + \delta + \beta \delta (-1 + (4 - 3 \delta) \delta)}. \quad (19)$$

When $\beta > \beta''$, $p_n^* = \frac{2-2\alpha+c_c+2c_n}{4-\alpha}$, $p_c^* = \frac{\alpha^2-2c_c-\alpha(1+c_n)}{-4+\alpha}$ *and* $p_r^* = \frac{1+c_r}{2}$.

In terms of structure, the competitive case is similar to the benchmark case in that two pricing regimes are used. The important difference is the impact of cannibalization. Cannibalization from remanufactured products not only affects the manufacturer's new product sales but also the competitor's. Obviously, the remanufactured product brings competitive strength to the manufacturer since it captures the green segment consumers. However, remanufacturing can still be profitable without a green segment since the cost advantage required for profitable remanufacturing can be lower under competition than in the benchmark (monopoly) case (see Corollary 10 of the Appendix).

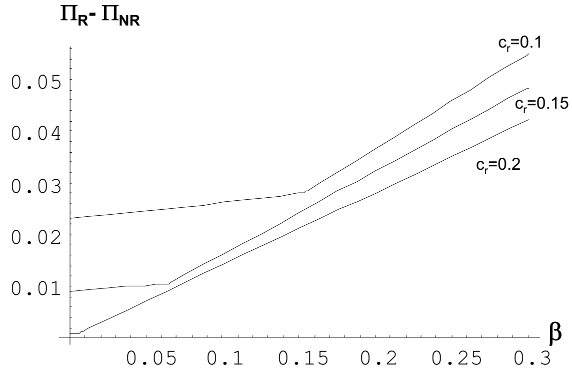
We are interested in finding the conditions where remanufacturing is profitable under competition. Unfortunately, due to the complex structure of the pricing strategy described in Proposition 3, analytical investigation of the general case is tedious and not insightful. However, critical insights can be obtained from some special cases. For instance, one can show that when the competitor has high brand power, the manufacturer's extra profit from remanufacturing is decreasing in the competitor's manufacturing costs. (see corollary 11 in Appendix.) In other words, remanufacturing can be a better strategy against a competitor with low manufacturing costs. Considering this observation, assume a worst-case scenario for the manufacturer where the competitor has no cost advantage from new product manufacturing, i.e. his brand power is proportional to his manufacturing cost. Corollary 4 states the condition for profitable remanufacturing.

Corollary 4 *Assume* $\beta > 0$ *and* $c_c = \alpha c_n$. *Then, remanufacturing is profitable* ($\Pi_R \geq \Pi_{NR}$) *if remanufacturing costs are sufficiently low* ($c_r \leq c_r'' = c_n + \frac{\left(4-4\sqrt{1-\alpha}-\alpha\right)(1-c_n)}{4-\alpha}$).

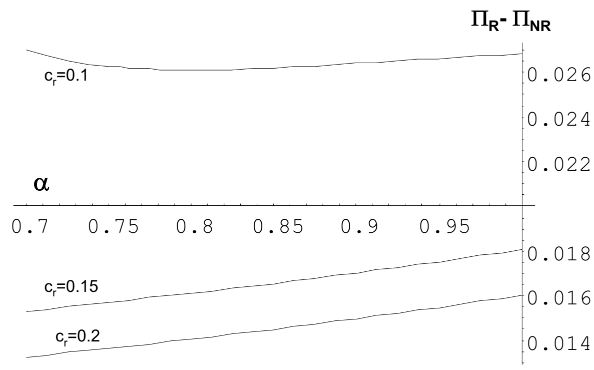
Corollary 4 shows that, for remanufacturing to be profitable, the remanufacturing cost should be below a certain threshold. By simple algebra: $c_r'' - c_n = \frac{\left(4-4\sqrt{1-\alpha}-\alpha\right)(1-c_n)}{4-\alpha} > 0$. This shows two important facts: *(i) Cost advantage required for remanufacturing is lower under competition than under the benchmark (monopoly) case, (ii) Under competition, remanufacturing can be profitable even without a cost advantage.*

We now turn our attention to the general situation under competition. We are interested in the impact of the competitor's brand power, the green segment size and their interaction. To isolate the impact of the competitor's manufacturing cost we first assume that the latter has no cost advantage, i.e.

$c_c = \alpha c_n$, in Figures 7 and 8. Figure 7 (8) describes the situation when manufacturer's cost savings from remanufacturing are high (low). Figure 9 shows the case where the competitor has a cost advantage, i.e. $c_c < \alpha c_n$.



(a) $\alpha = 0.8$, $c_c = \alpha c_n$

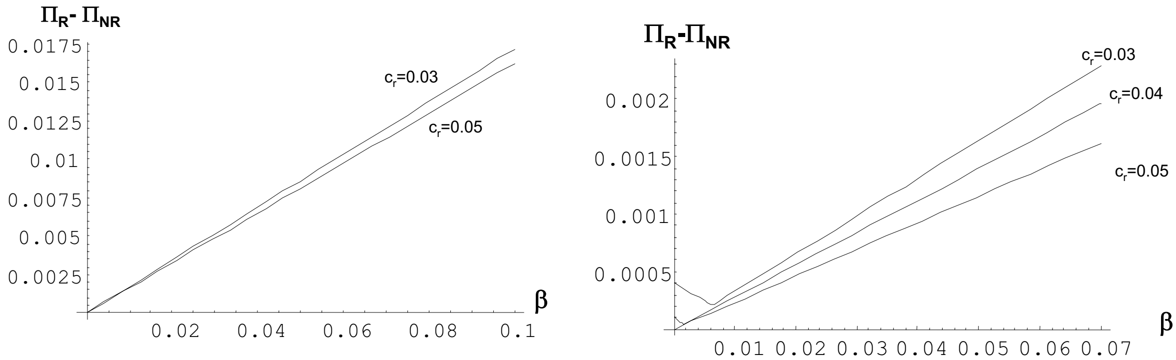


(b) $\beta = 0.1$, $c_c = \alpha c_n$

Figure 7: Extra profit from remanufacturing when $c_n = 0.5$, $\delta = 0.5$ and $\Delta = 1$

Figure 7a illustrates the impact of the green segment size, while Figure 7b illustrates the impact of the competitor's brand power on the profitability of remanufacturing. According to Figure 7a the extra profit from remanufacturing is increasing in β when the absolute cost savings from remanufacturing are high, similar to the benchmark case scenario. Comparing Figure 7a with Figure 1b shows that the *extra profit from remanufacturing is higher under competition than under monopoly.* It is important to note that *even when there is no green segment ($\beta = 0$) remanufacturing is more profitable under competition.* This is basically because the manufacturer steals from the competitor's share via offering the remanufactured product. Figure 7b supports this observation by showing that the extra profit from remanufacturing is increasing in the competitor's brand power. In other words, the stronger the competitor, the higher his market share and thus the higher his brand power, the more his sales are cannibalized by the remanufactured product.

In contrast to Figure 7, Figure 8 considers the case where the manufacturer's absolute cost savings from remanufacturing are low. The important observation is that the manufacturer only uses the high pricing regime, i.e. sells remanufactured products to the green segment only. Note that there is no kink in the profit difference in Figure 8a unlike Figure 8b (which is the same as Figure 1b). This means that remanufacturing with low cost savings against a competitor without cost advantage, has substantial benefits only if the green segment is large. Furthermore, comparing Figures 8a and 8b, we observe that when the green segment is small, extra profit from remanufacturing under competition is close to

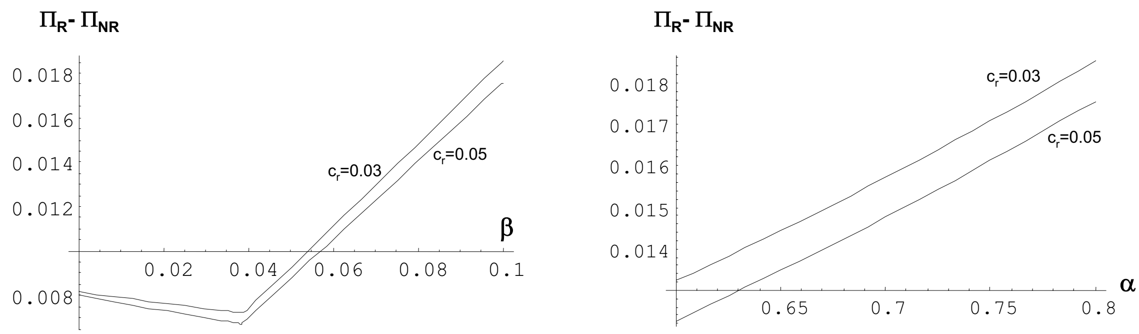


(a) Competition: $\alpha = 0.8$, $c_c = \alpha c_n$

(b) Monopoly

Figure 8: Extra profit from remanufacturing when $c_n = 0.1$, $\delta = 0.5$ and $\Delta = 1$

that under monopoly. In other words, remanufacturing is not a profitable strategy against a high cost competitor unless: (i) the green segment is large or (ii) the cost savings from remanufacturing are high.



(a) $\alpha = 0.8$, $c_c = \alpha c_n/4$

(b) $\beta = 0.1$, $c_c = \alpha c_n/4$

Figure 9: Extra profit from remanufacturing when $c_n = 0.1$, $\delta = 0.5$ and $\Delta = 1$

Figure 9 represents extra profits from remanufacturing under different remanufacturing cost and green segment levels similar to Figure 8. The only difference between these two figures is that Figure 9 represents the case when the competitor's manufacturing costs are lower. Comparing these two figures we observe that the extra profit from remanufacturing is significantly higher under competition than under monopoly. When the competitor has a cost advantage, remanufacturing is a better strategy under competition than under monopoly. The additional market to be captured via the remanufactured product is even higher now. Similar to Figure 7b, Figure 9b illustrates that the profits from remanufacturing are increasing in the competitor's brand power. These observations, combined with earlier results, reinforce the main message of the paper: *Remanufacturing is a profitable strategy against a strong competitor with low cost and/or with high brand power.*

5.4 Extension: Competition with a Local Remanufacturer

In a remanufacturing context, OEM competition is not the only type of competition. As the remanufacturing literature points out, local remanufacturers can come into the market and compete with the new product. The local remanufacturer can use a low pricing strategy and steal market share from the manufacturer from both primary and green segments, which results in high cannibalization.

To explore this case, let us consider the best situation for the manufacturer: assume that the local remanufacturer's cost (c_L) is so high that he uses the high pricing strategy only. Technically, this means: $p_r^L \geq \delta p_n \Rightarrow c_L \geq \delta c_n - (1-\delta))$. In case the local remanufacturer can price lower to capture the primary market's low valuation customers, the manufacturer will be even worse off. Let us denote the manufacturer's profit under remanufacturer competition by Π_{NR}^L.

Proposition 4 *Remanufacturing is profitable* ($\Pi_R \geq \Pi_{NR}^L$) *under local remanufacturer competition. Furthermore, the profitability of remanufacturing* ($\Pi_R - \Pi_{NR}^L$) *is increasing in* β.

We have shown that the manufacturer is worse off without remanufacturing even in the best case scenario. Thus, when there is a threat of competition from local remanufacturers, the remanufacturing strategy is necessarily better. However, as a caveat, we have to note that remanufacturing is not the only entry deterrent strategy. A preemptive collection strategy may be better than the remanufacturing strategy when the potential cannibalization impact is stronger, as shown by Ferguson and Toktay (2005).

6 Remanufacturing under OEM Competition and Product Life Cycle

This section combines the previous two sections to consider the impact of market growth and competition on the profitability of remanufacturing. Following the previous notation, the manufacturer's two-period objective under remanufacturing can be written as:

$$\max_{p_n,p_r,p_1} \Pi_R = (p_1 - c_n)q_1 + (p_n - c_n)q_n + (p_r - c_r)q_r \tag{20}$$

$$s.t. \quad q_r \leq q_1\rho. \tag{21}$$

Recall that the first period sales determine the amount of remanufacturable supply for the second period. Competition in the first period would require additional assumptions on the return access in the second period, i.e. whose returns are available and to whom. This increases the complexity of the analysis significantly.[7] Therefore, without loss of generality we assume that the second period starts when the competitor enters the market. In other words, the demand in the first period will be the same as in the monopoly case, i.e. $q_1 = 1 - p_1$. The second period demand will be described by (10), (11), (12), (14), (15) and (16). The competitor's objective is $\max_{p_c} \Pi_C = (p_c - c_c)q_c$. Note that under these assumptions the manufacturer's profit under the no remanufacturing scenario is trivial, i.e. the first period profit is the same as in section 4.1 and the second period profit is the same as in section 5.2.

It is important to clarify the meaning of Δ here, which depends on the definition of the second period. In this scenario, we define the second period as the time where the remanufactured products are used against a competitor. Therefore, Δ can be defined as the market growth rate at the time when the OEM introduces remanufactured products against a competing OEM. This parameter is a significant driver of the remanufacturing decision as before:

Lemma 2 *Constraint (21) is binding if $\Delta > \Delta''$ where*

$$\begin{cases} \Delta'' = \frac{\rho\,(-1+c_n)\,(\alpha-\delta)\,\delta\left(\alpha^2\,(1+\beta\,(-1+\delta))+2\,\alpha\,(1+\beta\,(-1+\delta))\,(-2+\delta)+\delta+\beta\,\delta\,(-1+(4-3\,\delta)\,\delta)\right)}{(\alpha\,(1+\beta\,(-1+\delta))-\beta\,\delta^2)K} & \textit{when } \beta < \beta'' \\ \Delta'' = \frac{-\rho+\rho\,c_n}{\beta\,(-1+c_r)} & \textit{otherwise,} \end{cases} \tag{22}$$

where $K = (-1 - 2\,c_c - c_n + \beta\,(-3 + 2\,c_c + c_n + 4\,c_r) + \alpha\,(1 + \beta\,(-3 + 2\,c_r)))\,\delta^2 - 3\,\beta\,(-1 + c_r)\,\delta^3 + (4 - \alpha)\,\alpha\,(-1 + \beta)\,c_r + \delta\left(-2\,(-1 + \beta)\,(c_c + c_r) + \alpha^2\,(-1 + \beta\,c_r) + \alpha\,(1 + c_n + c_r - \beta\,(-3 + c_n + 5\,c_r))\right)$.

Lemma 2 shows that there exists a market growth (diffusion) level below which remanufacturing is unconstrained under competition. In that case, Proposition 3 applies. However, when the remanufacturable supply is constrained, the pricing rules in Proposition 3 do not apply. In this case Proposition 5 identifies the manufacturer's optimal decision. Due to the complex structure of the solution, we provide the equations to calculate the equilibrium prices, i.e. the best response functions of the manufacturers in Appendix. Since the best responses are linear and the constraint is convex, there is a unique equilibrium.

Proposition 5 *When $\Delta > \Delta''$, there is a unique Nash equilibrium. Characterization of the equilibrium outcome is provided in Appendix.*

As before, we would like to identify the profitability conditions for remanufacturing. In particular, we want to understand the joint impact of competition and market growth rate. Corollary 5 considers the profit impact of remanufacturing under supply constraints and competition.

[7] If we assumed competition, the demand in the first period would be presented via (8) and (9) with $\Delta = 1$. Then, determining the amount of remanufacturable supply would require additional assumptions, e.g. whether the manufacturer could remanufacture his competitor's product returns or what the rate of reusability of the competitor's product returns is.

Corollary 5 *Assume $\beta > 0$. The condition $c_r < c_r''$ is not sufficient for profitable remanufacturing when $\Delta > \Delta''$. There exists a τ'' (detailed in Appendix) such that remanufacturing is profitable ($\Pi_R \geq \Pi_{NR}$) if $\Delta\beta \leq \tau''$.*

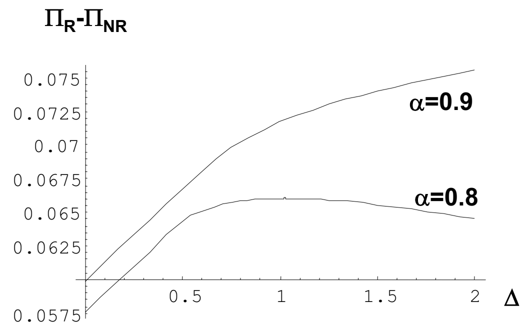


Figure 10: Optimal market growth level at $\delta = 0.7$, $\beta = 0.1$, $c_n = 0.5$, $c_c = 0.3$, $c_r = 0.2$ and $\rho = 0.6$

According to Corollary 5, the intuition behind the competition scenario is the same as in the monopoly case: remanufacturing is not profitable under high cannibalization and constrained remanufacturable product supply. But there is more to that: profitability of remanufacturing is affected significantly by the interaction between the market growth rate and the degree of competition. Figure 10 illustrates the impact of the competitor's brand power and the rate of market growth on the profitability of remanufacturing. According to Figure 10, the manufacturer is better off with remanufacturing when the market growth rate is high at the time of entry of the competitor with a strong brand image. When the competitor's brand image is lower, the manufacturer is better off with remanufacturing when the market growth rate is low at the time of entry.

7 Concluding Remarks and Future Research

Based on real-life examples, like the Bosch case we have constructed a model in which a manufacturer can collect returns with recoverable value potential and has the option to sell remanufactured products. We have shown that the manufacturer's remanufacturing decision is driven by factors like cannibalization, consumer segment sizes, competition and diffusion.

Our core result says that remanufacturing is more beneficial under competition than in a monopoly setting. The tougher the competition, the more profitable remanufacturing is. In particular, remanufacturing is best against a strong brand image competitor with low manufacturing costs. This is because remanufactured products help the manufacturer compete for the low valuation consumer segments, that would otherwise be lost to low cost OEM competitors. Obviously, remanufacturing seems to be a better alternative under competition as it captures the green segment. However, we have highlighted that remanufacturing is profitable under competition even in the absence of a green segment. For small green segments, the profit savings from remanufacturing are even decreasing in the green segment size.

Our results suggest that manufacturers' cannibalization concerns are valid. Nevertheless, the negative impact of cannibalization can be overcome by using a smart pricing strategy. Correct identification of the market segments is crucial to achieve this. When the ratio of customers who are indifferent between the new and remanufactured products (or even value the remanufactured product more because of its environmental attributes) is expected to be high, a high pricing regime should be used. Otherwise, the remanufactured products should be priced low to capture the low valuation customers.

Naturally, cost saving is one major cause for the remanufacturing option. Basically, there exists cost thresholds that make remanufacturing a profitable alternative. It is important to understand how these thresholds are shaped under different conditions though. For instance, high cost savings are required when the demand for the remanufactured product is low while low cost savings suffice under high demand for the remanufactured product. The competition is also an important factor that determines these cost thresholds. Remanufacturing cost savings that are not sufficient under monopoly can be sufficient under competition.

We have found that the supply constraint - a special feature of remanufacturing systems - combined with life cycle effects is also an important driver of remanufacturing profit. Constrained remanufacturable product availability under fast market growth makes remanufacturing a worse alternative, i.e., requires higher consumer valuations and is more vulnerable to cannibalization. This can be controlled by a smart selection of the remanufactured product introduction time. There is an optimal market growth rate for introducing remanufacturable products: the market size should be sufficiently low to match the supply with the demand and it should be sufficiently high to maximize sales and thus profit from remanufacturing.

Figure 11 summarizes all our main results but the impact of competition which has been discussed in detail above. Remanufacturing can be profitable if the cost savings from remanufacturing, the green segment size and the market growth rates are matched correctly. Fast market growth rates at the moment of remanufactured product introduction should be avoided unless the demand for the remanufactured product is small and there are high cost savings associated with remanufacturing. In addition, even under slow market growth rates, remanufacturing may result in profit loss if the cost savings from remanufacturing are not sufficiently high and the demand for the remanufactured product is low. Otherwise, remanufacturing is a profitable strategy as a low cost alternative.

	Small Green Segment		Large Green Segment	
	High Cost Savings	Low Cost Savings	High Cost Savings	Low Cost Savings
Slow Growth	✓	✗	✓	✓
Fast Growth	✓	✗	✗	✗

Figure 11: When is remanufacturing a profitable strategy?

The remanufacturing literature is small compared to the importance of the managerial issues that it represents. With new regulations adopted by various developed countries and in the presence of strong consumer pressure it is likely that remanufacturing will increase in importance on most firms agendas. Our focus on the demand side of this problem leaves many questions for future research. For example, some of the exogenous parameters in our models are interdependent. It is likely for example that diffusion rates and green segment size are interlinked with green segment sizes being larger near the end of the product life cycle. How would this affect our results? Similarly, one could consider the issue of how to increase consumer valuations for remanufactured products, keeping green segment sizes constant. Considering multiple markets is also an issue that could be studied in more detail. The competition level or the market structure can be different for different markets, leading to market specific remanufacturing and pricing decisions with possible transfers of collected products across markets. Capacity constraints on new product manufacturing can also be an issue. When such problems exist remanufacturing may be an even better alternative. Future research on these issues may provide interesting new insights for firms that consider remanufacturing.

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8 Appendix

Proof of Proposition 1

Assume that $p_r \leq \delta p_n$. Then:
$\Pi_R = -\left(\frac{(-1+\beta)\,\Delta\,(-c_n+p_n)\,(-1+\delta+p_n-p_r)}{-1+\delta}\right) + \left(\frac{(-1+\beta)\,\Delta\,(\delta\,p_n-p_r)}{(-1+\delta)\,\delta} - \beta\,\Delta\,(-1+p_r)\right)(-c_r+p_r)$.
The first order conditions are: $\frac{\partial \Pi_R}{\partial p_n} = -\left(\frac{(-1+\beta)\,\Delta\,(-1-c_n+c_r+\delta+2\,p_n-2\,p_r)}{-1+\delta}\right) = 0$ and
$\frac{\partial \Pi_R}{\partial p_r} = \frac{\Delta\,(c_r\,(-1+\beta\,(1+(-1+\delta)\,\delta))+\delta\,(c_n-2\,p_n+\beta\,(-1-c_n+\delta+2\,p_n))+2\,p_r-2\,\beta\,(1+(-1+\delta)\,\delta)\,p_r)}{(-1+\delta)\,\delta} = 0$.
The second order derivatives can be written as:

$$\frac{\partial^2 \Pi_R}{\partial p_n^2} = \frac{-2\,(-1+\beta)\,\Delta}{-1+\delta} < 0,$$

$$\frac{\partial^2 \Pi_R}{\partial p_r^2} = -2\,\beta\,\Delta - \frac{2\,(-1+\beta)\,\Delta}{(-1+\delta)\,\delta} < 0,$$

$$\frac{\partial^2 \Pi_R}{\partial p_n \partial p_r} = \frac{2\,(-1+\beta)\,\Delta}{-1+\delta}.$$

The determinant of the Hessian can be written as: $|H| = \frac{4\,(-1+\beta)\,(1+\beta\,(-1+\delta))\,\Delta^2}{(-1+\delta)\,\delta} > 0$. Thus, the solution to the first order conditions gives the unique maximizer. The monopolist's optimal sales quantities under the low pricing strategy can be obtained as:

$$q_n^* = \frac{-((-1+\beta)\ (-1+c_n-c_r+\delta)\ \Delta)}{2\ (-1+\delta)}, \tag{23}$$

$$q_r^* = \frac{-((\delta\ (c_n-\beta\ (-1+c_n+\delta))+c_r\ (-1+\beta\ (1+(-1+\delta)\ \delta)))\ \Delta)}{2\ (-1+\delta)\ \delta}, \tag{24}$$

where the prices are given as:

$$p_n^* = \frac{-\left(-1+\beta-c_n+\beta\, c_n-2\,\beta\,\delta-\beta\, c_n\,\delta+\beta\,\delta^2\right)}{2\ (1-\beta+\beta\,\delta)}, \tag{25}$$

$$p_r^* = \frac{-(-c_r+\beta\, c_r-\delta-\beta\, c_r\,\delta)}{2\ (1-\beta+\beta\,\delta)}. \tag{26}$$

Let us denote the optimal profit under this solution by Π_R^l.

Now assume $p_r \geq \delta p_n$. Then:

$\Pi_R = (-1+\beta)\ \Delta\ (-1+p_n)\ (-c_n+p_n) - \beta\,\Delta\ (-1+p_r)\ (-c_r+p_r)\,.$

The first order conditions are: $\frac{\partial \Pi_R}{\partial p_n} = (-1+\beta)\ \Delta\ (-1+p_n) + (-1+\beta)\ \Delta\ (-c_n+p_n) = 0$ and $\frac{\partial \Pi_R}{\partial p_r} = -\left(\beta\,\Delta\ (-1+p_r)\right) - \beta\,\Delta\ (-c_r+p_r) = 0.$

The second order derivatives can be written as:

$\frac{\partial^2 \Pi_R}{\partial p_n^2} = 2\ (-1+\beta)\ \Delta < 0$, $\frac{\partial^2 \Pi_R}{\partial p_r^2} = -2\beta\Delta < 0$ and $\frac{\partial^2 \Pi_R}{\partial p_n \partial p_r} = 0.$

The determinant of the Hessian is positive. Thus, the solution to the first order conditions gives the unique maximizer. The monopolist's optimal decisions under the high pricing strategy can be obtained as:

$$q_n^* = \Delta(1-\beta)\frac{1-c_n}{2},\ \ q_r^* = \Delta\beta\frac{1-c_r}{2},\ \ p_n^* = \frac{1+c_n}{2},\ \ p_r^* = \frac{1+c_r}{2}$$

Let us denote the optimal profit under this solution by Π_R^h.

$\beta^* = \frac{-{c_r}^2+2\,c_n\,c_r\,\delta-{c_n}^2\,\delta^2}{(-1+\delta)\,({c_r}^2+\delta-2\,c_n\,c_r\,\delta-2\,\delta^2+{c_n}^2\,\delta^2+\delta^3)}$ is obtained by solving the equation $\Pi_R^h = \Pi_R^l$.

This threshold is decreasing in c_r, since $\frac{\partial \beta^*}{c_r} = \frac{2\,(-1+\delta)\,\delta\,(-c_r+c_n\,\delta)}{({c_r}^2+\delta-2\,c_n\,c_r\,\delta+\delta^2\,(-2+{c_n}^2+\delta))^2}$. This derivative is negative when $c_r < \delta c_n$ which holds when $\beta \leq \beta^*$. ■

Corollary 6 *When $\beta = 0$, $q_r \geq 0$ if $c_r \leq \delta c_n$ and $q_n \geq 0$ if $c_r > c_n - 1 + \delta$.*

Proof of Corollary 6 These conditions can be obtained by solving $q_r = 0$ and $q_n = 0$. See also (Ferguson and Toktay 2006). ■

Corollary 7 *When $0 < \beta \leq \beta^*$, if $c_r \leq \bar{c}_r = -\left(\frac{\delta\left(-c_n+\beta\left(c_n+(-1+\delta)^2-c_n\,\delta\right)\right)}{1+\beta\,(-1+\delta)}\right)$, then selling remanufactured products to the primary market is optimal (i.e. $q_r^{P*} \geq 0$). Furthermore $\bar{c}_r$ is decreasing in β. Otherwise, no remanufactured products will be sold in the primary market.*

Proof of Corollary 7

At Optimality,

$$q_r^{P*} = \frac{\beta\ \left(c_r\ (1+\beta\ (-1+\delta))+\delta\ \left(-c_n+\beta\ \left(1+c_n-(2+c_n)\ \delta+\delta^2\right)\right)\right)\ \Delta}{2\ (1+\beta\ (-1+\delta))\ (-1+\delta)\ \delta}.$$

One can show that $q_r^{P*} \geq 0 \Leftarrow c_r \leq \bar{c}_r = -\left(\frac{\delta\left(-c_n+\beta\left(c_n+(-1+\delta)^2-c_n\,\delta\right)\right)}{1+\beta\,(-1+\delta)}\right)$. It is easy to see that $\bar{c}_{r\{\beta=0\}} = \delta c_n$ and that $\frac{\partial \bar{c}_r}{\partial \beta} = -\left(\frac{(-1+\delta)^2\,\delta}{(1+\beta\,(-1+\delta))^2}\right) \leq 0$. Therefore, the cost threshold for selling remanufactured products to the primary market is decreasing in β.

Or equivalently, $q_r^{P*} \geq 0 \Leftarrow \beta \leq \bar{\beta} = \frac{1}{1-\delta} + \frac{\delta}{c_r+\delta\,(-1-c_n+\delta)}$. This suggests that there exists a $\bar{\beta}$ above which the primary market is not served with a remanufactured product. ■

Corollary 6 states that when there is no green segment, remanufactured products are sold to the primary market when the remanufacturing cost is lower than the new product manufacturing cost multiplied by the discount rate (δ) for the remanufactured products ($\bar{c}_r|\{\beta = 0\} \leq \delta c_n$). This implies that the profit margin from the remanufactured product should be at least as high as the new product margin, i.e. $p_r - cr \geq p_n - c_n$ in order to overcome the cannibalization impact. Otherwise, remanufacturing is not profitable. Moreover, to sell remanufactured and new products together, the remanufacturing cost should not be too low ($c_r > c_n - 1 + \delta$). Otherwise, no new products will be sold. For the rest of our analysis, we assume that the second condition holds. The degree of cannibalization in the primary segment again depends on the cost margins involved in remanufacturing. When $\beta > 0$, the cost margins required to overcome the cannibalization impact in the primary segment is different.

Corollary 7 identifies the remanufacturing cost thresholds when there is a green segment. To overcome the negative cannibalization impact of selling remanufactured products in the primary market, the remanufacturing cost should be even **lower** than δc_n. This is because the remanufactured product sales prices will be higher to extract the consumer surplus from the green segment, which decreases the profit savings from the primary market. This way the low valuation consumers in the primary market cannot be captured via price discrimination. To avoid cannibalization in the primary segment, Proposition 1 suggests changing the pricing strategy, i.e. selling remanufactured products to the green segment only, when the green segment ratio becomes sufficiently high.

Proof of Corollary 2

Assuming that $c_r > -1 + c_n + \delta$, when $\beta < \beta^*$ (which guarantees that $c_r \leq \bar{c}_r$ under profitable remanufacturing.):

1. $\frac{\partial q_r}{\partial \delta} = \frac{-\left((-1+\beta)\left(c_r - 2\,c_r\,\delta + c_n\,\delta^2\right)\Delta\right)}{2\,(-1+\delta)^2\,\delta^2} \geq 0.$
2. $\frac{\partial q_r}{\partial \beta} = \frac{\Delta\,(\delta\,(-1+c_n+\delta) - c_r\,((-1+\delta)\,\delta+1))}{2\,(-1+\delta)\,\delta} \leq 0.$
3. $\frac{\partial^2 \Pi_R}{\partial \beta^2} = \frac{\delta\,\Delta\,(\delta-1)^2}{2\,(\beta\,(\delta-1)+1)^3} \geq 0.$ The profit is convex in β.
4. $\frac{\partial q_n}{\partial \delta} = \frac{-((-1+\beta)\,\Delta)}{2\,(-1+\delta)} + \frac{(-1+\beta)\,(-1+c_n-c_r+\delta)\,\Delta}{2\,(-1+\delta)^2} = \frac{(-1+\beta)\Delta(c_n-c_r)}{2(-1+\delta)^2} \leq 0.$
5. $\frac{\partial q_n}{\partial \beta} = \frac{-((-1+c_n-c_r+\delta)\,\Delta)}{2\,(-1+\delta)} \leq 0.$

When $\beta > \beta^*$ and $c_r \leq c_n$, it is trivial to show that the profit is increasing in β under the high pricing regime. ■

Proof of Corollary 1

Let us consider the suboptimal strategy where the profit is given by Π_R^h only, where, $\Pi_R^h = \Delta(1-\beta)\frac{(1-c_n)^2}{4} + \Delta\beta\frac{(1-c_r)^2}{4}$. It is easy to show that $\Pi_r^h - \Pi_{NR} = \frac{-(\beta\,(c_n-c_r)\,(-2+c_n+c_r)\,\Delta)}{4} > 0$, when $c_r \leq c_n$ and $\beta > 0$. The remaining condition follows from Corollaries 6 and 7. ■

Proof of Lemma 1

Recall the proof of Proposition 1. When $p_r \leq \delta p_n$, the optimal sales in the first period would be given as $q_1^* = \frac{1-c_n}{4}$ if $q_r \leq \rho q_1^*$, and $q_r^*$is given by (24). Solving for $q_r \leq q_1\rho$ the bounds are obtained for $\beta \leq \beta^*$. When $p_r \geq \delta p_n$, $q_1^* = \frac{1-c_n}{4}$ if $q_r \leq \rho q_1^*$, and q_r^* is given by $\frac{1+c_r}{2}$. Solving for $q_r \leq q_1\rho$, the above conditions are obtained. ■

Proof of Proposition 2

When $p_r \leq \delta p_n$, the Lagrangean can be written as:

$L = \Pi_R + \lambda(q_1\rho - q_r) = (1-p_1)\,(-c_n+p_1) + \lambda\left(\rho\,(1-p_1) - \frac{(-1+\beta)\,\Delta\,(\delta\,p_n-p_r)}{(-1+\delta)\,\delta} + \beta\,\Delta\,(-1+p_r)\right) - \frac{(-1+\beta)\,\Delta\,(-c_n+p_n)\,(-1+\delta+p_n-p_r)}{-1+\delta} + \left(\frac{(-1+\beta)\,\Delta\,(\delta\,p_n-p_r)}{(-1+\delta)\,\delta} - \beta\,\Delta\,(-1+p_r)\right)(-c_r+p_r)\,.$

The first order conditions are: $\frac{\partial L}{\partial p_n} = -\left(\frac{(-1+\beta)\,\Delta\,(-1-c_n+c_r+\delta+\lambda+2\,p_n-2\,p_r)}{-1+\delta}\right) = 0,$

$\frac{\partial L}{\partial p_r} = \frac{\Delta\left(c_n\,\delta + c_r\,(-1+\beta\,(1+(-1+\delta)\,\delta)) - \lambda - 2\,\delta\,p_n + \beta\left(\lambda+\delta^2\,(1+\lambda-2\,p_r) - \delta\,(1+c_n+\lambda-2\,p_n-2\,p_r) - 2\,p_r\right) + 2\,p_r\right)}{(-1+\delta)\,\delta} = 0,$

$\frac{\partial L}{\partial p_1} = 1 + c_n - \rho\,\lambda - 2\,p_1 = 0$ and $\frac{\partial L}{\partial \lambda} = \rho - \rho\,p_1 - \frac{(-1+\beta)\,\Delta\,(\delta\,p_n-p_r)}{(-1+\delta)\,\delta} + \beta\,\Delta\,(-1+p_r) = 0.$

Concavity of the objective follows from Proposition 1 since the only constraint is convex. Thus, the solution to the first order conditions give the unique maximizer to the problem. At optimality:

$p_1^* = \frac{2\,\rho^2\,(-1+\delta)\,\delta + (1+c_n)\,(-1+\beta\,(1+(-1+\delta)\,\delta))\,\Delta + \rho\,(\delta\,(c_n-\beta\,(-1+c_n+\delta)) + c_r\,(-1+\beta\,(1+(-1+\delta)\,\delta)))\,\Delta}{2\,(-1+\beta)\,\Delta + 2\,(-1+\delta)\,\delta\,(\rho^2+\beta\,\Delta)},$

$p_r^* = \frac{\rho\,(-1+\delta)\,\delta\,(-1+c_n+\beta\,(-1+c_n+\rho\,c_r)\,(-1+\delta) + \rho\,(c_r+\delta)) + \delta\left(-1-c_n+\beta\left(-(c_n\,(-2+\delta)) + \delta^2 + \beta\,(-1+\delta)\,(-1+c_n+\delta)\right)\right)\Delta}{2\,(1+\beta\,(-1+\delta))\,((-1+\beta)\,\Delta + (-1+\delta)\,\delta\,(\rho^2+\beta\,\Delta))},$

$p_n^* = \frac{1+c_n+\beta\,(1+c_n-\delta)\,(-1+\delta)}{2+2\,\beta\,(-1+\delta)}$, and $\lambda^* = \frac{\rho\,(-1+c_n)\,(-1+\delta)\,\delta - (-1+\beta)\,c_r\,\Delta - \delta\,(c_n-\beta\,(-1+c_n+c_r+\delta-c_r\,\delta))\,\Delta}{(-1+\beta)\,\Delta + (-1+\delta)\,\delta\,(\rho^2+\beta\,\Delta)}.$

Finally, at optimality:

$$q_1^* = \frac{\left((1-\beta)\,(-1+c_n+\rho\,c_r)+(\beta\,(1+\rho)\,(-1+c_n)-\rho\,c_n+\beta\,\rho\,c_r)\,\delta-\beta\,(-1+c_n+\rho\,(-1+c_r))\,\delta^2\right)\,\Delta}{2\,(-1+\beta)\,\Delta+2\,(-1+\delta)\,\delta\,(\rho^2+\beta\,\Delta)}, \tag{27}$$

$$q_n^* = \frac{(-1+\beta)\,\Delta\,(-(\rho\,\delta\,(1-c_n+\rho\,(-1+c_n-c_r+\delta)))-(-1+c_n+\beta\,(-1+\delta)\,(-1+c_n+\delta))\,\Delta)}{2\,(-1+\beta)\,\Delta+2\,(-1+\delta)\,\delta\,(\rho^2+\beta\,\Delta)}, \tag{28}$$

$$q_r^* = \frac{\rho\,\left((1-\beta)\,(-1+c_n+\rho\,c_r)+(\beta\,(1+\rho)\,(-1+c_n)-\rho\,c_n+\beta\,\rho\,c_r)\,\delta-\beta\,(-1+c_n+\rho\,(-1+c_r))\,\delta^2\right)\,\Delta}{2\,(-1+\beta)\,\Delta+2\,(-1+\delta)\,\delta\,(\rho^2+\beta\,\Delta)}. \tag{29}$$

Let us define the manufacturer profit at this solution as Π_R^l.

When $p_r \geq \delta p_n$, the Lagrangean can be written as:

$L = \Pi_R + \lambda(q_1\rho - q_r) = (1-p_1)\,(-c_n+p_1) + (-1+\beta)\,\Delta\,(-1+p_n)\,(-c_n+p_n) + \lambda\,(\rho\,(1-p_1)+\beta\,\Delta\,(-1+p_r)) - \beta\,\Delta\,(-1+p_r)\,(-c_r+p_r)$

The first order conditions are:

$\frac{\partial L}{\partial p_n} = -((-1+\beta)\,\Delta\,(1+c_n-2\,p_n)) = 0$, $\frac{\partial L}{\partial p_r} = \beta\,\Delta\,(1+c_r+\lambda-2\,p_r) = 0$, $\frac{\partial L}{\partial p_1} = 1+c_n-\rho\,\lambda-2\,p_1 = 0$ and $\frac{\partial L}{\partial \lambda} = \rho-\rho\,p_1+\beta\,\Delta\,(-1+p_r) = 0$

Concavity of the objective follows from Proposition 1 since the only constraint is convex. Thus, the solution to the first order conditions give the unique maximizer to the problem.

The optimal prices are given as:

$p_1^* = \frac{-\left(-2\,\rho^2-\beta\,\Delta+\beta\,\rho\,\Delta-\beta\,c_n\,\Delta-\beta\,\rho\,c_r\,\Delta\right)}{2\,(\rho^2+\beta\,\Delta)}$, $p_n^* = \frac{1+c_n}{2}$, $p_r^* = \frac{-\left(\rho-\rho^2-\rho\,c_n-\rho^2\,c_r-2\,\beta\,\Delta\right)}{2\,(\rho^2+\beta\,\Delta)}$ and $\lambda^* = \frac{\rho\,(-1+c_n)-\beta\,(-1+c_r)\,\Delta}{\rho^2+\beta\,\Delta}$

Finally the optimal sales quantities are given by:

$$q_1^* = \frac{-\,(\beta\,(-1+c_n+\rho\,(-1+c_r))\,\Delta)}{2\,(\rho^2+\beta\,\Delta)}, \tag{30}$$

$$q_n^* = \frac{(-1+\beta)\,(-1+c_n)\,\Delta}{2}, \tag{31}$$

$$q_r^* = \frac{-\,(\beta\,\rho\,(-1+c_n+\rho\,(-1+c_r))\,\Delta)}{2\,(\rho^2+\beta\,\Delta)}. \tag{32}$$

Let us define the manufacturer profit at this solution as Π_R^h.

To show that there exists a β' we need to solve the inequality $\Pi_R^h \leq \Pi_R^l$. The solution to this problem requires finding the roots of a fifth-order polynomial which cannot be analytically solved. Without loss of generality however, we can show that the roots of this polynomial exists for any numerical setting. Assume that $\Delta = 1$, $\delta = 0.5$, $c_n = 0.5$, and $c_r = 0.2$. Figure 12 illustrates the β' for this numerical setting.

■

Corollary 8 *Assume $\beta = 0$. The monopolist sells both new and remanufactured products to the primary segment ($q_r^P * \geq 0$, $q_n^* \geq 0$), if $\frac{\rho\,\delta\,(1-c_n+\rho\,(-1+c_n+\delta))+(-1+c_n)\,\Delta}{\rho^2\,\delta} = c_r' \leq c_r \leq \bar{c}_r' = \frac{1-c_n}{\rho} + c_n\,\delta$. Furthermore, c_r' is smaller than the unconstrained threshold, i.e. $c_r' \leq -1 + c_n + \delta$.*

Proof of Corollary 8

The lower bound cost threshold can be calculated by replacing the optimal prices in the equation for q_r^P and solving for $q_r^P = 0$. With straightforward algebra, one can show that $\Delta > \rho\delta$ for the constrained case, since by assumption $c_r \geq -1 + c_n + \delta$. The difference between the constrained threshold and the unconstrained threshold is $c_r' - (-1 + cn + \delta) = \frac{(1-c_n)\,(\rho\,\delta-\Delta)}{\rho^2\,\delta}$, which is negative, because when the constraint is binding $\Delta > \rho\delta$ holds.

Similarly, when $\beta > 0$, the upper bound cost threshold can be calculated as:

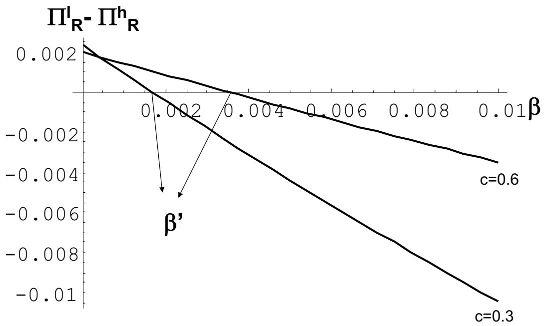

Figure 12: β': When low pricing dominates.

$\bar{c}_r|\{\beta > 0\} = \frac{\rho^2\,(c_n+\beta\,(1+c_n-\delta)\,(-1+\delta))\,\delta+\rho\,(-1+c_n)\,(-1+\beta-\beta\,\delta)+\beta\,(-2+\delta+c_n\,\delta+\beta\,(-1+\delta)\,(-2+(1+c_n-\delta)\,\delta))\,\Delta}{\rho^2\,(1+\beta\,(-1+\delta))}$. Replacing $\beta = 0$ gives the formula in the corollary. ■

According to Corollary 8, the cost threshold to sell remanufactured products in the primary market ($\bar{c}_r$) is larger than the unconstrained threshold δc_n. This is basically because, when the remanufacturable supply is limited, the price for the remanufactured product will be higher to reduce the demand. When the price is higher, the required profit margin will be attained at higher cost levels. Corollary 8 also states that under fast diffusion, q_n is positive for even lower c_r values. Basically, even though the cost advantage of remanufactured products is higher, new product sales are still valuable since the remanufacturable supply is lower.

Corollary 9 *The cost threshold to sell remanufactured products is lower when there exists green segments, i.e.* $\bar{c}'_{r|\beta=0} \geq \bar{c}'_{r|\beta>0}$.

Proof of Corollary 9

$\bar{c}_r|\{\beta > 0\} - \bar{c}_r|\{\beta = 0\} = \frac{\beta\left(-\left(\rho^2\,(-1+\delta)^2\,\delta\right)+(-2+\delta+c_n\,\delta+\beta\,(-1+\delta)\,(-2+(1+c_n-\delta)\,\delta))\,\Delta\right)}{\rho^2\,(1+\beta\,(-1+\delta))}$. Note that this difference is negative when $\beta > \frac{-2+\left(1+c_n-\rho^2\,(-1+\delta)^2\right)\delta}{(-1+\delta)\,(2-(1+c_n)\,\delta+\delta^2)}$. Since $\frac{-2+\left(1+c_n-\rho^2\,(-1+\delta)^2\right)\delta}{(-1+\delta)\,(2-(1+c_n)\,\delta+\delta^2)} < 0$, this condition always holds. ■

Proof of Corollary 3

Similar to corollary 1 let us consider the suboptimal strategy where the profit is given by:

$$\Pi^h_R = \frac{\Delta\left(\rho^2\,(-1+c_n)^2 - \beta^2\,(-1+c_n)^2\,\Delta + \beta\left(2\,\rho\,(-1+c_n)\,(-1+c_r) - \rho^2\,(c_n-c_r)\,(-2+c_n+c_r) + (-1+c_n)^2\,(1+\Delta)\right)\right)}{4\,(\rho^2+\beta\,\Delta)}$$

$\Rightarrow \Pi_R - \Pi_{NR} \geq 0 \Leftarrow \Delta\beta \leq \tau = \frac{(-1+c_n)\,(-1+c_n+2\,\rho\,(-1+c_r))}{(-1+c_r)^2}$. Since we know that there exists a β' above which Π^h_R is the optimal profit, a necessary condition for profitability is $\Delta\beta < \tau$. Note that in corollary 1, the only condition required for profitable remanufacturing was that $c_n > c_r$. However, when there is limited supply, an additional condition is required, i.e. $\beta \leq \tau$.

τ is decreasing in c_n since $\frac{\partial\tau}{\partial c_n} = \frac{2\,(-1+c_n+c\,(-1+c_r))}{(-1+c_r)^2} \leq 0$. τ is increasing in c_r since $\frac{\partial\tau}{\partial c_r} = \frac{-2\,(-1+c_n)\,(-1+c_n+\rho\,(-1+c_r))}{(-1+c_r)^3\,\Delta} \geq 0$. ■

Proof of Proposition 3

Let us first consider the case that $p_c/\alpha \geq p_r/\delta$ and denote the optimal profit at this solution by Π^l_R:

Manufacturer

$\Pi_R = \frac{(-1+\beta)\,\Delta\,(c_n-p_n)\,(-1+\alpha-p_c+p_n)}{-1+\alpha} + (-c_r+p_r)\left(-(\beta\,\Delta\,(-1+p_r)) + \frac{(-1+\beta)\,\Delta\,(-(\delta\,p_c)+\alpha\,p_r)}{(\alpha-\delta)\,\delta}\right)$

The first order conditions are:

$\frac{\partial\Pi_R}{\partial p_n} = -\left(\frac{(-1+\beta)\,\Delta\,(-1+\alpha-c_n-p_c+2\,p_n)}{-1+\alpha}\right) = 0,$

$\frac{\partial\Pi_R}{\partial p_r} = \frac{\Delta\,(\alpha\,(c_r\,(1+\beta\,(-1+\delta))+\beta\,\delta-2\,(1+\beta\,(-1+\delta))\,p_r)-\delta\,(-p_c+\beta\,(\delta+c_r\,\delta+p_c-2\,\delta\,p_r)))}{(\alpha-\delta)\,\delta} = 0$

Competitor

$\Pi_C = (1-\beta)\ \Delta\ (-c_c + p_c)\ \left(\frac{p_c - p_n}{-1+\alpha} + \frac{p_c - p_r}{-\alpha+\delta}\right)$

The first order condition is: $\frac{\partial \Pi_C}{\partial p_c} = \frac{(-1+\beta)\,\Delta\,(c_c - c_c\,\delta + 2\,(-1+\delta)\,p_c + \alpha\,p_n - \delta\,p_n + p_r - \alpha\,p_r)}{(-1+\alpha)\,(\alpha-\delta)} = 0.$

It is easy to see that the best response functions are linear in the other player's prices. Thus, there exists a unique equilibrium given by the prices:

$$p_n^* = \frac{\alpha\ \left((-1+\beta)\ (4+2\,c_c+4\,c_n+c_r) - (-2-2\,c_c-3\,c_n+\beta\ (7+4\,c_c 7\,c_n+c_r))\ \delta + \beta\ (-1+2\,c_c+4\,c_n-c_r)\ \delta^2 + 4\,\beta\,\delta^3\right)}{2\ (\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta))}$$

$$+\frac{\alpha^2\ (4+c_r+\beta\ (4+c_r-4\,\delta)\ (-1+\delta) - 3\,\delta) + \delta\ \left(1+c_n+\beta\ \left(-1-c_n+(5+2\,c_c+4\,c_n+c_r)\ \delta - 2\ (2+c_c+2\,c_n)\ \delta^2\right)\right)}{2\ (\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta))}, \tag{33}$$

$$p_r = \frac{\alpha\ (\delta\ (-1-c_n-\delta+\beta\ (-3+c_n+3\,\delta)) + c_r\ (-4+3\,\delta+\beta\ (4+\delta\ (-7+2\,\delta))))}{2\ (\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta))}$$

$$+\frac{\delta\ (2\,c_c\ (-1+\beta+\delta-\beta\,\delta) + \delta\ (1+c_n-\beta\ (-3+c_n-4\,c_r+3\ (1+c_r)\ \delta))) + \alpha^2\ (c_r\ (1+\beta\ (-1+\delta)) + \delta) +}{2\ (\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta))}, \tag{34}$$

$$p_c = \frac{\alpha\ \left(1+\alpha^2\,\beta+2\,c_c+c_n-\beta\ (2+4\,c_c+c_n+c_r) + \alpha\ (-1+\beta\ (1-c_n+c_r))\right)\ \delta}{\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta)}$$

$$+\frac{-\,(\alpha\ (-1+\beta)\ (-2\,c_c-c_r+\alpha\ (-1+\alpha-c_n+c_r))) + \beta\ (1+2\,c_c+\alpha\ (1-2\,\alpha+2\,c_c+2\,c_n-c_r)+c_r)\ \delta^2 + \beta\ (-1+\alpha-2\,c_c-c_n)\ \delta^3}{\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta)}. \tag{35}$$

Replacing the equilibrium prices in the demand equations, we obtain:

$$q_n^*/(\Delta(1-\beta)) = \frac{\left(-1+\beta+c_n-\beta\,c_n+\alpha^2\ (3+\beta\ (-8+2\,c_n-c_r)) + \alpha\ (-2-2\,c_c+c_n+\beta\ (7+4\,c_c-5\,c_n+c_r))\right)\ \delta}{2\ (-1+\alpha)\ (\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta))}$$

$$+\frac{-\,(\alpha\ (-1+\beta)\ (4+2\,c_c-4\,c_n+\alpha\ (-4+2\,c_n-c_r)+c_r)) + \beta\ (-5-2\,c_c+4\,c_n-c_r+\alpha\ (1+4\,\alpha-2\,c_c+c_r))\ \delta^2 - 2\,\beta\ (-2+2\,\alpha-c_c+c_n)\ \delta^3}{2\ (-1+\alpha)\ (\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta))},$$

$$q_r^* = \frac{\left((-1-2\,c_c-c_n+\beta\ (-3+2\,c_c+c_n+4\,c_r) + \alpha\ (1+\beta\ (-3+2\,c_r)))\ \delta^2 - 3\,\beta\ (-1+c_r)\ \delta^3\right)\ \left(\beta\,\delta^2+\alpha\ (-1+\beta-\beta\,\delta)\right)\ \Delta}{2\ (\alpha-\delta)\ \delta\ (\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta))}$$

$$+\frac{\left((4-\alpha)\ \alpha\ (-1+\beta)\ c_r + \left(-2\ (-1+\beta)\ (c_c+c_r) + \alpha^2\ (-1+\beta\,c_r) + \alpha\ (1+c_n+c_r-\beta\ (-3+c_n+5\,c_r))\right)\ \delta\right)\ \left(\beta\,\delta^2+\alpha\ (-1+\beta-\beta\,\delta)\right)\ \Delta}{2\ (\alpha-\delta)\ \delta\ (\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta))},$$

$$q_c^*/(\Delta(1-\beta)(1-\delta)) = \frac{\alpha^3\ (1+\beta\ (-1+\delta)) - \alpha^2\ (1+c_c+c_n-c_r+\delta+\beta\ (-1+\delta)\ (1+c_c+c_n-c_r+2\,\delta)) +}{(1-\alpha)\ (\alpha-\delta)\ (\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta))}$$

$$+\frac{\delta\ \left(c_c\ \left(-1+\beta\,(-1+\delta)^2\right) + \beta\,\delta\ (1+c_r-(1+c_n)\ \delta)\right) + \alpha\ \left(-c_r+2\,c_c\ (1+\beta\ (-1+\delta)) + \delta + c_n\,\delta + \beta\ \left(c_r-(2+c_n+c_r)\ \delta + (1+2\,c_n-c_r)\ \delta^2 + \delta^3\right)\right)}{(1-\alpha)\ (\alpha-\delta)\ (\alpha^2\ (1+\beta\ (-1+\delta)) + 2\,\alpha\ (1+\beta\ (-1+\delta))\ (-2+\delta) + \delta + \beta\,\delta\ (-1+(4-3\,\delta)\ \delta))}.$$

Now, let us consider the case that $p_c/\alpha \leq p_r/\delta$ and denote the optimal profit at this solution by Π_R^h:

Manufacturer

$\Pi_R = \frac{(-1+\beta)\,\Delta\,(c_n-p_n)\,(-1+\alpha-p_c+p_n)}{-1+\alpha} - \beta\,\Delta\ (-1+p_r)\ (-c_r+p_r)$

The first order conditions are: $\frac{\partial \Pi_R}{\partial p_r} = \beta\,\Delta\ (1+c_r-2\,p_r) = 0$ and $\frac{\partial \Pi_R}{\partial p_n} = \frac{(1-\beta)\,\Delta\,(-1+\alpha-c_n-p_c+2\,p_n)}{-1+\alpha} = 0.$

Competitor

$\Pi_C = \frac{(-1+\beta)\,\Delta\,(c_c-p_c)\,(p_c-\alpha\,p_n)}{(-1+\alpha)\,\alpha}$

The first order condition is: $\frac{\partial \Pi_C}{\partial p_c} = \frac{(-1+\beta)\,\Delta\,(c_c-2\,p_c+\alpha\,p_n)}{(-1+\alpha)\,\alpha} = 0.$

It is easy to see that the best response functions are linear in the other player's prices. Thus, there exists a unique equilibrium given by the prices $p_r = \frac{1+c_r}{2}$, $p_n = \frac{2-2\,\alpha+c_c+2\,c_n}{4-\alpha}$ and $p_c = \frac{\alpha-\alpha^2+2\,c_c+\alpha\,c_n}{4-\alpha}$. Replacing these prices in the demand equations, the equilibrium quantities are obtained as $q_n^* = -\left(\frac{(-1+\beta)\,(2+c_c+\alpha\,(-2+c_n)-2\,c_n)\,\Delta}{4-5\,\alpha+\alpha^2}\right)$ and $q_r^* = \Delta\beta\frac{1+c_r}{2}$.

It is easy to see that the first solution is valid when $q_r^P \geq 0$ and $\Pi_R^l \geq \Pi_R^h$. Otherwise the second solution is the equilibrium outcome. The exact characterization of β'' is very hard but we show the existence of such β'' via Figure 13 assuming that $\Delta = 1,\ \delta = 0.5,\ c_n = 0.5$, and $c_r = 0.1$.

■

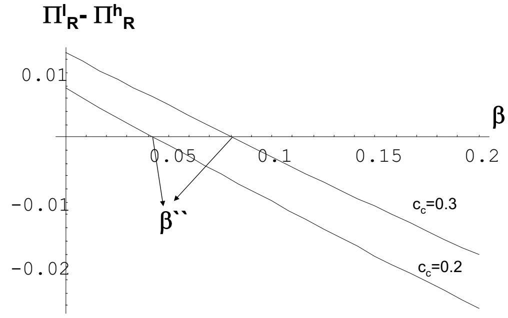

Figure 13: β'': When low pricing dominates.

Corollary 10 *Assume* $\beta = 0$*. If* $c_r \le \bar{c}_r = \frac{c_n\,\delta\,(-\alpha+\delta)}{2\,\delta+\alpha\,(-4+\alpha+\delta)} + \frac{\delta\left(\alpha^2+2\,c_c\,(-1+\delta)+\delta-\alpha\,(1+\delta)\right)}{2\,\delta+\alpha\,(-4+\alpha+\delta)}$*, then the manufacturer (M) serves the primary market with remanufactured products, i.e.* $q_r \ge 0$*.*

Proof of Corollary 10

$q_r|\{\beta = 0\} = \frac{-\left(\alpha\left((-4+\alpha)\,\alpha\,c_r+(2\,(c_c+c_r)+\alpha\,(1-\alpha+c_n+c_r))\,\delta+(-1+\alpha-2\,c_c-c_n)\,\delta^2\right)\Delta\right)}{2\,(\alpha-\delta)\,\delta\,(\delta+\alpha\,(-4+\alpha+2\,\delta))}$.

Note that $q_r|\{\beta = 0\} \ge 0$ if $c_r|\{\beta = 0\} \le \bar{c}_r = \frac{c_n\,\delta\,(-\alpha+\delta)}{2\,\delta+\alpha\,(-4+\alpha+\delta)} + \frac{\delta\left(\alpha^2+2\,c_c\,(-1+\delta)+\delta-\alpha\,(1+\delta)\right)}{2\,\delta+\alpha\,(-4+\alpha+\delta)}$. Now let us compare this bound with the monopoly case. One can show that $\bar{c}_r \ge \delta c_n$ if $c_c \ge \frac{(1-\alpha)\,(\alpha-\delta)}{2\,(-1+\delta)} + \frac{c_n\,(\alpha\,(3-\alpha-\delta)+\delta)}{2\,(1-\delta)}$. This condition is always satisfied if $c_c \ge \alpha c_n$, since $\frac{(1-\alpha)\,(\alpha-\delta)}{2\,(-1+\delta)} \le 0$ and $\frac{c_n\,(\alpha\,(3-\alpha-\delta)+\delta)}{2\,(1-\delta)} \le \alpha c_n$. Figure 14 demonstrates the idea. ■

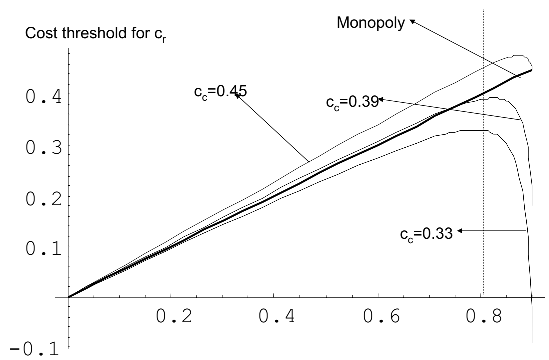

Figure 14: Remanufacturing cost (c_r) thresholds for $c_n = 0.5$, $\alpha = 0.8$ and $\beta = 0$.

Using Corollary 10, one can show that, when the competitor's cost is sufficiently high the remanufacturing cost threshold under competition is higher than the monopoly threshold, i.e. ($c_c \ge \frac{(1-\alpha)\,(\alpha-\delta)}{2\,(-1+\delta)} + \frac{c_n\,(\alpha\,(3-\alpha-\delta)+\delta)}{2\,(1-\delta)} \Rightarrow \bar{c}_r \ge \delta c_n$). Furthermore, a sufficient condition for $\bar{c}_r \ge \delta c_n$ is that $c_c \ge \alpha c_n$. Therefore, Corollary 10 states that the cost advantage required to serve the primary segment with remanufactured products is less restrictive if the competitor's manufacturing cost is high. Using our experience from the monopoly case, $\bar{c}_r$ would be lower for positive β, since a higher green segment ratio allows for higher p_r which allows for positive remanufactured sales at higher remanufacturing costs.

Corollary 11 *Assume $\beta > \beta''$. Extra profit from remanufacturing ($\Pi_R - \Pi_{NR}$) is decreasing in the competitor's manufacturing cost (c_c) if the competitor has a sufficiently high brand power, i.e., $\alpha \geq \frac{2(1-c_n)+c_c}{2-c_n}$.*

Proof of Corollary 11

In this case the manufacturer's total profit will be given by $\Pi_R^h = \frac{(-1+\beta)\,(2+c_c+\alpha\,(-2+c_n)-2\,c_n)^2\,\Delta}{(-4+\alpha)^2\,(-1+\alpha)} + \frac{\beta\,(-1+c_r)^2\,\Delta}{4}$. Note that this case can only be true if: $p_r/\delta \geq p_c/\alpha \Rightarrow c_r \geq -1 + \frac{2\,\alpha\,(1-\alpha+3\,c_n)\,\delta}{4-\alpha}$. Otherwise, the solution is not valid, since some of the green customers may be buying the remanufactured product. If we compare the remanufacturing scenario with the no remanufacturing scenario:

$$\Pi_R - \Pi_{NR} = \frac{(-1+\beta)\ (2+c_c+\alpha\ (-2+c_n)-2\,c_n)^2\,\Delta}{(-4+\alpha)^2\ (-1+\alpha)} + \frac{\beta\,(-1+c_r)^2\,\Delta}{4} + \frac{(2+c_c+\alpha\ (-2+c_n)-2\,c_n)^2\,\Delta}{(-4+\alpha)^2\ (-1+\alpha)}.$$

One can show that $\frac{\partial(\Pi_R-\Pi_{NR})}{\partial c_c} = \frac{2\,\beta\,(2+c_c+\alpha\,(-2+c_n)-2\,c_n)\,\Delta}{(-4+\alpha)^2\,(-1+\alpha)}$ is negative if $\alpha \geq \frac{2(1-c_n)+c_c}{2-c_n}$ and positive otherwise.

■

Proof of Corollary 4

When $c_c = \alpha c_n$, in order for remanufacturing to be profitable, it has to be true that $\Pi_R \geq \Pi_{NR}$. A sufficient condition for this is that $\Pi_R^h \geq \Pi_{NR}$.

With straightforward algebra this is equivalent to $c_r \leq c_r'' = 1 - \frac{4\sqrt{1-\alpha}\,(-1+c_n)}{-4+\alpha}$.

Or, equivalently, it has to be true that $\alpha \geq \frac{-4\left(1+2\,(-2+c_n)\,c_n-(-2+c_r)\,c_r+\sqrt{(-1+c_n)^2\,(1+4\,(-2+c_n)\,c_n-3\,(-2+c_r)\,c_r)}\right)}{(-1+c_r)^2}$. It is easy to show that $c_n \leq c_r''$. Thus, when $c_r \leq c_r''$, $\Pi_R \geq \Pi_{NR}$ for any $\beta > 0$, which means that $\Pi_R = \max(\Pi_R^h, \Pi_R^l)$ is always larger than Π_{NR}. ■

Proof of Proposition 4

The remanufacturer prices for the green segment only at $p_r^L = \frac{1+c_r}{2}$ and sells $q_r^L = \Delta\beta\frac{1-c_r}{2}$. The manufacturer prices at $p_n = \frac{1+c_n}{2}$ and sells $q_n = \Delta(1-\beta)\frac{1-c_n}{2}$ with a profit of $\Pi_{NR}^L = \Delta(1-\beta)\frac{(1-c_n)^2}{4}$. It is easy to see that the high pricing strategy of the monopolist $\Pi_R^h = \Delta(1-\beta)\frac{(1-c_n)^2}{4} + \Delta\beta\frac{(1-c_r)^2}{4}$ is always greater than this profit.

■

Proof of Lemma 2

Let us consider first the case where $p_c/\alpha \geq p_r/\delta$. The manufacturer is constrained when $q_r^* > q_1^*\rho$, where $q_1^* = \frac{1+c_n}{2}$ and q_r^* is given by Proposition 2. Solving for $q_r^* = q_1^*\rho$ gives the boundary condition on Δ.

Now, let us consider the case where $p_c/\alpha \leq p_r/\delta$. The proof for this case follows the proof of Lemma 1. ■

Proof of Proposition 5

Let us consider first the case where $p_c/\alpha \geq p_r/\delta$:

Manufacturer

The Lagrangean can be written as:

$L = \Pi_R + \lambda(q_1\rho - q_r) = (1-p_1)\ (-c_n+p_1) - \frac{(-1+\beta)\,\Delta\,(-c_n+p_n)\,(-1+\alpha-p_c+p_n)}{-1+\alpha} + \lambda\left(\rho\ (1-p_1) + \beta\,\Delta\ (-1+p_r) - \frac{(-1+\beta)\,\Delta\,(-(\delta\,p_c)+\alpha\,p_r)}{(\alpha-\delta)\,\delta}\right) + (-c_r+p_r)\left(-(\beta\,\Delta\ (-1+p_r)) + \frac{(-1+\beta)\,\Delta\,(-(\delta\,p_c)+\alpha\,p_r)}{(\alpha-\delta)\,\delta}\right)$.

The first order conditions for the manufacturer are:

$\frac{\partial L}{\partial p_n} = -\left(\frac{(-1+\beta)\,\Delta\,(-c_n+p_n)}{-1+\alpha}\right) - \frac{(-1+\beta)\,\Delta\,(-1+\alpha-p_c+p_n)}{-1+\alpha} = 0$,

$\frac{\partial L}{\partial p_r} = \left(\beta\,\Delta - \frac{\alpha\,(-1+\beta)\,\Delta}{(\alpha-\delta)\,\delta}\right)\lambda - \beta\,\Delta\ (-1+p_r) + \left(-(\beta\,\Delta) + \frac{\alpha\,(-1+\beta)\,\Delta}{(\alpha-\delta)\,\delta}\right)(-c_r+p_r) + \frac{(-1+\beta)\,\Delta\,(-(\delta\,p_c)+\alpha\,p_r)}{(\alpha-\delta)\,\delta} = 0$, $\frac{\partial L}{\partial p_1} = 1 + c_n - \rho\,\lambda - 2\,p_1 = 0$ and $\frac{\partial L}{\partial \lambda} = \rho\ (1-p_1) + \beta\,\Delta\ (-1+p_r) - \frac{(-1+\beta)\,\Delta\,(-(\delta\,p_c)+\alpha\,p_r)}{(\alpha-\delta)\,\delta} = 0$.

Competitor:

$\Pi_C = (1-\beta)\ \Delta\ (-c_c+p_c)\left(\frac{p_c-p_n}{-1+\alpha} + \frac{p_c-p_r}{-\alpha+\delta}\right)$

The first order condition for the competitor is:

$\frac{\partial\Pi_C}{\partial p_c} = (1-\beta)\left(\frac{1}{-1+\alpha} + \frac{1}{-\alpha+\delta}\right)\Delta\ (-c_c+p_c) + (1-\beta)\ \Delta\left(\frac{p_c-p_n}{-1+\alpha} + \frac{p_c-p_r}{-\alpha+\delta}\right)$.

For an investigation of uniqueness of the equilibrium, Facchinei and Pang (2000) consider the problem of the form

$$\begin{aligned} \max \quad & \pi^i(p_i, p_{-i}) \\ \text{s.t.} \quad & p_i \in X_i \end{aligned}$$

and show that, when X_i is a nonempty convex set, π_i is continuously differentiable and concave for every p_{-i}, there exists a unique Nash Equilibrium if $\bigtriangledown_p \pi(p)$ is monotone $\forall p \in X$.

By checking the Hessian of the objective functions, it is easy to see that the objectives for both the manufacturer and the competitor are strictly concave. The feasible set for the manufacturer is linear, therefore convex. Moreover, the derivatives of the objective functions are linear in prices, therefore strictly monotone. Therefore, the Nash Equilibrium is unique.

We do not provide the equilibrium outcome in closed form since it is too complex. But it is relatively easy to calculate the equilibrium outcome for any numerical setting.

Now, let us consider the case where $p_c/\alpha \leq p_r/\delta$.

Manufacturer

The Lagrangean can be written as:

$L = \Pi_R + \lambda(q_1\rho - q_r) = -\left((-1+p_1)(-c_n+p_1)\right) + \frac{(-1+\beta)\,\Delta\,(c_n-p_n)\,(-1+\alpha-p_c+p_n)}{-1+\alpha} - \lambda\left(\rho\,(-1+p_1) - \beta\,\Delta\,(-1+p_r)\right) - \beta\,\Delta\,(-1+p_r)\,(-c_r+p_r)$.

The first order conditions for the manufacturer are:

$\frac{\partial L}{\partial p_r} = \beta\,\Delta\,(1+c_r+\lambda-2\,p_r) = 0$, $\frac{\partial L}{\partial p_n} = -\left(\frac{(-1+\beta)\,\Delta\,(-1+\alpha-c_n-p_c+2\,p_n)}{-1+\alpha}\right) = 0$, $\frac{\partial L}{\partial p_1} = 1+c_n-\rho\,\lambda-2\,p_1 = 0$ and $\frac{\partial L}{\partial \lambda} = \rho - \rho\,p_1 + \beta\,\Delta\,(-1+p_r) = 0$.

Competitor:

$\Pi_C = \frac{(-1+\beta)\,\Delta\,(-c_c+p_c)\,(-p_c+\alpha\,p_n)}{(-1+\alpha)\,\alpha}$

The first order condition for the competitor is:

$\frac{\partial \Pi_C}{\partial p_c} = \frac{(-1+\beta)\,\Delta\,(c_c-2\,p_c+\alpha\,p_n)}{(-1+\alpha)\,\alpha} = 0.$

Uniqueness follows the argument in Proposition 5 since the objectives are concave, best response functions are linear and the constraint forms a convex set.

The equilibrium prices are given as:

$$p_1^* = \frac{2\,\rho^2 + \beta\,(1+c_n+\rho\,(-1+c_r))\,\Delta}{2\,(\rho^2+\beta\,\Delta)}, \tag{36}$$

$$p_n^* = \frac{2-2\,\alpha+c_c+2\,c_n}{4-\alpha}, \tag{37}$$

$$p_c^* = \frac{\alpha^2-2\,c_c-\alpha\,(1+c_n)}{-4+\alpha}, \tag{38}$$

$$p_r^* = \frac{\rho\,(-1+\rho+c_n+\rho\,c_r)+2\,\beta\,\Delta}{2\,(\rho^2+\beta\,\Delta)}. \tag{39}$$

The resulting equilibrium sales quantities are given by:

$$q_1^* = \frac{-\left(\beta\,\rho\,(-1+c_n+\rho\,(-1+c_r))\,\Delta\right)}{2\,(\rho^2+\beta\,\Delta)}, \tag{40}$$

$$q_n^* = -\left(\frac{(-1+\beta)\,(2+c_c+\alpha\,(-2+c_n)-2\,c_n)\,\Delta}{4-5\,\alpha+\alpha^2}\right), \tag{41}$$

$$q_c^* = \frac{(-1+\beta)\,\left(\alpha^2+2\,c_c-\alpha\,(1+c_c+c_n)\right)\,\Delta}{\alpha\,(4-5\,\alpha+\alpha^2)}, \tag{42}$$

$$q_r^* = \frac{-\left(\beta\,\rho\,(-1+c_n+\rho\,(-1+c_r))\,\Delta\right)}{2\,(\rho^2+\beta\,\Delta)}. \tag{43}$$

■

Proof of Corollary 5

Similar to Corollary 4, the profit Π_R^h at optimality is given by:

$$\frac{\Delta}{4}\left(\frac{4\,\beta\,(2+c_c+\alpha\,(-2+c_n)-2\,c_n)^2}{(-4+\alpha)^2\,(-1+\alpha)} + \frac{\beta\,(-1+c_n+\rho\,(-1+c_r))^2}{\rho^2+\beta\,\Delta}\right).$$

One can show that $\Pi_R - \Pi_{NR} \geq 0 \Leftarrow \Delta\beta \leq \tau'' =$

$$-\frac{-\left(\alpha^3\left(-1+c_n+\rho\,(-1+c_r)\right)^2\right)-\alpha^2\,(3-3\,c_n+\rho\,(-1+2\,c_n-3\,c_r))\,(3\,(-1+c_n)+\rho\,(-7+2\,c_n+3\,c_r))}{4\left(2+c_c+\alpha\,(-2+c_n)-2\,c_n\right)^2}$$

$$+\frac{-8\,\alpha\left(3\left(-1+c_n\right)^2+6\,\rho\,(-1+c_n)\,(-1+c_r)+\rho^2\,(-1+c_c\,(-2+c_n)-2\,(-3+c_n)\,c_n+3\,(-2+c_r)\,c_r)\right)}{4\left(2+c_c+\alpha\,(-2+c_n)-2\,c_n\right)^2}$$

$$+\frac{-4\,(2\,(-1+c_n)+\rho\,(c_c-2\,c_n+2\,c_r))\,(2-2\,c_n+\rho\,(c_c-2\,(-2+c_n+c_r)))}{4\left(2+c_c+\alpha\,(-2+c_n)-2\,c_n\right)^2}$$

Thus, a necessary condition for profitable remanufacturing is that $\Delta\beta < \tau''$, where τ'' is defined as above. ■

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