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Abstract

In order to explain cross-country differences in the effects of capital market liberalization, this paper proposes a model of international asset markets in which investors in different countries each face constraints on portfolio choice. The model demonstrates that liberalization, i.e. the lifting of constraints, can increase or decrease the liberalized stock market's volatility, depending on how severely the constraint was binding before being removed, and whether markets are fully or only partially liberalized. The same factors also determine whether a market's correlation with world markets increases or decreases, thus linking correlation effects to the magnitude of capital inflows post-liberalization.

Key words: Asset Pricing Theory; Portfolio Constraints; International Finance; Correlation; Volatility

JEL Classification: G11; G12; G15; G18

Recent decades have seen many countries steadily liberalize their capital markets, relaxing restrictions on capital flows into or out of their stock markets. Empirical studies show that the subsequent changes in stock market behavior post-liberalization have varied considerably across countries. For instance, Bekaert and Harvey (1997) find that approximately half the countries studied see their market's return correlation with the world increase post-liberalization, and that most of them experience a drop in return volatility. Miles (2002) on the other hand finds the opposite for the countries in his sample — a volatility increase after liberalization is more common than a decrease.

The concerted regional efforts in the 1990s to liberalize markets in Latin America and Asia have been studied in some detail. The impact of liberalization on the magnitude and composition of capital flows differed across the regions, potentially leading to the disparate effects on stock markets: Latin America saw its markets' volatility decrease, while market correlations with the world increased. Asia in contrast saw volatility increase, with mostly stable correlations.[1]

Existing theories of asset pricing have difficulties explaining these cross-country differences in liberalization experiences. This paper proposes a model that can accommodate such disparities by taking into account investors' circumstances at the time of liberalization. It shows that liberalization can indeed have opposite effects on stock market dynamics, depending on how severely constraints are binding and whether liberalization is partial or complete. It provides distinct testable implications, linking capital flows and the relative attractiveness of countries' investment opportunities at the time of liberalization, with subsequent changes in markets' volatility and correlation with world markets.

I model investors in different countries facing different constraints that limit their portfolio choices. The assumption of multiple constraints is new, most equilibrium models of portfolio constraints consider a single constraint on one investor. This assumption is critical to the analysis, as it allows us to study partial vs. full liberalization. When multiple constraints bind, the one that binds more severely will dominate the overall effect on these market characteristics. Accordingly, partially liberalizing by removing only one of the constraints can indeed have the opposite effect on the stock market's dynamics than removing all capital constraints at once. This interaction between

[1]See e.g. the study by Edwards, Biscarri, and de Gracia (2003).

how severely a constraint distorts portfolio choice, and whether liberalization remains incomplete, proves crucial for the main results of the paper, which are as follows.

First, liberalization can either reduce or increase the correlation between countries' stock market returns. Lifting a severely binding constraint triggers large capital flows into the newly liberalized market, leading to a drop in cross-country return correlations. Small capital inflows, following the removal of a mildly binding constraint, will coincide with an increase in correlation. The net flow of capital following the simultaneous removal of multiple constraints is dominated by the more severely binding constraint. This result implies that markets without capital restrictions exhibit more variation in cross-country correlations over time, due to more volatile capital flows into and out of these markets.

Second, eliminating a constraint can increase or decrease volatility, depending on whether another constraint remains in place, and how much it distorts portfolio choice. The less diversified countries' portfolio holdings are, the higher is stock market volatility, due to a feedback between goods markets and financial markets. Binding constraints distort investors' portfolios by restricting large investments into some assets, so wealth must be reallocated among the remaining, accessible, assets. When liberalization exacerbates portfolio differences across investors, volatility increases in response. Volatility decreases when liberalization leads to more similar portfolio holdings across countries' investors.

The composition of stock volatility is likewise affected by this distortion. A constrained investor aims to compensate for the unattainable asset in his portfolio by investing into a 'substitute' asset — a composite portfolio of the accessible assets that is highly correlated with the restricted asset. This reallocation exacerbates the contribution of extraneous sources of risk on stock volatility — beyond their impact on the fundamental economy — creating the impression of 'excess' volatility. This distortion of portfolios also leads to the dominance of local rather than 'worldwide' risk factors in stock markets that restrict foreign capital inflow, in line with empirical findings.

The model proposes a theory of stock market dynamics before and after capital market liberalization. It cannot as such speak to the welfare implications of liberalizing or restricting capital mobility. Indeed, markets with different degrees of incompleteness cannot generally be Pareto-ranked, as shown e.g. by Hart (1975). This holds true in the setting of this paper also.

The model's specifics are as follows. I consider a continuous-time pure-exchange economy with two countries, *home* and *foreign*, whose respective goods are produced by separate Lucas trees, subject to country-specific supply shocks. The investors consume both goods but have a preference for their respective domestic good. Each country has one stock (market) that is a claim on domestic output, and a zero-net supply, locally riskless bond.

I make three main assumptions. First, the two stock markets are claims to distinct goods. Investors' respective consumption preferences across goods gives a role to the wealth distribution across countries, feeding into equilibrium stock market valuation.

Second, both investors face portfolio constraints: The *home* investor faces a leverage constraint, i.e. the total amount he can invest in *home* and *foreign* stocks jointly is limited. The *foreign* investor's holdings abroad, in the *home* stock, are limited. The assumption that both investors are constrained contrasts with much of the literature, and its distinct implications are linked to the asymmetry of the constraints. Both affect the stock market in the *home* country directly — one functions as a capital inflow constraint, the other a type of capital outflow constraint — allowing us to study a market when there exists no unrestricted liquidity provider to take up extra supply.

The third assumption is that the two investors can disagree about the countries' expected economic growth rates. This dispersion in beliefs about fundamentals can potentially vary over time, thus determining how severely a constraint binds: a given constraint on an investor's portfolio position in a particular stock is said to bind more severely when this investor is more optimistic about that stock. The constraint then results in a more severe distortion from the portfolio he would ideally want to hold. This modeling device provides a tractable way of separating two concepts: how strict the imposed constraints are, and how much these restrictions affect investors' portfolios. For example, limiting an investor's position in a country's stock market to 10% of his portfolio is a very severe restriction if he is sufficiently optimistic and would like to hold, say, 40%, but is a less severe limitation if his desired stake is only 20%. This is conceptually different from loosening or tightening the constraint itself from 10% down to 5% or up to 15%.

Allowing beliefs to vary over time also provides a role for 'push' vs. 'pull' factors in capital flows. These terms have been used to describe the empirical finding that capital flows into a stock market from foreign investors are determined by perceived changes in investment opportunities not only in that market ('pull' factor), but also in the foreign investor's own local market ('push' factor). Allowing for differences in beliefs thus leads to more precise implications on the link between existing portfolio holdings, capital flows in response to liberalization, and the resulting changes to volatility and correlation.

I briefly illustrate the feedback mechanism driving the results. In this framework, stock returns depend on both output and terms-of-trade effects from goods markets. When an investor becomes wealthier, he will allocate part of this new wealth to current consumption. His innately stronger preference for his domestic good will increase relative demand for it, thus raising its relative price, boosting his domestic stock market. Both the initial shock to economic fundamentals that led to the wealth increase, as well as the second 'feedback effect' into the stock market via consumption choices, will be reflected in the volatility and correlation of stock market returns.

Differences in portfolio holdings determine how a shock to economic fundamentals translates into a change in relative wealth. An investor becomes *relatively* wealthier only if his portfolio exhibits higher realized returns than that of the other investor. A stock's realized returns will more strongly affect the wealth of the investor holding a larger position in it. Any disagreement about countries' expected growth rates, as well as binding constraints, will be reflected in investors' asset holdings. So the more the investors disagree, the more their relative wealth changes in response to an economic shock, generating stronger feedback effects.[2]

When constraints bind, investors cannot choose their portfolio according to their beliefs. A binding constraint on a long position in a stock means that the investor's holdings correspond to a less optimistic view of the stock than he actually has. Whether this distortion of the portfolio amplifies or dampens the feedback effect depends on how severely the constraint binds. Assume

[2]The most extreme example of this mechanism is a scenario of complete market separation: each investor holds exclusively his own domestic stock, therefore no risk insurance takes place and wealth effects are exacerbated compared to a scenario of perfect integration, where all hold the world market portfolio.

that the investor is strongly optimistic regarding the restricted stock. He would ideally choose to hold a larger portion of his wealth in the stock than the other investor. The constraint prevents the strong tilt, making the two investors' holdings *more similar*. Relative wealth is less affected by any fundamental shocks — the constraint dampens the feedback effect. Now assume instead that the constrained investor is the less optimistic of the two. In that case, he would choose to invest a smaller portion of his wealth into the stock regardless, but the constraint limits his holdings further, making the two portfolios *more different* than in absence of the constraint. Accordingly, the constraint exacerbates the wealth transfer from a fundamental shock, and thereby the feedback effect. When both investors' constraints bind, they mitigate one another's impact: The constraint that binds more severely will dominate in the aggregate feedback effect, shaping the market's reaction to the lifting of a constraint.

The paper proceeds as follows. The next section describes the related literature. Section II develops the model. Section III derives the equilibrium. Section IV discusses implications for stock market dynamics. Section V concludes. Proofs are in the appendix.

I Literature Review

The paper provides a unified theory that can help explain the mixed evidence found in cross-country studies on the effects of liberalization on stock markets. The analysis characterizes conditions under which market volatilities or correlations increase or decrease in response to liberalization, linking the effects of capital flows and stock market moments. Here I mention just some examples of the large empirical literature on liberalization effects. Despite the rescissions of various international capital market regulations over the last decades, Bekaert, Harvey, and Lumsdaine (2002) show that significant constraints on international investment remain.[3]

[3]The survey by Stulz (1995) as well as Karolyi and Stulz (2002) provide a good overview of the empirical findings on international integration and asset pricing. Bekaert, Harvey, and Lundblad (2003) provide a synthesis of empirical methods to allow a differentiated study of the different liberalization paths countries have followed.

Edison and Warnock (2003) confirm the results of Bekaert and Harvey (2000) that on average, correlations slightly increase, but some countries saw a significant decrease.[4] Harvey (1995), while not looking at liberalization events in particular, shows that emerging markets exhibit time-varying levels of correlation, and their risk is determined primarily locally. The model's implications are consistent with the latter finding, and suggests that time variation in moments can be exacerbated as markets are increasingly liberalized. Calvo, Leiderman, and Reinhart (1993) and Taylor and Sarno (1997) link the time-variation in cross-country correlations to more volatile capital flows into and out of these markets.

DeSantis and Imrohoroglu (1997) find similarly mixed evidence on liberalization's effects on markets' return volatilities. Comparing restricted and unrestricted stocks within one market, Bae, Chan, and Ng (2004) show that volatility is higher among assets that are more accessible to non-domestic investors, compared to assets whose ownership is restricted. While the authors explain the finding with investible stocks having a higher exposure to world risk, this explanation may be strained in some young and emerging stock markets where local risk tends to be higher than world market risk. Levine and Zervos (1998) also link higher volatility with stronger integration of markets.

Miles (2002) shows that investors' expectations regarding their own local investment opportunities play a large role in explaining capital flows into Emerging Markets, and Kim and Singal (2000) link capital inflows to increases in volatility.[5]

The event studies on the 'A-B share premium' in the Chinese market also illustrate the idea discussed in this paper, that constraints' impact on a market can vary depending on the presence of other constraints. This regulation separated the market for a stock, making Class A shares available to local investors, Class B shares to foreign investors. In China, local investors paid on average higher prices for a stock than foreign investors, which is in contrast to findings in other countries with such a dual market. Bailey, Chung, and Kang (1999) and Bailey and Jagtiani (1994) suggest

[4] Carrieri, Errunza, and Hogan (2007) likewise do not find consistent patterns of correlation changes.

[5] Brennan and Cao (1997) also deal with international capital flows, and look mainly at the impact of information advantage for local investors.

that this was due to local Chinese investors facing stringent restrictions on investing abroad, thus pushing up local prices for lack of investment alternatives.[6]

This paper provides a tractable and relatively flexible model that can accommodate various types of heterogeneity among investors, but it contributes to the theoretical literature on asset market imperfections along one dimension in particular. To date, the literature on portfolio constraints' impact on equilibrium market outcomes has generally assumed that only a subset of investors is constrained, while the marginal (and thus price-setting) investor is free to provide any amount of liquidity necessary to clear the market (often characterized by a single asset).[7] While these assumptions have provided interesting findings and tractability, the models have had little to say on how constraints interact, and thus how partial liberalization affects stock market dynamics, and how these effects can differ from full liberalization.

Pavlova and Rigobon (2008) show how portfolio constraints imposed on investors of large developed markets lead to high correlations among the developing stock markets, while lowering correlation between the large market and the cluster of developing markets. They maintain tractability by retaining the investors of the developing markets as the universally unconstrained price-setting investors. The analysis in this paper is similar in spirit, but emphasizes that this effect crucially relies on the investors in developed markets being the only constrained group, and all investors holding the same beliefs.[8]

[6]Local investors' information advantage (see, e.g. Fernald and Rogers (2002)) could potentially also explain a local price premium, but has more difficulties explaining the local price discount in other stock markets with similar systems of separate share classes.

[7]One exception is Basak and Croitoru (2000), who study mispricing in a derivative market in a setting of limited arbitrage.

[8]Other models of segmentation include Basak (1996), who studies welfare and interest rate effects of segmentation, Errunza and Losq (1989), He and Modest (1995), Heaton and Lucas (1996), Zapatero (1998), Caballero and Krishnamurthy (2001) (who focus on crises), and Bhamra (2007). Soumare and Wang (2006) also study constraints in a multiple-good economy, but again focus on individual constraints.

II Model

II.A Output in the Economy

Within a pure exchange economy, two countries, *home* and *foreign*, are each represented by one investor $i = H, F$, respectively. Each country produces one good, $j = h, f$, which both investors have access to. There are no transportation costs or other trade frictions — goods markets are assumed to be perfectly integrated. The endowment economy setup imposes exogenously the production or dividend processes

$$dY_t^j = \mu_t^{Y_j} Y_t^j dt + \sigma_t^{Y_j} Y_t^j dW_t^j, \qquad j = h, f \tag{1}$$

where parameters $\mu_t^{Y_j}$ and $\sigma_t^{Y_j}$ are adapted processes, representing the expected economic growth rate of country j and its volatility.[9] The Brownian Motions representing countries' production shocks, dW_t^h and dW_t^f, are assumed to be uncorrelated.

II.B Investor Preferences

Both the *home* (H) as well as the *foreign* (F) investor's utility functions are separable and additive over these two goods, and both display a preference for their respectively domestic good:

$$u_H\left(C_{Ht}^h, C_{Ht}^f\right) = \alpha_t^H \log C_{Ht}^h + (1-\alpha_t^H)\log C_{Ht}^f, \tag{2}$$

$$u_F\left(C_{Ft}^h, C_{Ft}^f\right) = (1-\alpha^F)\log C_{Ft}^h + \alpha^F \log C_{Ft}^f. \tag{3}$$

The preference parameters α_t^H and α^F are assumed to be greater than 0.5, implying the home bias in consumption. Demand shocks in the economy are driven by changes in α_t^H, the extent of the *home* investor's preference for his local good. α_t^H follows a martingale, uncorrelated with production shocks:

$$d\alpha_t^H = \sigma_t^\alpha dW_t^*. \tag{4}$$

[9] As a special case, the production processes can be assumed to follow geometric Brownian motions.

To ensure that α_t^H remains larger than 0.5, this process must have an appropriate form, and σ_t^α will necessarily be time-varying.[10] For tractability, α^F is assumed to be constant. Rather than interpreting this as only one country's representative investor being fickle in his relative demand, it may be more intuitive to think about α_t^H representing a relative shift in demands. The importance of such demand shifts in a multiple-good economy was established by Dornbusch, Fischer, and Samuelson (1977), whose modeling approach I follow here.[11] The empirical pervasiveness of a home bias in consumption across the world has been linked to reasons of familiarity, transportation costs, and non-tradability of certain goods, especially services.[12]

The set (C_{it}^h, C_{it}^f) describes the optimal consumption of goods h and f by $i = H, F$, the *home* and the *foreign* investors. Investor i maximizes expected utility $E\left[\int_0^T u_i\left(C_{it}^h, C_{it}^f\right) dt\right]$, subject to his budget constraint, dividing up wealth among consumption and investment into the financial assets available.

In equilibrium, the supply of and demand for each of the goods j at any time t determines their market-clearing prices p_t^j. The relative price of the goods is captured throughout the paper by $\bar{p}_t = p_t^f / p_t^h$.[13] The numeraire good is composed of a goods basket, where, without loss of generality, weight $\beta \in [0, 1]$ is put on the *home* good, and $(1 - \beta)$ on the *foreign* good.

II.C Financial Markets

Each country's financial markets consist of a locally riskless country bond and a stock market. The stock traded in the *home* country's market, S_t^h, is a claim to the future output of the *home* good, Y_t^h. Accordingly S_t^f is a claim to output Y_t^f.

[10] One example of such a process would be to fix the terminal time-T value of $\alpha_T^H \in (0.5, 1)$ and the martingale property will ensure that $\alpha_t^H = E[\alpha_T^H | \mathcal{F}_t]$.

[11] It could be interpreted as an ad-hoc way to take into account exogenous events that change demand. For example, hurricane warnings cause large spikes in the demand for timber, and import negotiations between countries can also trigger changes in demand, like the so-called Banana Wars between the U.S. and the EU. These events do not affect production of timber or bananas. In order to endogenize the effects of such demand shocks on the supply, it would be necessary to model a production economy.

[12] As Dumas and Uppal (2001) have shown, imperfections in goods markets do not alter the benefits of financial integration significantly. Treating this consumption bias as exogenous therefore does not seem to be a critical shortcoming.

[13] Both consumers in this model face the same price of goods, as there are no frictions in the goods market. Accordingly, there is no 'market price of exchange rate risk' as in e.g. the model of Dumas and Solnik (1995).

The two stocks have the following dynamics:

$$dS_t^h = \mu_t^{S_h} S_t^h dt + \vec{\sigma}_t^{S_h} S_t^h d\vec{W}_t, \tag{5}$$

$$dS_t^f = \mu_t^{S_f} S_t^f dt + \vec{\sigma}_t^{S_f} S_t^f d\vec{W}_t, \tag{6}$$

where expected return and volatility parameters $\mu_t^{S_j}$ and $\vec{\sigma}_t^{S_j}$ are determined in equilibrium. The three-dimensional vector $\vec{\sigma}_t^{S_j}$ comprises the sensitivities of stock S_t^j with respect to the mutually uncorrelated Brownian Motions representing supply and demand shocks $d\vec{W}_t = \left(dW_t^h, dW_t^f, dW_t^*\right)^\top$.

Together with the two stocks, the two local bonds in zero net supply provide the necessary assets to span the market.[14]

$$dB_t^h = r_t^h B_t^h dt \qquad \text{in terms of good } Y_t^h, \tag{7}$$

$$dB_t^f = r_t^f B_t^f dt \qquad \text{in terms of good } Y_t^f. \tag{8}$$

In terms of the numeraire, bond prices follow $d(p_t^j B_t^j)$, so for any choice of $\beta \in [0,1]$, at least one of them will not be truly riskfree. The truly riskless bond B_t, that postpones consumption of one unit of the numeraire, will consist of a portfolio of both bonds with weights β and $(1-\beta)$, as assigned to the numeraire basket: $B_t = \beta p_t^h B_t^h + (1-\beta)\, p_t^f B_t^f$.[15]

Both investors divide their investment portfolio between these four assets, subject to their budget constraint

$$dX_t^i = X_t^i \left[\sum_{j=h}^{f} \pi_{it}^{S_j} (dS_t^j + p_t^j Y_t^j dt)/S_t^j + \sum_{j=h}^{f} \pi_{it}^{B_j} d(p_t^j B_t^j)/(p_t^j B_t^j) \right] - \sum_{j=h}^{f} p_t^j C_{it}^j dt \tag{9}$$

where agent i's wealth X_t^i must satisfy $X_t^i \geq 0$, and p_t^j is the price of good $j = h, f$. $\pi_{it}^{S_j}$ is the

[14]Market completeness does not follow necessarily, but can be shown to hold in this setting.

[15]Note that the notion of a 'riskfree' bond B_t here is strictly in terms of the numeraire. Unless $\beta = \alpha^i$, the bond is not completely riskfree in terms of actual consumption choices an agent makes. I allow β to be different from either preference parameter α^i for the sake of generality. This assumption is akin to inflation-indexed bonds such as TIPS, where the consumption basket used to calculate inflation may differ from the actual basket of goods consumed by an individual who invests in such a bond.

fraction of investor i's wealth that he chooses to invest in stock S_t^j, $\pi_{it}^{B_j}$ the fraction invested into bond B_t^j.

II.D Portfolio Constraints on *Home* and *Foreign* Investor

The investors' portfolio decisions are complicated by the fact that both face constraints on their positions in the stock market(s). In this paper I study two particular examples of constraints. The *home* investor H is subject to a leverage constraint: He can hold both long and short positions in the two country bonds individually, but his net position in the riskfree asset cannot be negative.[16] In contrast, the *foreign* investor F is limited in the amount he invests into the stock abroad, S_t^h of the *home* stock market. These constraints can be expressed as

$$\iota_H^\top \pi_{H,t} \leqslant 1, \qquad \text{and} \qquad \iota_F^\top \pi_{F,t} \leqslant \varphi, \tag{10}$$

where $\iota_H = [1,1,0]^\top$, $\iota_F = [1,0,0]^\top$, and $\pi_{i,t} = [\pi_{i,t}^{S_h}, \pi_{i,t}^{S_f}, \pi_{i,t}^{B_h}]^\top$ denotes the three-dimensional vector of investor i's portfolio.[17]

It is not difficult to find examples of both limits on leverage as well as restrictions on capital allocation into foreign markets. We can observe similar limitations imposed internally within the asset management industry, both for leverage as well as for asset allocation across regions or between emerging vs. developed markets. More broadly, some countries' legislation also constrains the flow of capital into or out of their markets. But the reasons for using these particular examples in the context of a two-country model need to be described in more detail.[18]

The paper focuses on how constraints interact when they bind simultaneously, i.e. when there exists no liquidity provider with unlimited investment capacity. To date, the literature on portfolio constraints in equilibrium generally retains at least one investor who is not constrained and

[16] This is the simplest form of analyzing borrowing restrictions, where no borrowing is allowed. Conceptually, a less strict leverage constraint should lead to similar implications for wealth transfers but would impede tractability.

[17] Satisfaction of the budget constraint implies $\pi_{i,t}^{B_f} = 1 - \mathbf{1}^\top \pi_{i,t}$.

[18] In reality, restrictions on the proportion of the *asset* held, rather than on the proportion of *wealth* held in the asset, are more prevalent. The reason the latter type of constraint is generally used in equilibrium models is due to tractability. The former type of constraint introduces a feedback loop between investment strategy and stock prices, an endogenous quantity, thus not generally allowing for a tractable solution.

will thus be the price setting investor: he can absorb any extra asset supply not taken up by the constrained investor. As Basak and Croitoru (2000) show, an investor with unbounded pockets will function as an arbitrageur and prevent mispricing in markets even in the presence of other investors' constraints. Within a two-country, two-asset setup I study constraints that, when they bind, affect the *same* stock, in this case the *home* stock S_t^h: the *foreign* investor's holding is restricted, and the local investor H cannot easily take up the excess supply of stock, due to his own leverage constraint. A 'marginal investor' that sets the price does not naturally emerge. This would not be the case if the constraints were the mirror image of one another, e.g. investor F being constrained in his holding of S_t^h, while investor H is constrained in his holding of S_t^f — the other investor would always be free to act as liquidity provider.

Among types of constraints that can capture this feature described above, the market clearing requirement limits the examples further. In order to compare the market equilibria with and without binding constraints, the entire available supply of any asset must be held by one of the market participants in all situations, including when both investors' constraints are binding. This restricts the possible combinations of constraints that can be studied in a two-country-two-investor model.[19] The setup allows for generalizable insights beyond these particular examples of portfolio restrictions, based on how constraints on the two investors interact when they bind — whether they mitigate or exacerbate one another.

I make no particular assumption on who imposes these constraints, whether a country is responsible for imposing restrictions on their own citizens (capital outflow restrictions) or on the non-domestic investors (capital inflow restrictions). In the remainder of the paper, 'liberalization' (partial or full) is always assumed to be simply an exogenous lifting of one or both constraints, not the result of optimal decision making on behalf of an entity within the model. For the investors' portfolio decisions, the origin of the constraints is irrelevant. The analysis of the effects of liberalization looks at how capital flows and changes to portfolios affect conditional stock price dynamics, keeping economic fundamentals the same.

[19] Technically, expanding the model to n countries is not a problem, but the increase in the number of parameters is substantial.

II.E Information Structure

Uncertainty is characterized by the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P})$. But investors are assumed to have different beliefs about the growth rate of output in the two countries: their information set is the incomplete filtration $\{\mathcal{F}_t^{Y_{i,j}}\}$, generated by processes Y_t^h and Y_t^f, rather than the full information set $\{\mathcal{F}_t^{\vec{W}}\}$. Both investors have access to the same public information, they can observe output of and demand for goods at every point in time. But if they have different priors on the expected growth rate of a country, observing changes in output over time will not allow beliefs to converge instantaneously: Based on their different expectations, the investors will continue to 'agree to disagree' about expected growth rates.[20]

$$
\begin{aligned}
dY_t^j &= \mu_t^{Y_j} Y_t^j dt + \sigma_t^{Y_j} Y_t^j dW_{j,t} \\
&= m_{Y^j,t}^{(H)} Y_t^j dt + \sigma_t^{Y_j} Y_t^j dW_{j,t}^{(H)} \\
&= m_{Y^j,t}^{(F)} Y_t^j dt + \sigma_t^{Y_j} Y_t^j dW_{j,t}^{(F)} \quad \text{for j= h,f}
\end{aligned} \tag{11}
$$

$dW_{j,t}^{(i)}$ represents the innovation process perceived by agent $i = H, F$ and $m_{Y^j,t}^{(i)}$ is his belief regarding country j's expected production growth rate. The dispersion in investors' beliefs can be captured by the difference in perceived innovation processes.

$$
dW_{h,t}^{(F)} = dW_{h,t}^{(H)} - \Delta m_t^{Y_h} dt \quad ; \quad dW_{f,t}^{(F)} = dW_{f,t}^{(H)} - \Delta m_t^{Y_f} dt \quad ; \quad dW_{*,t}^{(F)} = dW_{*,t}^{(H)} \tag{12}
$$

where $\Delta m_t^{Y_j} = \frac{m_{Y^j,t}^{(F)} - m_{Y^j,t}^{(H)}}{\sigma_t^{Y_j}}$ for $j = h, f$. This definition implies that values of $\Delta m_t^{Y_j}$ will be positive if F is the more optimistic of the two investors regarding the growth rate of country j. Accordingly a negative value implies that H is the more optimistic investor.[21] These differences in beliefs are captured in the vector $\Delta \vec{m}_t^Y = [\Delta m_t^{Y_h}, \Delta m_t^{Y_f}, 0]^\top$. The third element is zero, since α_t^H is known to follow a martingale, there is no growth rate to disagree on.

[20] Through quadratic variation, both can draw exact inferences about the diffusion terms of output processes dY_t^j and demand shocks $d\alpha_t^H$.

[21] As an example, home bias in beliefs about investment opportunities would be reflected in the scenario $\Delta m_t^{Y_f} > 0$ and $\Delta m_t^{Y_h} < 0$.

The foundations for the assumption of different priors have been discussed at length for the general case in Morris (1994) and similar setups can be found, e.g. in Basak (2000) who includes extraneous risk, and Gallmeyer and Hollifield (2008). In this paper I remain agnostic about the microeconomic foundations of the differences in beliefs across countries.[22] I abstract from issues of asymmetric information, neither investor here has reason to try to infer information from the actions of the other.

The time-subscripts in $m^{(i)}_{Y^h,t}$ and $m^{(i)}_{Y^f,t}$ reflect the potential for time variation in investors' beliefs. The model allows for incorporating a learning process about the uncertain growth rates. In a typical Bayesian setup, investors would use the information in observed output to update their priors of the economies' growth rates: a high realization of output Y^h_t for example causes investors to shift their beliefs and expect higher growth rates in the *home* country going forward. By how much this new information shifts an investor's prior depends on the prior's precision. Many models of international finance argue that investors have better information about their local economy, implying that their priors on local growth rates are more precise than those of foreign investors.[23] Different precisions of priors let investors' beliefs diverge in the interim, even though rational updating ultimately leads to beliefs converging in the long run. Changes in beliefs over time cause the constraints in this paper to become binding, or cause an already binding constraint to bind more or less severely. For parsimony, I abstract from detailing a model of learning explicitly. The testable implications of this model do not rely on a particular assumption about *how* investors learn and update their beliefs about economic fundamentals, as long as such a learning process ensures the boundedness of $\Delta\vec{m}^Y_t$.

[22]Other papers studying the effect of differences in beliefs are, e.g. Harrison and Kreps (1978) and Basak (2005). The literature on differences in risk attitude, e.g. Kogan and Uppal (2003), focuses on a different source of heterogeneity in agents.

[23]We have less intuition on what the cross-country correlation of priors would be, e.g. how a high realization of local output would affect investors' expectation of growth rates abroad. This concept of the correlation of priors is only partly related to the fundamental correlations between economic fundamentals.

III Equilibrium

The competitive equilibrium is established via the aggregated 'social planner' utility function

$$U(C_H, C_F) = u_H\left(C^h_{Ht}, C^f_{Ht}\right) + \lambda_t u_F\left(C^h_{Ft}, C^f_{Ft}\right), \tag{13}$$

where the two investors' utility functions $u_H(\cdot)$ and $u_F(\cdot)$ are as stated in eqs. (2) and (3).

The progressively measurable state variable λ_t is central in characterizing the equilibrium and the effects of constraints and belief dispersion on the financial markets. Normalizing, without loss of generality, the weight of investor H to 1, λ_t represents the relative weight of investor F in the economy. This relative weight is the ratio of investors' state price densities, and can be interpreted as the importance a social planner would give to F, or alternatively the impact he has on the competitive equilibrium in consumption and financial markets.

If all investors held identical portfolios, wealth gains and losses from portfolio returns would be distributed symmetrically and λ_t would be constant, reflecting only the distribution of initial endowments. In this model, λ_t is stochastic since investors hold different portfolios for two reasons: differences in beliefs and portfolio constraints. The latter distort portfolio choice — investors' equilibrium holdings may be *less or more* similar than they would be in absence of the constraints. λ_t is reflected in market-clearing consumption and investment choices:

	investor H	investor F
home good:	$C^h_{Ht} = \frac{\alpha^H_t}{\alpha^H_t + (1-\alpha^F)\lambda_t} Y^h_t$	$C^h_{Ft} = \frac{\lambda_t\left(1-\alpha^F\right)}{\alpha^H_t + (1-\alpha^F)\lambda_t} Y^h_t$
foreign good:	$C^f_{Ht} = \frac{\left(1-\alpha^H_t\right)}{1-\alpha^H_t + \alpha^F\lambda_t} Y^f_t$	$C^f_{Ft} = \frac{\lambda_t \alpha^F}{1-\alpha^H_t + \alpha^F\lambda_t} Y^f_t$

As log-investors, they consume a fraction of their wealth X^i_t , but this fraction is not constant over time, it varies with changes in goods' prices.

$$X_t^H = C_{Ht}^h \cdot \frac{p_t^h}{\alpha_t^H}(T-t) = C_{Ht}^f \cdot \frac{p_t^f}{1-\alpha_t^H}(T-t), \tag{14}$$

$$X_t^F = C_{Ft}^h \cdot \frac{p_t^h}{1-\alpha^F}(T-t) = C_{Ft}^f \cdot \frac{p_t^f}{\alpha^F}(T-t). \tag{15}$$

Eqs. (14) and (15) show that investors' marginal propensity to consume is stochastic only through the effects of demand and price shifts. Future changes in investors' investment opportunity set — through constraints and time-varying expected growth rates — do not affect the savings motive.

The methodology to assess the impact of investment constraints on portfolio choice was introduced by Cvitanic and Karatzas (1992).[24] The constraints as given by eq. (10) prevent the investors from holding portfolio positions that reflect their true beliefs about investment opportunities. Portfolio choice and equilibrium asset prices will thus reflect distorted state price densities ξ_t^i, which take into account investors' perceived risk–return tradeoff as well as the unattainability of some consumption sets.

$$d\xi_t^i = -\left(r_t + \delta(\upsilon_t^i)\right)\xi_t^i dt - \vec{\kappa}_t^{i\top}\xi_t^i d\vec{W}_t^{(i)}, \tag{16}$$

where $\vec{\kappa}_t^i$ is the market price of risk of investor $i = H, F$, as reflected in portfolios:

$$\vec{\kappa}_t^i = \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(i)} + \upsilon_t^i \iota_i - r_t \mathbf{1}\right) = \vec{\kappa}_{o,t}^i + \vec{\sigma}_{S,t}^{-1}\upsilon_t^i \iota_i. \tag{17}$$

Both the risk-free interest rate r_t and the true beliefs about market prices of risk, $\vec{\kappa}_{o,t}^i = \vec{\sigma}_{S,t}^{-1}(\vec{m}_{S,t}^{(i)} - r_t\mathbf{1})$, are augmented by functions of υ_t^i — a scalar parameter that makes i's portfolio choice permissible under the imposed constraint. The investor's problem is restated in terms of these 'fictitious' market parameters. Investing as if expecting the rates of return to be $\vec{m}_{S,t}^{(i)} + \upsilon_t^i \iota_i$ rather than $\vec{m}_{S,t}^{(i)}$, he

[24] The literature on incorporating constraints also include e.g. He and Pearson (1991), Karatzas, Lehoczky, Shreve, and Xu (1991), and Liptser and Shiryaev (2001).

will *choose* to hold a portfolio permissible within the confines of the constraint.[25]

$$\pi_{Ht} = (\vec{\sigma}_{S,t}^{-1})^\top \left(\vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(H)} - r_t\mathbf{1}\right) + \vec{\sigma}_{S,t}^{-1}\upsilon_t^H \iota_H\right), \tag{18}$$

$$\pi_{Ft} = (\vec{\sigma}_{S,t}^{-1})^\top \left(\vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t\mathbf{1}\right) + \vec{\sigma}_{S,t}^{-1}\upsilon_t^F \iota_F\right). \tag{19}$$

Constraints on long positions in an asset imply that the constrained investor is not able to express the full extent of his optimism regarding investment opportunities. υ_t^is will accordingly be negative whenever the constraint on investor i binds, and zero otherwise. Portfolio choice in all assets, not just those that investor i faces a direct constraint in, will reflect this adjustment. The terms $(\vec{\sigma}_{S,t}^{-1})^\top(\vec{\sigma}_{S,t}^{-1}\upsilon_t^H\iota_H)$ and $(\vec{\sigma}_{S,t}^{-1})^\top(\vec{\sigma}_{S,t}^{-1}\upsilon_t^F\iota_F)$ in eqs. (18) and (19) capture this notion: The investor seeks to compensate for his binding constraint by investing into a combination of the remaining assets that is highly correlated with that of the desired, but inaccessible, asset.

The model explicitly distinguishes between a home bias in consumption and a home bias in portfolio choice.[26] In the nested 'benchmark' equilibrium of identical beliefs across investors and no constraints, both investors would hold the world market portfolio, despite a home bias in consumption.

Differences in beliefs as well as binding constraints will shift investors' holdings away from the world market portfolio. The investor that is more optimistic about a particular country's growth rate tends to invest a larger portion of his wealth in the associated stock market. However, the imposed constraints limit the extent to which investors' portfolios reveal their true beliefs, thereby driving a wedge between the true dispersion of beliefs and the dispersion reflected in asset prices via portfolio choice. Depending on whether the constrained investor is the more or the less opti-

[25]Recall from section II.D that π_{it} denotes the fractions of wealth investor i holds in the assets S_t^h, S_t^f and B_t^h. Accordingly, $\vec{m}_{S,t}^{(i)}$ represents the vector of expected returns in these assets from the perspective of investor i. The equilibrium expected stock returns will be discussed in more detail in later sections.

[26]The model of Uppal (1993) shows that even when goods market imperfections like transportation costs are modeled explicitly, a portfolio home bias only results when agents are less risk-averse than log utility investors. This implies that imperfections of that type seem an unlikely source of home bias in portfolios. Chabakauri (2009) studies constraints under more general CRRA preferences. In this paper I use the simpler log utility as a benchmark and focus on different types of constraints. This should not imply that that log utility is the more realistic assumption about preferences. Rather, that variations in regulation — rather than variations in utility functions across countries — provide potential explanations for the different effects of liberalization on countries' stock markets.

mistic, portfolios may be more or less similar to one another than in absence of the constraint.

Methodologically, I follow the approach of Detemple and Murthy (1997) and attain closed-form solutions for equilibrium stock prices. The potentially binding constraints that an investor is faced with will be reflected in his state price density through the described adjustment term υ_t^i. A model that retains at least one unconstrained investor can make use of the simpler functional form of his state price density to price all assets in equilibrium. Observational equivalence then dictates that all investors are unwilling to trade (within their means) at the equilibrium price. This implies that the unconstrained investor will be setting prices at the margin, by supplying the necessary capital to clear markets, when other investors are already bound by their constraints. In this model, both investors' state price densities will reflect constraints, making equilibrium valuation more complicated:[27]

$$S_t^j = \frac{1}{\xi_t^H} E_t \left[\int_t^T \xi_s^H p_s^j Y_s^j ds \right] + \frac{1}{\xi_t^H} E_t \left[\int_t^T \left(\upsilon_t^H + \delta(\upsilon_t^H) \right) S_s^j \xi_s^H ds \right] \qquad j = h, f. \tag{20}$$

The first term in eq. (20) is the appropriately discounted value of the expected future dividend stream, the same as in models with unconstrained investors. The remaining two terms inside the second integral have been interpreted by Detemple and Murthy (1997) as speculative and collateral premia. An example in their paper shows that in a single-asset economy, the types of constraints that can simultaneously bind while still allowing markets to clear are somewhat restricted. Indeed, in that equilibrium, the multiple constraints neutralize each other, rendering stock prices and dynamics identical to those in a perfect frictionless market.

Extending their model to a two-asset setting broadens the set of available assets. This opens up an additional channel for constraints to affect investment, by allowing constraints to propagate across stocks, while retaining a tractable solution. Delineating groups of investors by the constraints they face is typically easier in an international setting, which may make the empirical testing of the paper's implications more straightforward. The stocks being claims to two distinct goods, whose

[27] The valuation and thus the remainder of the paper are done using investor H's perception of growth rates. This is without loss of generality, as results that are affected by this, such as expected returns, can easily be translated into investor F's perception by applying the relationships known from section II.E.

consumption markets clear separately, is critical for tractability. Settings in which investors are indifferent to which Lucas tree's output they consume present the problem of stock price indeterminacy: only the *total* value of all stocks jointly can be uniquely determined.[28] The following proposition describes in detail the four potential equilibria of the model, and the parameter conditions under which they occur, respectively.

Proposition 1. *Equilibrium stock prices in the* home *and* foreign *countries are*

$$
\begin{aligned}
S_t^h &= \frac{1}{\beta + (1-\beta)\,\bar{p}_t} Y_t^h (T-t), && (21)\\
S_t^f &= \frac{\bar{p}_t}{\beta + (1-\beta)\,\bar{p}_t} Y_t^f (T-t) \quad \forall t, && (22)\\
where \;\; \bar{p}_t &= \frac{(1-\alpha_t^H) + \alpha^F \lambda_t}{\alpha_t^H + (1-\alpha^F)\lambda_t} \frac{Y_t^h}{Y_t^f}. && (23)
\end{aligned}
$$

Depending on the investors' beliefs about growth rates, 4 possible equilibria can arise: neither investor is constrained, only the home investor's constraint is binding, only the foreign investor's constraint is binding, or both investors' constraints are binding. Equilibrium adjustment terms υ_t^i, where

$$
\upsilon_t^H = \min\left(\frac{1-\iota_H^\top (\vec{\sigma}_{S,t}^{-1})^\top \vec{\kappa}_{o,t}^H}{\iota_H^\top (\vec{\sigma}_{,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \iota_H}, 0\right) \;\; \& \;\; \upsilon_t^F = \min\left(\frac{\varphi - \iota_F^\top (\vec{\sigma}_{S,t}^{-1})^\top \vec{\kappa}_{o,t}^F}{\iota_F^\top (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \iota_F}, 0\right), \qquad (24)
$$

are non-positive at all times t when i's constraint is binding, and zero otherwise. λ_t follows dynamics $d\lambda_t = \lambda_t \Delta\vec{\kappa}_t^\top d\vec{W}_t^{(H)}$, where $\Delta\vec{\kappa}_t^\top = \left[\Delta\kappa_t^h, \Delta\kappa_t^f, \Delta\kappa_t^\alpha\right]$ is the difference in investors' market prices of home, foreign, and demand risk. How closely these market prices of risk reflect actual differences in beliefs will vary in the 4 equilibria, depending on which constraints bind: $\Delta\vec{\kappa}_t = \Delta\vec{m}_t^Y + \vec{\sigma}_{S,t}^{-1}\left(\upsilon_t^F \iota_F - \upsilon_t^H \iota_H\right)$.

case U (neither investor constrained)

$$
d\lambda_t^{\mathtt{U}} = \lambda_t \Delta\vec{m}_t^{Y\top} d\vec{W}_t^{(H)},
$$

[28]In absence of a preference shock, perfect correlation of the two stocks would also render markets incomplete in this setting.

when $\Delta m_t^{Y_h} < \left(\varphi\left(1+\lambda_t\right) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_t^{Y_h}$ *&* $\Delta m_t^{Y_f} > -\frac{\sigma_t^{Y_f}}{\sigma_t^{Y_h}}\Delta m_t^{Y_h}$

case F (investor F constrained in holdings of S_t^h)

$$d\lambda_t^{\mathtt{F}} = \lambda_t\left[\Delta\vec{m}_t^Y + \vec{\sigma}_{S,t}^{-1}\upsilon_t^F\iota_F\right]^\top d\vec{W}_t^{(H)},$$

when $\Delta m_t^{Y_h} > \left(\varphi\left(1+\lambda_t\right) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_t^{Y_h}$ *&*
$\Delta m_t^{Y_f} > \frac{-\Delta m_t^{Y_h}\left[\lambda_t(1-\varphi)(1-\alpha_t^H-\alpha^F)+\alpha_t^H\left(1-\alpha_t^H+\alpha^F\lambda_t\right)\right]^2\sigma_t^{Y_h}\sigma_t^{Y_f}-\left(\varphi(1+\lambda_t)-\alpha_t^H-(1-\alpha^F)\lambda_t\right)\sigma_t^{Y_f}(\sigma_t^\alpha)^2}{\left[\lambda_t(1-\varphi)(1-\alpha_t^H-\alpha^F)+\alpha_t^H\left(1-\alpha_t^H+\alpha^F\lambda_t\right)\right]^2(\sigma_t^{Y_h})^2+(\sigma_t^\alpha)^2}$

case H (investor H leverage constrained)

$$d\lambda_t^{\mathtt{H}} = \lambda_t\left[\Delta\vec{m}_t^Y - \vec{\sigma}_{S,t}^{-1}\upsilon_t^H\iota_H\right]^\top d\vec{W}_t^{(H)}$$

when $\Delta m_t^{Y_h} < \left(\varphi\left(1+\lambda_t\right) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_t^{Y_h}$ *&*
$-\left(\varphi\left(1+\lambda_t\right) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\frac{(\sigma_t^{Y_h})^2+(\sigma_t^{Y_f})^2}{\sigma_t^{Y_f}} + \frac{\sigma_t^{Y_h}}{\sigma_t^{Y_f}}\Delta m_t^{Y_h} < \Delta m_t^{Y_f} < -\frac{\sigma_t^{Y_f}}{\sigma_t^{Y_h}}\Delta m_t^{Y_h}$

case FH (both investors constrained)

$$d\lambda_t^{\mathtt{FH}} = \lambda_t\left[\Delta\vec{m}_t^Y + \vec{\sigma}_{S,t}^{-1}\left(\upsilon_t^F\iota_F - \upsilon_t^H\iota_H\right)\right]^\top d\vec{W}_t^{(H)}$$

when

$$\begin{aligned}
\Delta m_t^{Y_h} &< \left(\varphi\left(1+\lambda_t\right) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_t^{Y_h} \text{ \& } \\
\Delta m_t^{Y_f} &< -\left(\varphi\left(1+\lambda_t\right) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\frac{(\sigma_t^{Y_h})^2+(\sigma_t^{Y_f})^2}{\sigma_t^{Y_f}} + \Delta m_t^{Y_h}\frac{\sigma_t^{Y_h}}{\sigma_t^{Y_f}} \\
&\quad \textit{or} \\
\Delta m_t^{Y_h} &> \left(\varphi\left(1+\lambda_t\right) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_t^{Y_h} \text{ \& } \\
\Delta m_t^{Y_f} &< \frac{-\Delta m_t^{Y_h}\left[\lambda_t(1-\varphi)(1-\alpha_t^H-\alpha^F)+\alpha_t^H\left(1-\alpha_t^H+\alpha^F\lambda_t\right)\right]^2\sigma_t^{Y_h}\sigma_t^{Y_f}-\left(\varphi(1+\lambda_t)-\alpha_t^H-(1-\alpha^F)\lambda_t\right)\sigma_t^{Y_f}(\sigma_t^\alpha)^2}{\left[\lambda_t(1-\varphi)(1-\alpha_t^H-\alpha^F)+\alpha_t^H\left(1-\alpha_t^H+\alpha^F\lambda_t\right)\right]^2(\sigma_t^{Y_h})^2+(\sigma_t^\alpha)^2}
\end{aligned}$$

The appendix gives the technical details and shows υ_t^i in terms of fundamentals, thereby clos-

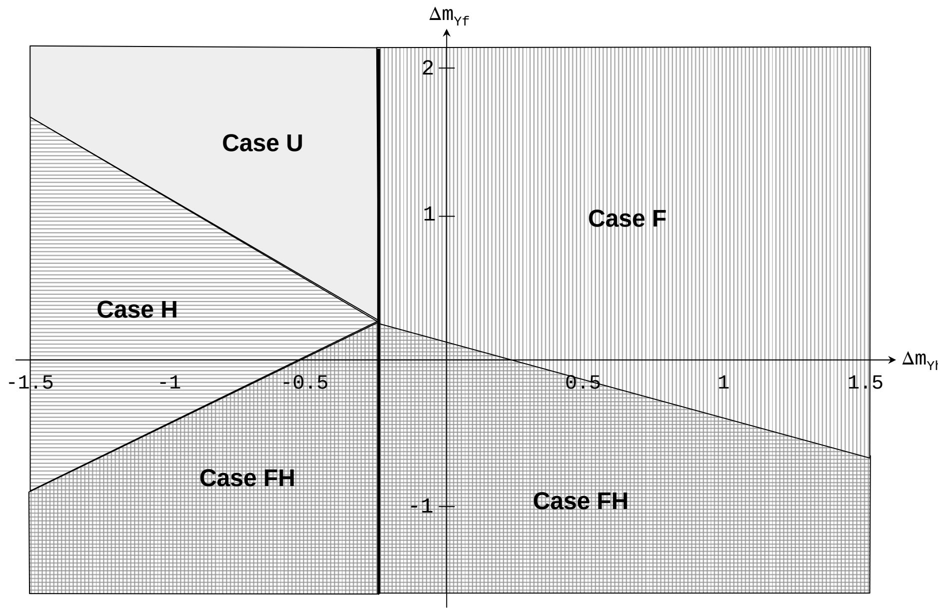

Figure 1: **Equilibria with two constraints:** The constraints imposed on investors F and H can each bind individually (case F, case H), jointly (case FH), or not at all (case U). Which of the four possible equilibria holds, depends on the belief dispersion regarding fundamental economic growth rates in both countries, $\Delta m_t^{Y_h}$ and $\Delta m_t^{Y_f}$. In the special case of homogeneous beliefs ($\Delta m_t^{Y_h} = \Delta m_t^{Y_f} = 0$), both constraints will always bind.

ing the model. Fig. (1) illustrates the parameter conditions for the four possible equilibria. As investors' beliefs change, portfolios shift until constraints become binding. At that point, trade in constrained assets ceases and shifts into the markets of the remaining unconstrained assets. Comparing to this the graphs in fig. (2) shows that imposing multiple constraints expands the parameter space under which a constraint binds. The lower left quadrant of fig. (1) illustrates that in the presence of another restriction, investor F's constraint can bind even when he is quite pessimistic about growth rates in the *home* country — a strongly negative $\Delta m_t^{Y_h}$. Under the same beliefs, it would not bind when imposed in isolation, as demonstrated in the second graph of fig. (2). In the presence of both constraints, H's simultaneously binding constraint restricts him from taking on as much of either stock as he would like. F thus soaks up additional supply of S_t^h until he himself hits his own portfolio limit φ. The relative severity of the constraints when they bind jointly, determines which of the two constraints dominates the effect on equilibrium market dynamics.

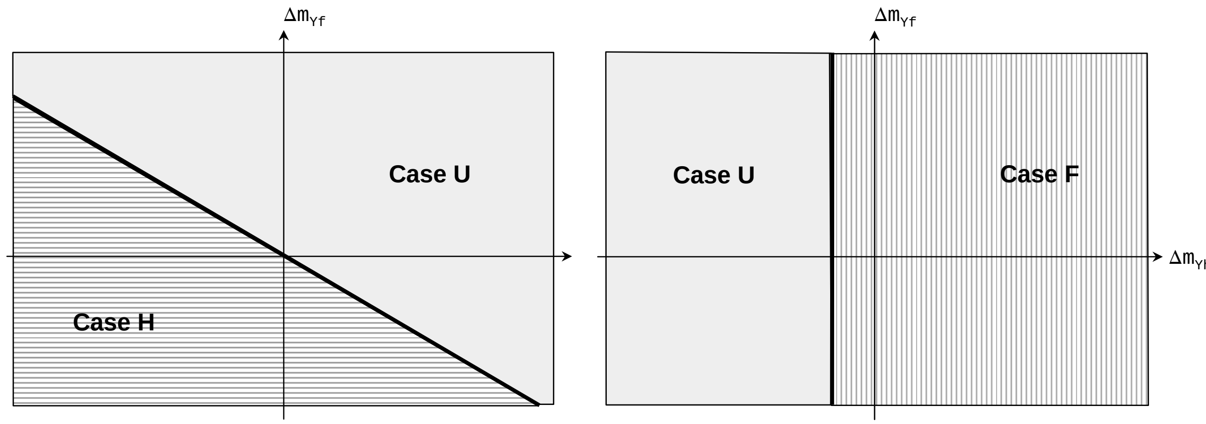

Figure 2: **Economies with a single constraint:** The left graph shows the possible equilibria for an economy where only investor H's leverage constraint is imposed. This constraint will bind when H is relatively optimistic regarding at least one country. The right graph shows possible equilibria when only investor F faces his constraint, on holdings of S_t^h. This constraint will bind when he is relatively optimistic regarding the *home* country, i.e. when $\Delta m_t^{Y_h}$ is not too negative.

IV Stock Prices and Dynamics

While output of the *home* and *foreign* economies are uncorrelated, their stock markets are not. Equilibrium stock prices are driven by consumption and investment choices of both investors, linking the two financial markets. The relative price of consumption goods, $\bar{p}_t = p_t^f / p_t^h$, is the conductor for shocks to propagate through the system. Perfect integration of goods markets implies that $\bar{p}_t$ is determined by the relation of marginal utilities with respect to the two goods, $\bar{p}_t = u^i_{C^F}(\cdot) / u^i_{C^H}(\cdot)$. The equilibrium price is as stated in eq. (23) of the proposition: $\bar{p}_t = \frac{(1-\alpha_t^H)+\alpha^F \lambda_t}{\alpha_t^H + (1-\alpha^F)\lambda_t} \frac{Y_t^h}{Y_t^f}$.

$\bar{p}_t$ reflects supply and demand of goods. Supply is exogenously given by the Lucas trees. A positive supply shock to *home* country's good Y_t^h lowers its price p_t^h relative to p_t^f — $\bar{p}_t$ will rise. A positive supply shock to good Y_t^f will lower $\bar{p}_t$.

Demand is driven by consumption preferences and the distribution of wealth. α_t^H and $(1-\alpha^F)$ are the investors' respective preferences for *home* good Y_t^h. Higher relative demand for Y_t^h lowers $\bar{p}_t$. The ratio of investors' state prices, λ_t, determines the relative impact of the two investors on equilibrium consumption, linking the goods markets to the financial markets. For log-utility investors, λ_t is uniquely captured by their relative wealth, X_t^F / X_t^H. Changes to this state variable are determined by differences in investment decisions. The less similar portfolio composition is across investors, the more strongly a fundamental economic shock leads to 'wealth transfers', which feed back into stock markets. An example of this wealth effect is the development of copper prices. Copper is a commodity heavily used in industrial economies. As these economies became more wealthy, demand for copper rose, driving up its price. The rising prices meant a windfall for economies like Chile that have a large copper industry.

From equilibrium stock prices as determined in the proposition, stock price dynamics follow

$$dS_t^h = (\cdot)dY_t^h + (\cdot)dY_t^f + (\cdot)d\alpha_t^H - (\cdot)d\lambda_t^{(\mathsf{case})}, \tag{25}$$

$$dS_t^f = (\cdot)dY_t^f + (\cdot)dY_t^h - (\cdot)d\alpha_t^H + (\cdot)d\lambda_t^{(\mathsf{case})}, \quad \text{all } (\cdot) > 0 \tag{26}$$

where the dynamics of λ_t are endogenous and depend on which, if any, constraints are binding. The first term on the right side of eqs. (25) and (26) respectively, is the direct supply or 'dividend' effect: a positive production shock to *home* good Y_t^h leads to an appreciation of its associated stock price S_t^h, and likewise for the *foreign* market. The second terms capture the 'terms-of-trade' effect: a positive supply shock to Y_t^h raises the relative price of the *foreign* good (whose supply did not increase), which pushes up the price of the *foreign* stock, S_t^f.[29]

From the demand side, a higher preference α_t^H for the *home* good Y_t^h will increase its relative price, thus increasing the value of output, pushing up stock price S_t^h. Conversely, a higher α_t^H puts downward pressure on the *foreign* stock price, as relative demand for Y_t^f is lowered. Both output and preference shocks are exogenous, so constraints affect stock price dynamics only through relative wealth dynamics $d\lambda_t$.

The effect of λ_t on stock prices can be decomposed: $\partial S_t^i/\partial\lambda_t = \partial S_t^i/\partial\bar{p}_t \cdot \partial\bar{p}_t/\partial\lambda_t$. The first fraction is again the 'terms-of-trade' effect discussed above, which is negative for S_t^h, and positive for S_t^f. $\partial\bar{p}_t/\partial\lambda_t$ is always positive: as investor F's relative wealth increases (higher λ_t), F will increase overall consumption — but predominantly of his local good Y_t^f. This raises $\bar{p}_t$, pushing up

[29] The dividend effect of a supply shock always dominates the negative terms-of-trade effect on its own stock, thus the positive sign on the first term.

S_t^f while pushing down S_t^h. Conversely, a wealth transfer to agent H (lower λ_t) will lower $\bar{p}_t$, as H channels more of his additional consumption into his own local good. Note that a wealth transfer in either direction has opposite effects on the two stock markets, decreasing correlation between the stock markets.

Wealth transfers establish a 'feedback effect' in financial markets, via changes of consumption in the goods markets. Whether the feedback effect reinforces or mitigates an initial shock to fundamentals depends on how λ_t is correlated with fundamental shocks. The investor that holds a larger proportion of wealth in a particular stock S_t^j will receive a wealth transfer from the other agent when this stock experiences positive returns, and part of this extra wealth will be allocated to consumption.

For example, if investor H exhibits, due to his beliefs or perhaps a binding constraint, a home bias in his portfolio, a positive return to his domestic stock S_t^h makes him relatively wealthier, thus allowing him to consume more. Most of this additional consumption is in his local good Y_t^h, thus pushing up the good's price, again boosting the local stock S_t^h. I refer to this as a positive feedback effect, as it reinforces the effect of the initial positive shock to the *home* stock. A negative feedback effect arises when wealth gains due to one country's high stock market returns disproportionately benefit the non-domestic stock holder, and are thus channeled out of the country to finance extra consumption.

The two-country model in this paper may seem to exaggerate the impact of wealth redistribution on market prices. Technically, this concern is easily alleviated by extending this model to n countries. The stock price of a good that is exported to multiple countries is less reactive to a redistribution of wealth than a good that is exported to only one other country. Wealth redistribution across n countries would be imperfectly correlated, generating a potential for diversification. However, the current debate about the repercussions of changes in countries' wealth on the real economies of the U.S. and China demonstrates that the impact can indeed be significant for large economies.

The main results of the paper derive from these feedback relationships. Removing one or more constraints will affect both investors's equilibrium portfolio composition, and as a result, the feedback effects. Under which circumstances liberalization strengthens or weakens a feedback effect will be discussed in greater detail in section IV.B, establishing the link to market correlation and volatility.

IV.A Stock Returns

Investors' different beliefs about growth rates will also be reflected in their beliefs regarding expected stock returns. Equations (27) and (28) describe the expected stock returns on both the *home* and the *foreign* stock from the viewpoint of the *home* investor H. The stock returns expected under the *foreign* investor's beliefs follow directly from the belief dispersion as defined in eq. (12).

$$m^{(H)}_{S_h,t} = -\frac{1}{T-t} + m^{(H)}_{Y^h,t} - \frac{(1-\beta)\bar{p}_t}{\beta+(1-\beta)\bar{p}_t}\mu_{\bar{p}_t} + \frac{(1-\beta)^2\bar{p}_t^2}{(\beta+(1-\beta)\bar{p}_t)^2}\sigma^2_{\bar{p}_t} - \frac{(1-\beta)\bar{p}_t}{\beta+(1-\beta)\bar{p}_t}\sigma_{(Y_h,\bar{p}),t}, \tag{27}$$

$$m^{(H)}_{S_f,t} = -\frac{1}{T-t} + m^{(H)}_{Y^f,t} + \frac{\beta}{\beta+(1-\beta)\bar{p}_t}\mu_{\bar{p}_t} - \frac{\beta(1-\beta)\bar{p}_t}{(\beta+(1-\beta)\bar{p}_t)^2}\sigma^2_{\bar{p}_t} + \frac{\beta}{\beta+(1-\beta)\bar{p}_t}\sigma_{(Y_f,\bar{p}),t}. \tag{28}$$

As in a single-good economy with a terminal date, the first two terms on the right side of eqs. (27) and (28) reflect time remaining and the positive impact of a high output growth rate on stock returns. Due to the two-good setup in this paper, instantaneous changes in the value of the output will also be reflected, by changes to relative prices $\bar{p}_t$. The drift and diffusion of price dynamics follow from eq. (23), represented here by the general terms $\mu_{\bar{p}_t}$ and $(\sigma_{\bar{p}_t})^2$. The returns on stocks are also affected by the covariance between the underlying output and its price, $\sigma_{(Y_j,\bar{p}),t}$: when high prices of a good tend to coincide with high output, returns are naturally exacerbated. The sign of this covariance between production and price can be positive or negative, and is determined by investors' consumption and investment choice.

$$\sigma_{(Y_h,\bar{p}),t} = \mathrm{cov}(\bar{p}_t, Y^h_t) = \bar{p}_t Y^h_t \sigma^{Y_h}_t + \frac{\lambda_t(\alpha^H_t+\alpha^F-1)}{(\alpha^H_t+(1-\alpha^F)\lambda_t)(1-\alpha^H_t+\alpha^F\lambda_t)}\bar{p}_t Y^h_t \sigma^{Y_h}_t \Delta\kappa^h_t, \tag{29}$$

$$\sigma_{(Y_f,\bar{p}),t} = \mathrm{cov}(\bar{p}_t, Y^f_t) = -\bar{p}_t Y^f_t \sigma^{Y_f}_t + \frac{\lambda_t(\alpha^H_t+\alpha^F-1)}{(\alpha^H_t+(1-\alpha^F)\lambda_t)(1-\alpha^H_t+\alpha^F\lambda_t)}\bar{p}_t Y^f_t \sigma^{Y_f}_t \Delta\kappa^f_t. \tag{30}$$

The first terms in eqs. (29) and (30) reflect, respectively, the supply effect — e.g. higher production of the *home* good leads to higher relative price $\bar{p}_t = p_t^f / p_t^h$. The second term reflects consumption demand — how changes in relative wealth affect aggregate demand, and thus prices.

IV.B Stock Market Volatility and Cross-Country Correlation

The stocks' equilibrium diffusion vectors $\vec{\sigma}_t^{S_h}$ and $\vec{\sigma}_t^{S_f}$ from eqs. (5) and (6) are three-dimensional, with both stocks exhibiting sensitivity with respect to all three sources of risk — production risk of the *home* good (dW_t^h), production risk of the *foreign* good (dW_t^f), and demand risk (dW_t^*)[30]:

$$\vec{\sigma}_t^{S_h} = \begin{pmatrix} \frac{\beta}{\beta+(1-\beta)\bar{p}_t}\sigma_t^{Y_h} - \frac{\bar{p}_t(1-\beta)}{\beta+(1-\beta)\bar{p}_t}\frac{\lambda_t(\alpha_t^H+\alpha^F-1)}{(1-\alpha_t^H+\lambda_t\alpha^F)(\alpha_t^H+\lambda_t(1-\alpha^F))}\Delta\kappa_t^h \\ \frac{(1-\beta)\bar{p}_t}{\beta+(1-\beta)\bar{p}_t}\sigma_t^{Y_f} - \frac{\bar{p}_t(1-\beta)}{\beta+(1-\beta)\bar{p}_t}\frac{\lambda_t(\alpha_t^H+\alpha^F-1)}{(1-\alpha_t^H+\lambda_t\alpha^F)(\alpha_t^H+\lambda_t(1-\alpha^F))}\Delta\kappa_t^f \\ \frac{(1-\beta)\bar{p}_t}{\beta+(1-\beta)\bar{p}_t}\frac{1+\lambda_t}{(1-\alpha_t^H+\lambda_t\alpha^F)(\alpha_t^H+\lambda_t(1-\alpha^F))}\sigma_t^{\alpha} - \frac{\bar{p}_t(1-\beta)}{\beta+(1-\beta)\bar{p}_t}\frac{\lambda_t(\alpha_t^H+\alpha^F-1)}{(1-\alpha_t^H+\lambda_t\alpha^F)(\alpha_t^H+\lambda_t(1-\alpha^F))}\Delta\kappa_t^{\alpha} \end{pmatrix}^{\top}, \quad (31)$$

$$\vec{\sigma}_t^{S_f} = \begin{pmatrix} \frac{\beta}{\beta+(1-\beta)\bar{p}_t}\sigma_t^{Y_h} + \frac{\beta}{\beta+(1-\beta)\bar{p}_t}\frac{\lambda_t(\alpha_t^H+\alpha^F-1)}{(1-\alpha_t^H+\lambda_t\alpha^F)(\alpha_t^H+\lambda_t(1-\alpha^F))}\Delta\kappa_t^h \\ \frac{(1-\beta)\bar{p}_t}{\beta+(1-\beta)\bar{p}_t}\sigma_t^{Y_f} + \frac{\beta}{\beta+(1-\beta)\bar{p}_t}\frac{\lambda_t(\alpha_t^H+\alpha^F-1)}{(1-\alpha_t^H+\lambda_t\alpha^F)(\alpha_t^H+\lambda_t(1-\alpha^F))}\Delta\kappa_t^f \\ -\frac{\beta}{\beta+(1-\beta)\bar{p}_t}\frac{1+\lambda_t}{(1-\alpha_t^H+\lambda_t\alpha^F)(\alpha_t^H+\lambda_t(1-\alpha^F))}\sigma_t^{\alpha} + \frac{\beta}{\beta+(1-\beta)\bar{p}_t}\frac{\lambda_t(\alpha_t^H+\alpha^F-1)}{(1-\alpha_t^H+\lambda_t\alpha^F)(\alpha_t^H+\lambda_t(1-\alpha^F))}\Delta\kappa_t^{\alpha} \end{pmatrix}^{\top}. \quad (32)$$

The volatility of fundamental output and demand contribute to stock volatility, as captured by the first terms in all three vector elements of $\vec{\sigma}_t^{S_h}$ and $\vec{\sigma}_t^{S_f}$, respectively. The second terms reflect feedback effects, as determined by differences in investors' portfolio holdings reflected by $\Delta\vec{\kappa}_t^{\top} = \left[\Delta\kappa_t^h, \Delta\kappa_t^f, \Delta\kappa_t^{\alpha}\right]$.

When one or more constraints are binding, portfolios are distorted. The constraints drive a 'wedge' between the true levels of belief dispersion $\Delta\vec{m}_t^Y$, and the level of dispersion reflected by portfolios and prices, $\Delta\vec{\kappa}_t = \Delta\vec{m}_t^Y + \sigma_{S,t}^{-1}\left(\upsilon_t^F\iota_F - \upsilon_t^H\iota_H\right)$. How severely a constraint binds determines the magnitude of this wedge characterized by υ_t^i. When beliefs change such that the constraint binds more severely, υ_t^i becomes more negative. As long as this constraint remains binding, actual holdings of the constrained stock do not change when beliefs do. Nonetheless stock market

[30] These two vectors represent the first two rows of the 3×3 covariance matrix $\vec{\sigma}_{S,t}$.

dynamics change, as trade shifts into the other asset markets. If, for example, the *foreign* investor's constraint on holdings of S_t^h binds, he will hold the maximum amount allowed, $\pi_t^{S_h} = \varphi$, regardless of whether his desired holding is just slightly higher than φ (the constraint binds less severely) or substantially higher (the constraint binds more severely). But what does change with the severity of the binding constraint are his holdings of the other available assets. Investor F aims to compensate for his lack of S_t^h by holding its closest substitute, a highly correlated portfolio of the remaining stock S_t^f and the two country bonds, B_t^h and B_t^f. The more severely the constraint binds, the more of this substitute 'asset' he will hold.

IV.B.1 Liberalization Effects on Stock Markets' Correlation

Beyond exogenous supply and demand factors, stock return correlation is also affected by endogenous wealth movements between the investors.[31] The feedback effects captured by $\Delta\vec{\kappa}_t$ in $\vec{\sigma}_t^{S_h}$ and $\vec{\sigma}_t^{S_f}$ above stem from wealth transfers $d\lambda_t$: an increase in λ_t pushes up S_t^f and pushes down S_t^h. This opposite sign implies that the stronger these feedback effects are, the lower is correlation between the two stocks. The less similar investors' portfolio compositions are, the more relative wealth will change in response to asset returns. The resulting change in λ_t is greater, and correlation between stocks is lower. Liberalization leads to higher cross-market correlations only if it induces investors' portfolio holdings to become more similar than before, i.e. both are closer to holding the world market portfolio.

Consider as an illustration case F, in which only the *foreign* investor is constrained. Liberalization in this case means eliminating the constraint on F from the economy. F's binding constraint on S_t^h makes him appear less optimistic about the *home* country's investment opportunity: $\Delta\kappa_t^h < \Delta m_t^{Y_h}$. Whether this means the *magnitude* of $\Delta\kappa_t^h$ is larger or smaller than that of $\Delta m_t^{Y_h}$ depends on the level of belief dispersion. For liberalization to have any effect, one must look at parameter values — beliefs in particular — where this constraint binds. This is depicted in the second

[31]Countries' output is assumed to be uncorrelated in the model. This will not affect findings on *changes* to correlation in response to liberalization much, but would certainly affect the *levels* of correlation one would want to calibrate the model to.

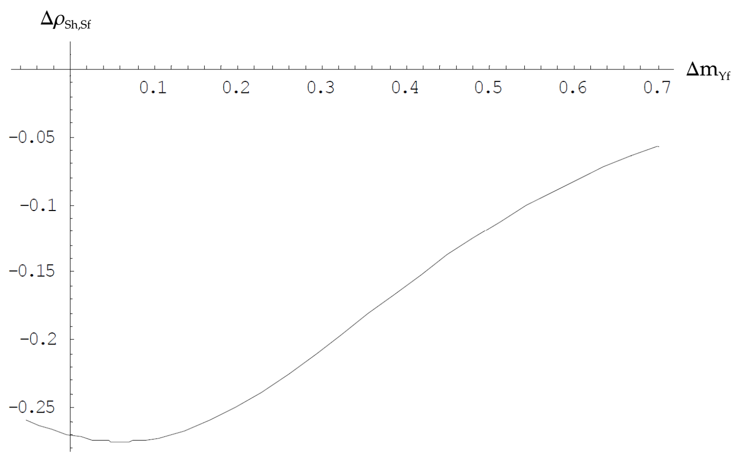

Figure 3: **Change in Correlation between Cases *F* and *U*** (small $\Delta m_t^{Y_h}$): When investor F's constraint on S_t^h binds only mildly, correlation between stock returns rises after liberalization (solid line, $\rho^{\text{caseF}}_{S^h,S^f} - \rho^{\text{caseU}}_{S^h,S^f}$).

quadrant of fig. (1). Immediately after liberalization, F will trade with H and purchase more of the previously restricted stock S_t^h. How much capital the *foreign* investor directs into the *home* stock market will determine the effect on correlation.

Assume first that belief dispersion about *home* country's growth rate is very small, indeed one can look at the special case of homogeneous beliefs about this country, $\Delta m_t^{Y_h} = 0$. H and F would like to hold similar proportions of wealth in S_t^h, but F's investment constraint does not allow this. F's portfolio reflects a more pessimistic view of *home* investment opportunities than he actually has: $\Delta\kappa_t^h$ is negative, even though $\Delta m_t^{Y_h} = 0$. The *magnitude* of belief dispersion is artificially increased by the constraint, and the 'domestic bias' in holdings of S_t^h generates a positive feedback effect for the stock. Now, when the constraint on F is lifted, portfolios adjust to the truly desired levels, $\Delta m_t^{Y_h} = 0$ will be reflected in prices. The feedback effect is eliminated, increasing correlation between the two markets. Fig. (3) provides a numerical illustration.

Now assume instead that F's constraint binds and he is much more optimistic than H about growth rates in *home*: $\Delta m_t^{Y_h}$ is strongly positive. If he were unconstrained, F would overweight the *home* stock heavily relative to the *home* investor H, thereby inducing a strong negative feedback effect in the stock. But the constraint prevents F from holding large positions of S_t^h, so holdings

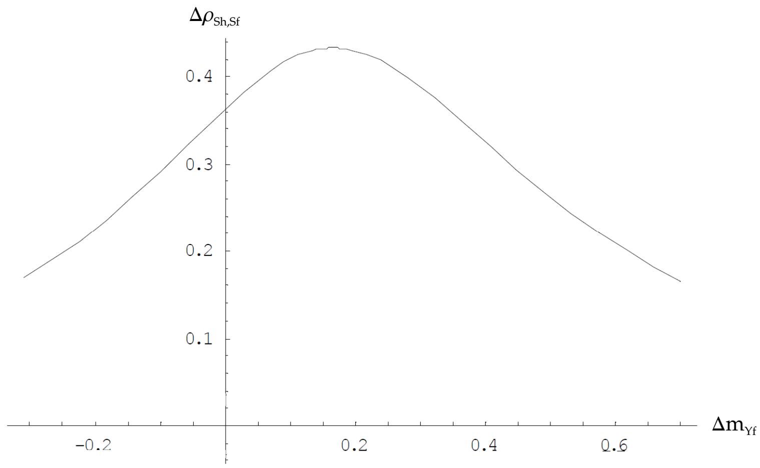

Figure 4: **Change in Correlation between Cases *F* and *U*** (large $\Delta m_t^{Y_h}$): When investor F is very optimistic and his constraint binds severely, liberalization leads to large capital flows into S_t^h, and correlation between stock returns falls (solid line, $\rho^{\texttt{caseF}}_{S^h,S^f} - \rho^{\texttt{caseU}}_{S^h,S^f}$).

are more similar across investors than they would be in absence of the constraint. This mitigates the feedback effect. Upon lifting the constraint, the newly liberalized *home* market receives large capital inflows from the *foreign* investor, who is very keen to invest in S_t^h. Portfolios diverge, and correlation across markets decreases. A numerical illustration of this can be seen in fig. (4).

The link between beliefs and the severity of binding constraints makes this hypothesis empirically testable, despite the fact that investor beliefs are not directly observable. Constraints bind most severely when investors' true beliefs are very different. Under those circumstances, opening, as in the above example, the *home* market to the *foreign* investor triggers large foreign capital inflows into the newly liberalized market, bringing down correlations between S_t^h and S_t^f post-liberalization. Conversely, market openings that are followed by weak capital inflows will see correlations rise post-liberalization.[32]

This result is not limited to the particular form of the *foreign* investor's constraint discussed here. The removal of any type of constraint that exacerbates portfolio differences will tend to raise correlations, as portfolios become more similar post-liberalization. This intuition remains the same

[32]Empirically, one would have to control for the fact that liberalization events often coincide across countries, thus redirecting capital flows into multiple countries simultaneously.

for the remaining cases where the other investor's or both constraints bind.[33]

The empirical studies mentioned earlier find that the effects of liberalization on stock markets' correlation with world markets varies across countries. This model provides a possible explanation for the disparity. Conditioning on the magnitude of post-liberalization capital inflows (or other related measures of how severely constraints were binding) should help explain the change in a country's correlation with world markets after liberalization. In interpreting the magnitudes of correlation changes in figures (3) and (4), it must be stressed that these are conditional moments. While the model is not calibrated, reasonable values for fundamental parameters lead to stock return volatilities in the unconstrained case of $10-15\%$ and positive correlations across the two markets.[34]

Another way of interpreting these results is along the time series dimension — how correlation changes as investors' beliefs change over time. Imposing constraints that restrict extreme portfolio positions of any investor lowers the variation of cross-market correlations over time. At low magnitudes of belief dispersion, where the unconstrained market would display high correlation, the binding constraint lowers correlation. At high magnitudes of belief dispersion, correlation would be low in absence of the constraint, and the binding constraint raises it. Any (rational) learning process of investors leads to variation in beliefs over time. The more uncertain investors are about a fundamental parameter, the more strongly beliefs change in response to signals. With large shifts in beliefs, portfolios would change swiftly in absence of constraints, causing relatively large swings in capital flows into and out of markets. Along with cross-country capital flows, stock markets' correlation would also be strongly time-varying. Portfolio constraints mitigate the extent to which portfolios in a particular market can react to changes in beliefs, thus mitigating the time-variation of correlation.[35] This effect is particularly strong in stock markets about which there is not only

[33]Again conditional on the parameter space where they bind, respectively.

[34]The two figures are based on the following fundamental economic parameters: $\alpha_t^H = \alpha^F = 0.7$, $\beta = 0.4$, $\varphi = 0.2$, $\lambda_t = 2$, $\sigma_t^{Y_h} = 0.12$, $\sigma_t^{Y_f} = 0.05$, $Y_t^h = 10$, $Y_t^f = 45$, $\sigma_t^{\alpha} = 0.02$,. For fig. (3), $\Delta m_t^{Y_h} = 0.2$, implying $m_{Y^h,t}^{(F)} - m_{Y^h,t}^{(H)} = 0.024$. For fig. (4), $\Delta m_t^{Y_h} = 1$, implying $m_{Y^h,t}^{(F)} - m_{Y^h,t}^{(H)} = 0.12$.

[35]The same intuition can apply for short-selling or margin constraints (though these equilibria are not determined in this paper): one investor's short position implies by market clearing a significant long position of another investor. Limiting short sales would prevent rapid swings in these positions, stabilizing correlation over time.

disagreement about growth rates, but a significant degree of uncertainty about these expectations.

One argument sometimes heard against financial market liberalization is that fickle — so-called 'hot money' — capital flows into a country destabilize its stock market. Inasmuch as time variation in correlations with world markets is considered to characterize instability, this model would support the notion. While the model predicts that liberalization's effect on correlation *levels* will depend on how severely the constraints were binding, the *variation* of correlations over time is predicted to unambiguously increase after liberalization. However, welfare implications about the desirability of such 'stability' cannot be directly drawn from this result.[36]

IV.B.2 Liberalization Effects on Stock Market Volatilities

The previous section showed that liberalization affects stock market correlations by strengthening or weakening the feedback effects. The same transmission channels also determine liberalization's effect on volatility, but for this the *magnitude* of the feedback effects play a bigger role. I will distinguish between implications for the composition of volatility — the sensitivity of a particular stock to different sources of risk — and its aggregate level.

When the *foreign* investor's constraint binds, restricting his access to S_t^h, the volatility of the *home* stock market is dominated by local risk factors. This supports the empirically robust finding that less integrated markets are more sensitive to local sources of risk than would be the case in a 'CAPM world', where only worldwide systematic risk is priced. A restriction on investor F's holdings of S_t^h tends to create (or exacerbate) a home bias in the holdings of that stock. The fact that H is forced to hold a larger portion of his wealth in S_t^h strengthens a positive feedback effect. A positive supply shock to Y_t^h pushes up the *home* stock price, and these wealth gains go disproportionately to investor H. His additional consumption stimulates his local economy, further boosting the stock market.[37] The positive feedback implies that S_t^h is very responsive to an initial local shock. The

[36]Much of the literature concerned with the effects of 'hot money' deals with the potentially adverse effects on the real economy and distortion of firms' investment decisions. This model is silent on these types of effects, due to the exchange economy setup.

[37]This effect is qualitatively the same also in absence of an intrinsic home bias. If F were more optimistic than H, the constraint prevents a strong 'foreign bias' in portfolios, weakening the associated negative feedback, which likewise increases the stock's sensitivity to local risk.

dominance of domestic risk in the stock market is consistent with Harvey (1995), who finds that emerging market returns are more likely than those of developed markets to be influenced by local information.[38]

A naive intuition for equilibrium effects of constraints centers around the idea that constraints prevent portfolio quantities from adjusting in response to exogenous shocks to the economy. Therefore, all adjustments in market clearing must go through prices, thus increasing volatility. This basic intuition, however, abstracts away from bond markets. A setup explicitly accounting for the presence of multiple assets including stocks and bonds allows us to study how investors deal with constraints when they can redirect investment, for which there is limited scope in a single-asset setup. Indeed, in most single-asset equilibrium models the riskfree asset plays a major role in accommodating market incompleteness for investors. The tight link between the consumption market in the single numeraire good and financial markets blocks this channel of transmission, and leaves the dynamics of the risky asset — the claim to the consumption good — largely unaffected. When allowing for multiple assets and multiple goods — where the world's aggregate consumption pattern across goods can change as wealth distribution changes — the link between the underlying good and its stock is not quite as rigid as in a single-good setup. As trade in one asset market is halted due to a binding constraint, trade moves into all the other asset markets, including bonds. This feature is appealing in light of empirical studies that find bond and stock markets react differently to international exposure and world market integration. While the previously mentioned series of papers by Harvey and Bekaert have shown that stock markets in less liberalized countries are strongly driven by local shocks and news, evidence on bond markets seem to indicate the opposite — bonds of countries that restrict access to their stock market tend to be more responsive to world shocks than to local shocks.[39]

The relationship $\Delta\vec{\kappa}_t = \Delta\vec{m}^Y_t + \sigma^{-1}_{S,t}(\upsilon^F_t \iota_F - \upsilon^H_t \iota_H)$ links beliefs with constrained investors' compensating investment choices. As investor i's constraint binds more severely, υ^i_t becomes more

[38]Harvey (1991) more generally finds that countries have a time-varying exposure to world covariance risk. While my model also supports this finding, it is surely not a unique link.

[39]See, e.g. Barr and Priestley (2004) for a study on international bond markets.

negative, and the covariance matrix $\vec{\sigma}_{S,t}$ determines how investment into all assets changes, to construct a correlated 'substitute' asset for the restricted stock. A side effect of this adjustment to the portfolio composition is that the investor's holdings of the restricted stock seem to reflect levels of belief dispersion that are inconsistent with his holdings of the other assets.

This effect can be seen particularly clearly for the demand shocks. Both investors know that preference shifts α_t^H follow a martingale, there is no disagreement about demand risk, which is therefore shared equally in an unconstrained market. However, when constraints bind, the distorted portfolios do reflect apparent disagreement: $\Delta\kappa_t^\alpha \neq 0$ in cases F, H, and FH.

Using again case F as an example, when the *foreign* investor's constraint is binding, he is underinvested in S_t^h relative to the beliefs reflected in his remaining portfolio of S_t^f and the two bonds. Since increases in α_t^H boost S_t^h, investor F's underweighting of this stock seems to imply that F is expecting a negative trend in α_t^H: $\Delta\kappa_t^{\alpha(\mathtt{caseF})} < 0$. This increases the impact of demand shocks on stock markets: any effect of $d\alpha_t^H$ on fundamentals is exacerbated by what looks to be priced disagreement.[40] Ghysels and Juergens (2001) show empirically that belief dispersion is a priced risk factor. This model suggests that when looking at stock markets that are subject to constraints (i.e. with a limited investor set), one can expect to see an exacerbated contribution of such extraneous risk factors, beyond their impact on variation of the firm's fundamentals. Investment constraints sever the link between beliefs and portfolio choice, making it more difficult to establish which underlying risk factor is priced, belief dispersion or limitations to risk sharing due to constraints.

A stock's total volatility is measured via the quadratic variation of the stocks with respect to all the sources of risk. This makes it more difficult to generalize implications from single to multiple binding constraints than in the previous sections on correlation and the composition of volatility. To contrast here the effects of partial and full liberalization, the interaction between multiple binding constraints is key. I therefore restrict the detailed analysis to the parameter space of $\Delta\vec{m}_t^Y$ where both constraints bind, in order to distinguish the impact of more and less extensive liberalization.[41]

[40]Formally, this is reflected in the stocks' volatility vectors in eqs. (31) and (32).

[41]Empirical studies of liberalization indicate that generally, constraints had been binding prior to being removed. Under non-logarithmic investors' utility, removing currently non-binding constraints would technically also have an impact on hedging portfolios, but these effects are likely to be quantitatively small.

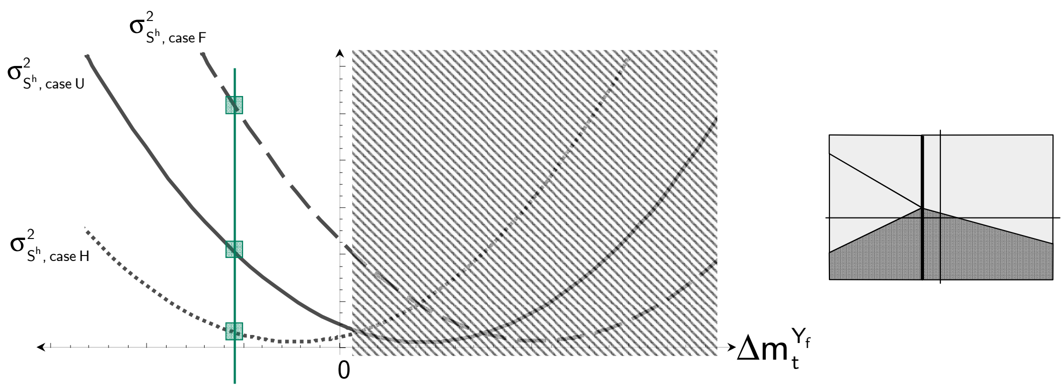

Figure 5: **Variance Under Constraints:** F's constraint shifts the parabola to the right ($\Delta\vec{\kappa}_t^{\text{case F}} < \Delta\vec{m}_t^Y$), meaning for any given level of belief dispersion, return variance is higher when the constraint binds. H's constraint shifts the parabola to the left ($\Delta\vec{\kappa}_t^{\text{case H}} > \Delta\vec{m}_t^Y$), implying a lower variance than in the unconstrained case for a given level of belief dispersion. The small inset graph recalls that both constraints can only bind in the indicated section of the parameter space, where $\Delta m_t^{Y_f}$ is negative (or only marginally positive). Therefore, the parabolas would not shift in the shaded area of the right quadrant, and are thus not displayed.

This is the case when $\Delta m_t^{Y_f}$ is relatively small or negative, as indicated in the two lower quadrants of fig. (1), and also recalled in the small inset graph of fig. (5).[42] In what follows I focus on the volatility effects for the *home* stock S_t^h. The effects of simultaneously binding constraints are best visible in this stock, as it is directly affected by both investors' constraints, whereas S_t^f is only directly affected by H's leverage constraint. The general intuition however applies analogously for S_t^f. For both stocks j, total variance of its returns, $\vec{\sigma}_t^{S_j}\vec{\sigma}_t^{S_j\top}$, is a parabolic function of belief dispersion, which follows directly from eqs. (31) and (32). When belief dispersion is zero, a stock's variance is determined exclusively by the variance of goods' fundamental output and demand shocks. As the *magnitude* of belief dispersion increases, the feedback effects due to wealth transfers become stronger, and variance increases. A binding constraint shifts this relationship, as variance then reflects $\Delta\vec{\kappa}_t$ rather than true beliefs $\Delta\vec{m}_t^Y$. To understand how the two constraints respectively shift this relationship, consider first only investor F's constraint. The fact that his portfolio reflects a less optimistic view about S_t^h shifts the parabola depicting variance as a function of belief dispersion

[42]The general intuition of how volatility is affected goes through for the parameter space where constraints bind individually. Details for these cases can be found in the appendix.

$\Delta\vec{m}_t^Y$ to the right. For any level of belief dispersion, e.g. the level of $\Delta m_t^{Y_f}$ indicated by the vertical green line in fig. (5), the stock's variance will reflect a seemingly more negative level of belief dispersion if the constraint binds. The point of intersection between the vertical green line and the shifted (dashed) parabola is consistent with a stock variance resulting from more negative levels of $\Delta m_t^{Y_f}$.

In contrast, consider the effects of H's constraint in isolation. Now H's optimism is not fully reflected in markets, the parabola is shifted to the left: the stock's variance is consistent with *less* negative levels of $\Delta m_t^{Y_f}$, as seen at the intersection of the vertical green line with the dotted parabola.

The two constraints of H and F have opposing effects on volatility, they 'compete' in a sense. When they are imposed simultaneously, both constraints prevent the respective investor from taking on very large portfolio positions. F's constraint prevents him from being heavily invested into S_t^h, but this potentially exacerbates an already existing 'home bias' of investor H. H's constraint, on the other hand, prevents this distortion from becoming extreme: large positions of investor H in stock markets are also restricted. In equilibrium, the two constraints mitigate one another when they bind simultaneously.

The interesting question for explaining — or predicting — a market's response to liberalization is which of the two constraints dominates the effect on volatility. Fig. (6) shows that the net effect on volatility depends on investors' relative assessments of investment opportunities: the more severely binding constraint will dominate. F's constraint tends to push up the volatility of S_t^h by exacerbating a home bias in holdings of the stock, while H's constraint tends to push it down relative to the unconstrained equilibrium, by limiting the extent of such a home bias. The grey parabolas depict the joint effect of the two constraints. In the left graph of fig. (6), when F is sufficiently optimistic about the *home* country's growth rate, his constraint dominates. F's optimism about growth rates in the *home* country means he would prefer to invest significantly more into S_t^h than he is allowed to. Accordingly, H is required to hold more of the stock in equilibrium than he would absent the constraint. However, he can only provide some liquidity to the market — until he reaches his own portfolio restriction. H's constraint binds because he acts as liquidity provider, rather than binding

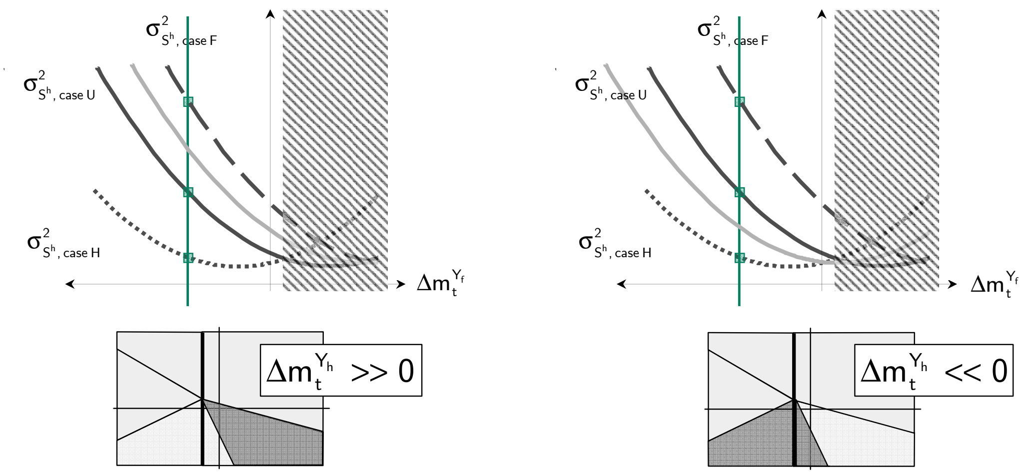

Figure 6: **The Dominant Constraint:** The more severely binding of the constraints will dominate the effect on volatility. For $\Delta m_t^{Y_h}$ large and positive (left graph), F is exceedingly optimistic regarding investment opportunities in *home*, thus his constraint binds more severely. When $\Delta m_t^{Y_h}$ is small or potentially negative (right graph), F is only constrained due to the liquidity he needs to provide in response to H's constraint binding. H's constraint binds more severely and thus dominates the effect on volatility.

because of his strong optimism regarding investment opportunities. During the times that both constraints remain binding, holdings of S_t^h and S_t^f must remain the same even when beliefs, and thus the perceived severity of the binding constraints, change. Any trade to compensate for this shifts into the international bond markets. Only through this channel will volatility in all markets be affected, despite lack of trade. As soon as constraints are lifted, F will seize the opportunity to buy more of the strongly desired *home* stock, and sell his local, the *foreign* stock. While the constraints were binding, he was holding more of S_t^f than optimal, because H could not hold it himself — he was providing liquidity to the *home* market.

Conversely, the graph on the right side of the same figure depicts the case where H's leverage constraint dominates the aggregate effect on volatility. Here, H is overall the more optimistic of the two investors, his constraint binds more severely. He would like to hold large positions in both stocks, but cannot lever up. F is required to provide liquidity and hold more stocks. As for H in the previous case, it is the demand for liquidity provision that causes F to ultimately hit his constraint. Being a rational investor who seeks some degree of diversification in line with his beliefs, F prefers to purchase more of both assets. However, his liquidity provision in the *home* market is restricted by his own constraint. When his constraint starts binding, trade in S_t^h ceases, and he can only provide additional liquidity in the market for S_t^f. If the two constraints are simultaneously removed under these circumstances, H will buy more of both the *home* and the *foreign* stocks. Importantly, this implies that even though F's constraint had been binding, he will, upon liberalization, choose to withdraw capital from the *home* market. This seems counterintuitive at first, and would not arise in a setup that looks only at individual constraints in isolation. This intuition is confirmed by recalling the parameter conditions under which constraints respectively bind. If F's constraint were imposed in isolation, it would not bind when H is the more optimistic investor: at strongly negative levels of $\Delta m_t^{Y_h}$. Jointly, the effect of the dominant leverage constraint on H implies that in response to full liberalization — the removal of both constraints — H will lever up his portfolio, inducing an increase in volatility.

Linking this result back to the empirical findings that this paper tries to explain, consider again the previously mentioned examples of liberalization in Latin America and Asia. Recall that volatility in Latin American markets dropped post-liberalization, which is consistent with the equilibrium displayed in the left graph of fig. (6) — volatility dropping from the grey to the solid black line. Asian markets on the other hand experienced a rise in volatility post-liberalization, consistent with the graph on the right, also shifting from the grey to the solid black line. The model suggests that restrictions on foreign capital inflows into Latin America were more severely binding than constraints on local investors' investment choices. In contrast, the investment restrictions on local Asian investors were more severely binding than the constraints preventing foreigners from investing into Asian stock markets.[43] This is consistent with the findings on the 'A-B share premium': Chinese investors were willing to pay a higher price than international investors for what

[43]Indonesia and Malaysia are two examples of Asian countries where funds, banks and insurance companies are limited in the amount of foreign securities they hold. Brazil opened up their stock markets to non-residents in 2000, though reiterated in 2003 limitations on foreign investment into certain industries, such as nuclear energy, health, media, rural property and banking. Chile for a long time had minimum-stay requirements for foreign capital and imposed an upper bound on the percentage of a firm's stock that can be owned by foreign investors. Many other examples exist, where in most cases constraints on domestic and foreign capital coexist, but the important issue is which type of constraint dominates.

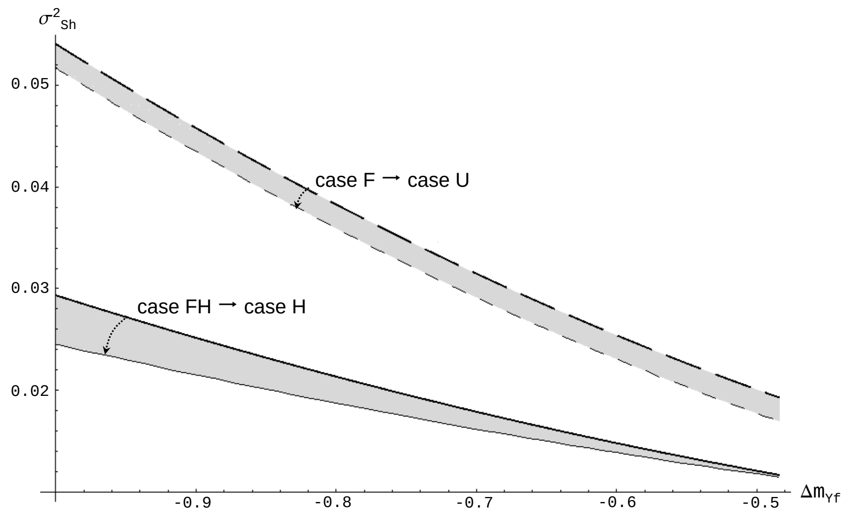

Figure 7: **Variance changes on removing constraint on investor F:** Partial liberalization (case $FH \rightarrow$ case H) decreases volatility of S^h_t. When F's constraint is the last one to remain, its elimination leads to fully liberalized markets, and volatility also decreases, albeit at a higher absolute level.

was really an identical claim. Taking into account the increase in volatility post-liberalization, the model suggests that this may have been due to lack of other investment opportunities for Chinese investors, rather than pure optimism.

Figures (7) and (8) illustrate in more detail how the volatility effects of partial and full liberalization will differ.[44] The first of these figures shows two scenarios where F's constraint is removed, with otherwise identical economic fundamentals. If F's constraint was the only remaining one, removing it moves the economy from *case F* to *case U*. If, prior to removing F's constraint, both investors were bound by constraints, removing F's constraints will result in partial liberalization, the economy moves from *case FH* to *case H*. In either case, the removal of F's constraint lowers volatility. However, when H's constraint remains in place, the drop in the stock's volatility shifts it even further away from the market's 'unconstrained' level of volatility: $\sigma^{S_h}_{t,\texttt{caseH}} < \sigma^{S_h}_{t,\texttt{caseFH}} < \sigma^{S_h}_{t,\texttt{caseU}}$.

The second graph, fig. (8), compares the effects of removing H's leverage constraint in the analogous situations — partial versus full liberalization. Removing only constraint H and leaving

[44]Parameter values of fundamentals are as in the previous illustrations, and beliefs here are such that H's constraint dominates.

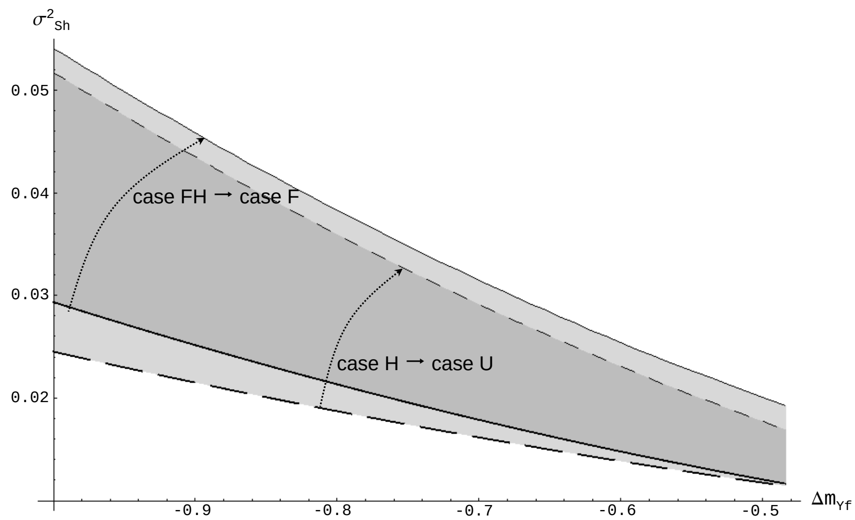

Figure 8: **Variance changes on removing constraint on investor H:** Both incidences of liberalization, case FH → case F and case H → case U increase volatility of S_t^h.

the constraint on F in place will make the volatility 'overshoot', rising above the unconstrained level of volatility: H will choose to buy stocks from investor F, exacerbating the disparity in portfolio holdings and thus feedback effects, increasing conditional volatility. If F's remaining constraint is now still binding, H will be disproportionately buying S_t^f, the stock investor F is more willing to give up. This exacerbates feedback effects more than if F were willing to sell a more even portfolio of the two stocks.[45]

The model is set up with two quite particular constraints, for technical reasons. But the main insights can be generalized beyond the types of constraints discussed. The feature that ultimately determines whether a particular binding constraint puts upward or downward pressure on volatility is its impact on portfolio differences across investors. Stock market volatility is determined purely by economic fundamentals and intrinsic correlation if expectations are identical, leading all investors across the world to hold the world market portfolio. As portfolio holdings deviate from this benchmark, the impact of fundamental shocks on stock prices is amplified. Constraints that

[45]If investor F is not particularly optimistic about S_t^h growth rates, he is happy to hold less than his limit φ once H is able to take on his desired portfolio. The equilibrium would then jump immediately to the unconstrained case U, investor F's constraint will no longer be binding when H's is removed.

exacerbate differences in portfolio holdings will therefore lead to higher volatility in that market. This relationship will hold, regardless of the nature of the constraint in question — short selling constraints, foreign investment constraints or leverage constraints: whether a particular restriction amplifies or a dampens the asset's volatility depends on how severely portfolio choice is distorted, and in which direction.

V Conclusion

The paper analyzes in a two-country setup how separate restrictions on international capital flows interact to affect stock market dynamics, specifically market volatility and correlation. Countries' liberalization efforts of the recent decades have taken place in an environment of incomplete liberalization, and some restrictions still remain. This model aims to help explain how the circumstances under which liberalization takes place affect the impact it has on stock markets.

How severely a constraint binds determines the aggregate effect on market dynamics post-liberalization. When the liberalization of a market results in investors' worldwide portfolios becoming more similar, markets' correlations rise, and total volatility falls post-liberalization. Liberalization also decreases a market's 'excess' sensitivity to extraneous risk factors, as constraints' distortions to portfolios are eliminated. But as investors' assessment of international investment opportunities varies over time, existing constraints may bind more or less severely. If a constraint is removed at a time when disagreement between investors about economic fundamentals is large, the newly liberalized market's volatility increases as investors' international portfolio holdings diverge.

Even though in the model, the constraints on the countries' investors are imposed exogenously, studying the interaction between multiple binding constraints can nevertheless give some intuition on the repercussions of 'step-wise' liberalization processes. While removing either one of the constraints individually can be seen as an event of 'partial liberalization', the effects of removing capital outflow restrictions is indeed opposite to those of removing capital inflow restrictions. When multiple constraints are lifted simultaneously, the net effect on stock dynamics will be dominated by the more severely binding of the constraints. Thus, in order to predict the reaction of a market to a liberalization event, one needs to be aware not only of which other constraints are in place, but how investors assess investment opportunities in different countries, i.e. how severely these different constraints distort portfolio choice.

The result that partial and full liberalization can have qualitatively very different results demonstrates that understanding countries' objectives for capital market regulation, and thus incorporating endogenous liberalization decisions into models, remains an important question for further research on capital market integration.

Appendix

A Optimal Consumption

Investors H and F maximize their respective expected utility, subject to budget constraints. Equilibrium is established by maximizing the aggregated utility function

$$U(C_H, C_F) = u_H\left(C^h_{H,t}, C^f_{H,t}\right) + \lambda_t u_F\left(C^h_{F,t}, C^f_{F,t}\right)$$

where

$$\begin{aligned} u_H\left(C^h_{H,t}, C^f_{Ht}\right) &= \alpha^H_t \log C^h_{Ht} + (1-\alpha^H_t)\log C^f_{H,t}, \\ u_F\left(C^h_{F,t}, C^f_{Ft}\right) &= (1-\alpha^F)\log C^h_{Ft} + \alpha^F \log C^f_{F,t}, \end{aligned}$$

and $\lambda_t = \frac{y_H \xi^H_t}{y_F \xi^F_t}$, the ratio of investors' state price densities.

FOC of optimal consumption of goods $j = h, f$, of investors $i = H, F$: $u^i_{C^j}(\cdot) = \frac{\partial u_i\left(C^i_{it}, C^j_{it}\right)}{\partial C^j_{it}} = y_i p^j_t \xi^i_t$, where p^j_t is the relative price of good j, ξ^i_t is investor i's state price density and y_i the associated Lagrange multiplier, reflecting initial endowment.

	investor H:	*investor F:*
good h:	$\frac{\alpha^H_t}{C^h_{Ht}} = y_H p^h_t \xi^H_t$	$\frac{1-\alpha^F}{C^h_{Ft}} = y_F p^h_t \xi^F_t$
good f:	$\frac{1-\alpha^H_t}{C^f_{Ht}} = y_H p^f_t \xi^H_t$	$\frac{\alpha^F}{C^f_{Ft}} = y_F p^f_t \xi^F_t$

Market clearing requires $\sum_i C^j_i = Y^j$ for both goods $j = h, f$, giving equilibrium total consumption in section 4.

B Optimal Wealth

Current wealth is an appropriately discounted value of all future consumption levels. Log utility in a finite horizon economy implies that both investors will consume a fixed portion of their wealth each period, as a function of the time remaining. The below is described for investor H, analogous values for investor F follow directly.

$$X_t^H = \frac{1}{\xi_t^H} E\left[\int_t^T \left(\xi_s^H p_s^h C_{Hs}^h + \xi_s^H p_s^f C_{Hs}^f\right) ds\right]$$

From FOC above, $\frac{\alpha_t^H}{y_H} = C_{Ht}^h p_t^h \xi_t^H$ and $\frac{1-\alpha_t^H}{y_H} = C_{Ht}^f p_t^f \xi_t^H$ holds, therefore:

$$X_t^H = \frac{1}{\xi_t^H} E\left[\int_t^T \left(\frac{\alpha_s^H}{y_H} + \frac{1-\alpha_s^H}{y_H}\right) ds\right] = \frac{1}{y_H \xi_t^H}(T-t).$$

Linking wealth X_t^i back to consumption above gives $X_t^H = C_{Ht}^h \cdot \frac{p_t^h}{\alpha_t^H}(T-t)$. Analogously for investor F: $X_t^F = \frac{1}{y_F \xi_t^F}(T-t)$.

C Relative Goods Prices

The relative price of the two goods is determined by their relative marginal utilities, which must be equal across the two agents, since both are faced with identical prices for goods, there are no frictions in goods markets: $\bar{p}_t = \frac{p_t^f}{p_t^h} = \frac{u^i_{C^f}(\cdot)}{u^i_{C^h}(\cdot)}$. The basket of goods $\beta p_t^h + (1-\beta)\, p_t^f = 1$ defines the numeraire. $\beta \in [0,1]$ and represents the weight of the *home* good in the basket. This weight does not represent either agent's de facto consumed basket. The levels of stock prices will be affected by the chosen β, but the relation between the two stocks will not be. Interesting special cases include $\beta = 0$, $\beta = 1$ or $\beta = \alpha^F$, denoting Y_t^f, Y_t^h or F's true consumption basket as the numeraire, respectively. The main insights from the paper are not sensitive to the choice of β.

Using the equilibrium marginal utilities from market clearing restrictions $\sum_i C_i^j = Y^j$ for goods $j = h, f$ gives:

$$\bar{p}_t = \frac{p_t^f}{p_t^h} = \frac{u^H_{C^f}(\cdot)}{u^H_{C^h}(\cdot)} = \frac{y_H p_t^f \xi_t^H}{y_H p_t^h \xi_t^H} = \frac{(1-\alpha_t^H) + \alpha^F \lambda_t}{\alpha_t^H + (1-\alpha^F)\lambda_t} \frac{Y_t^h}{Y_t^f}.$$

The dynamics of relative goods prices $\bar{p}_t$ follow

$$\begin{aligned} d\bar{p}_t \quad = \quad & (\cdot)dt + \frac{1-\alpha_t^H + \alpha^F \lambda_t}{\alpha_t^H + (1-\alpha^F)\lambda_t} \frac{1}{Y_t^f} dY_t^h - \frac{1-\alpha_t^H + \alpha^F \lambda_t}{\alpha_t^H + (1-\alpha^F)\lambda_t} \frac{Y_t^h}{(Y_t^f)^2} dY_t^f - \\ & - \frac{\lambda_t + 1}{(\alpha_t^H + (1-\alpha^F)\lambda_t)^2} \frac{Y_t^h}{Y_t^f} d\alpha_t^H + \frac{2\alpha_t^H - 1}{(\alpha_t^H + (1-\alpha^F)\lambda_t)^2} \frac{Y_t^h}{Y_t^f} d\lambda_t. \end{aligned}$$

D Auxiliary Market: Portfolio Choice in Constrained Markets

The constraints studied are limitations on the fraction of wealth $\pi_{i,t}^j$ that investor i places into one or more assets j. I assume that portfolio positions $\pi_{i,t}^j$ in assets $j = S_t^h, S_t^f, B_t^h, B_t^f$ are constrained to lie in a closed, convex, non-empty set K that contains the origin. The analysis here is based on the methodology developed in Cvitanic and Karatzas (1992).

The martingale analysis of incomplete markets requires the construction of a fictitious market that fictitiously augments the market parameters of the original constrained market. Under these augmented market parameters, the constrained investor will optimally choose a portfolio permissible within the constraints. This is then the optimal portfolio also under the original, constrained market.[46]

The set of admissible trading strategies is defined by the set K, the support function is $\delta(\upsilon_t^i) \equiv \delta(\upsilon_t^i|K) \equiv \sup\left(-\pi_{i,t}^\top \upsilon_t^i : \pi_{i,t} \in K\right)$ and the barrier cone of the set $-K$ is defined as $\bar{K} \equiv \left\{\upsilon_t^i \in \mathbb{R}^2 | \delta(\upsilon_t^i) < \infty\right\}$. υ_t^i is a square-integrable, progressively measurable process taking values in $\bar{K}$ to ensure boundedness.

Both investors' respective state price densities adjust to reflect these augmented market perceptions due to the constraints:

$$d\xi_t^i = -\left(r_t + \delta(\upsilon_t^i)\right)\xi_t^i dt - {\kappa_t^i}^\top \xi_t^i d\vec{W}_t^{(i)}, \tag{33}$$

where investor i's adjusted market price of risk is $\vec{\kappa}_t^i = (\sigma_{S,t}^{-1})\left(m_{S,t}^{(i)} + \upsilon_t^i \iota_i - r_t \mathbf{1}\right) = \kappa_{o,t}^i + \sigma_{S,t}^{-1}\upsilon_t^i$. $\kappa_{o,t}^i$ represents the market price of risk that the investor would base his portfolio decisions on, i.e. those reflecting his true beliefs. The second term, $+\sigma_{S,t}^{-1}\upsilon_t^i$, adjusts the market price of risk s.t. the investor does not violate his constraint, and at the same time captures the market price of risk that will be reflected in portfolio choice and thus equilibrium market prices. In this auxiliary market, based on

[46] This setting is a straightforward application of that in Cvitanic and Karatzas (1992), and it can be easily shown that their convex duality approach for convex constraint sets holds here.

which investor i makes his decisions, the prices of stocks and the 'world bond' follow:

$$\begin{aligned} dB_t^w &= \left(r_t^w + \delta(\upsilon_t^i)\right) B_t^w dt, \\ dS_t &= I_S \left(m_{S,t}^{(i)} + \upsilon_t^i \iota_i + \delta(\upsilon_t^i)\right) dt + I_S \sigma_{S,t} d\vec{W}_t^{(i)}, \end{aligned}$$

where I_S is a diagonal matrix whose entries are the time-t stock prices and $d\vec{W}_t^{(i)}$ is the 3-dimensional vector of investor i's perception of innovation processes, as defined by eq. (12).

Investor H's leverage constraint $\iota_H^\top \pi_t^H \leqslant 1$ paired with his optimal trading strategy $\pi_t^H = (\sigma_{S,t}^{-1})^\top \left[\kappa_{ot}^H + \sigma_S^{-1} \iota_H \upsilon_t^H\right]$ from eq. (18) gives

$$\upsilon_t^H = \min\left(\frac{1 - \iota_H^\top (\sigma_{S,t}^{-1})^\top \kappa_{ot}^H}{\iota_H^\top (\sigma_{S,t}^{-1})^\top \sigma_{S,t}^{-1} \iota_H}, 0\right), \qquad \delta(\upsilon_t^H) = -\upsilon_t^H = \max\left(-\frac{1 - \iota_H^\top (\sigma_{S,t}^{-1})^\top \kappa_{ot}^H}{\iota_H^\top (\sigma_{S,t}^{-1})^\top \sigma_{S,t}^{-1} \iota_H}, 0\right). \tag{34}$$

Note that a leverage constraint does not impact the two assets individually but rather only the joint holding, so investor H's adjustments υ_t^H are the same for both assets.

For investor F's constraint, the adjustments in the auxiliary market are:

$$\upsilon_t^F = \min\left(\frac{\varphi - \iota_F^\top (\sigma_{S,t}^{-1})^\top \kappa_{ot}^F}{\iota_F^\top (\sigma_{S,t}^{-1})^\top \sigma_{S,t}^{-1} \iota_F}, 0\right), \qquad \delta(\upsilon_t^F) = -\varphi \upsilon_t^F = \max\left(-\varphi \frac{\varphi - \iota_F^\top (\sigma_{S,t}^{-1})^\top \kappa_{ot}^F}{\iota_F^\top (\sigma_{S,t}^{-1})^\top \sigma_{S,t}^{-1} \iota_F}, 0\right). \tag{35}$$

Equilibrium υ_t^i's will be solved for in terms of fundamentals in a later section of the appendix.

E State Price Density

Investor H consumes a fraction $\frac{\alpha_t^H}{\alpha_t^H + (1-\alpha^F)\lambda_t}$ of good Y_t^h and a fraction $\frac{1-\alpha_t^H}{1-\alpha_t^H + \alpha^F \lambda_t}$ of good Y_t^f. This and equilibrium relative prices $\bar{p}_t$ gives

$$\xi_t^H = \beta \frac{\alpha_t^H + (1-\alpha^F)\lambda_t}{y_H Y_t^h} + (1-\beta)\,\frac{1 - \alpha_t^H + \alpha^F \lambda_t}{y_H Y_t^f}. \tag{36}$$

Analogously, investor F consumes a fraction $\frac{\lambda_t(1-\alpha^F)}{\alpha_t^H+(1-\alpha^F)\lambda_t}$ of good Y_t^h and a fraction $\frac{\lambda_t\alpha^F}{1-\alpha_t^H+\alpha^F\lambda_t}$ of good Y_t^f:

$$\xi_t^F = \beta\frac{\alpha_t^H + (1-\alpha^F)\lambda_t}{\lambda_t y_F Y_t^h} + (1-\beta)\,\frac{1-\alpha_t^H+\alpha^F\lambda_t}{\lambda_t y_F Y_t^f}. \tag{37}$$

F Asset Valuation

Proof of Proposition 1:

Valuing the stock under either investor i's information measure, there must be an $\mathcal{F}_t^i$ measurable process z_t^i such that $S_t^j \equiv E_t\left[\int_t^T \xi_s^i/\xi_t^i p_s^j Y_s^n ds\right] + E_t\left[\int_t^T z_s^{i,j}/\xi_t^i ds\right]$.[47]

Expanding this, $\xi_t^i S_t^j + \int_0^t \xi_s^i p_s^j Y_s^j ds + \int_0^t z_s^i ds = E_t\left[\int_0^T \xi_s^i p_s^j Y_s^j ds\right] + E_t\left[\int_0^T z_s^{i,j} ds\right]$ is a martingale for all $t \in [0,T]$.

Accordingly, discounted cum-dividend stock returns using the adjusted state price density will have an expected value of $E_t \int_t^T \left[d\xi_s^i S_s^j + \xi_s^i p_s^j Y_s^j ds\right] = -E_t \int_t^T z_s^{i,j} ds$.

Using the adjusted state price density in eq. (33) and the previously defined notation for stock dynamics $dS_t^j = m_{S^j,t}^{(i)} S_t^j dt + \sigma_t^{S_j} S_t^j d\vec{W}_t^{(i)}$ gives $z_s^{i,j} = \left(\delta(\upsilon_t^i) + \upsilon_{(j),t}^i\right) S_t^j \xi_t^i$. The j'th element of $\upsilon_t^i \iota_i$ is the speculative premium for asset j, and $\delta(\upsilon_t^i)$ the collateral premium.

Market clearing in asset markets requires

$$S_t^h + S_t^f = X_t^H + X_t^F = p_t^h Y_t^h (T-t) + p_t^f Y_t^f (T-t). \tag{38}$$

Each asset $j = h, f$ is valued as the sum of discounted dividends, taking into account the effects of future binding constraints — the second integral in the equation below.

$$S_t^j = \frac{1}{\xi_t^H} E_t\left[\int_t^T \xi_s^H p_s^j Y_s^j ds\right] + \frac{1}{\xi_t^H} E_t\left[\int_t^T \left(\upsilon_t^H + \delta(\upsilon_t^H)\right) S_s^j \xi_s^H ds\right] \qquad j = h, f.$$

Using $\frac{1}{p_t^h \xi_t^H} = \frac{Y_t^h y_H}{\alpha_t^H + (1-\alpha^F)\lambda_t}$ and goods market clearing, as well as $\lambda_t = \frac{y_H \xi_t^H}{y_F \xi_t^F}$ in the pricing function

[47] The proof of the stock price valuation closely follows that of Detemple and Murthy (1997).

of S_t^h:

$$S_t^h = p_t^h Y_t^h (T-t) + \frac{p_t^h Y_t^h}{\alpha_t^H + (1-\alpha^F)\lambda_t}(1-\alpha^F)\left[E_t \int_t^T \lambda_s ds - \lambda_t (T-t)\right] \tag{39}$$
$$+\frac{y_H p_t^h Y_t^h}{\alpha_t^H + (1-\alpha^F)\lambda_t} E_t \left[\int_t^T \left(\upsilon_t^H + \delta(\upsilon_t^H)\right) S_s^h \xi_s^H ds\right]$$

$$S_t^f = p_t^f Y_t^f (T-t) + \frac{p_t^f Y_t^f}{1-\alpha_t^1 + \alpha^2 \lambda_t}\alpha^F \left[E_t \int_t^T \lambda_s ds - \lambda_t (T-t)\right] \tag{40}$$
$$+\frac{y_H p_t^f Y_t^f}{1-\alpha_t^H + \alpha^F \lambda_t} E_t \left[\int_t^T \left(\upsilon_t^H + \delta(\upsilon_t^H)\right) S_s^f \xi_s^H ds\right]$$

Under the constraints on investors H and F as described by eq. (10), $E_t\left[\int_t^T \left(\upsilon_t^H + \delta(\upsilon_t^H)\right) S_s^i \xi_s^H ds\right] \leqslant 0$ can be shown to hold. $d\lambda_t$ is a supermartingale under all four possible equilibria. All terms in eqs. (40) and (41) except $p_t^j Y_t^j (T-t)$ are non-positive, thus for the equilibrium pinned down by eq. (38), they must all be zero in equilibrium. Therefore,

$$\begin{aligned} S_t^h &= p_t^h Y_t^h (T-t), \\ S_t^f &= p_t^f Y_t^f (T-t), \end{aligned} \tag{41}$$

where p_t^h and p_t^f can be rewritten in terms of $\bar{p}_t$.

In equilibrium, the adjustments to perceived investment opportunities (eqs.(34), (35)) are

case F:

$$\upsilon_t^F = \frac{\left[\Delta m_t^{Y_h} \sigma_t^{Y_h} - \left(\varphi(1+\lambda_t) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)(\sigma_t^{Y_h})^2\right](\sigma_t^\alpha)^2}{\left[\lambda_t(1-\varphi)\left(\alpha_t^H + \alpha^F - 1\right) - \left(1-\alpha_t^H + \alpha^F \lambda_t\right)\alpha_t^H\right]^2 (\sigma_t^{Y_h})^2 + (\sigma_t^\alpha)^2}; \qquad \upsilon_t^H = 0.$$

case H:

$$\upsilon_t^F = 0; \qquad \upsilon_t^H = \frac{\sigma_t^{Y_h}\sigma_t^{Y_f}\left(\Delta m_t^{Y_h}\sigma_t^{Y_f} + \Delta m_t^{Y_f}\sigma_t^{Y_h}\right)}{(\sigma_t^{Y_h})^2 + (\sigma_t^{Y_f})^2}.$$

case FH:

$$\upsilon_t^F = \frac{\left[\Delta m_t^{Y_f}\sigma_t^{Y_f} - \Delta m_t^{Y_h}\sigma_t^{Y_h} + \left(\varphi(1+\lambda_t) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)((\sigma_t^{Y_h})^2 + (\sigma_t^{Y_f})^2)\right](\sigma_t^\alpha)^2}{\left[\lambda_t(1-\varphi)\left(\alpha_t^H + \alpha^F - 1\right) - \left(1 - \alpha_t^H + \alpha^F\lambda_t\right)\alpha_t^H\right]^2((\sigma_t^{Y_h})^2 + (\sigma_t^{Y_f})^2) + (\sigma_t^\alpha)^2};$$

$$\begin{aligned}\upsilon_t^H = &\frac{\left[\lambda_t(1-\varphi)\left(\alpha_t^H + \alpha^F - 1\right) - \left(1 - \alpha_t^H + \alpha^F\lambda_t\right)\alpha_t^H\right]^2\sigma_t^{Y_h}\sigma_t^{Y_f}\left(\Delta m_t^{Y_h}\sigma_t^{Y_f} + \Delta m_t^{Y_f}\sigma_t^{Y_h}\right)}{\left[\lambda_t(1-\varphi)\left(\alpha_t^H + \alpha^F - 1\right) - \left(1 - \alpha_t^H + \alpha^F\lambda_t\right)\alpha_t^H\right]^2((\sigma_t^{Y_h})^2 + (\sigma_t^{Y_f})^2) + (\sigma_t^\alpha)^2}\\ &+ \frac{\Delta m_t^{Y_f}\sigma_t^{Y_f}(\sigma_t^\alpha)^2 + \left(\varphi(1+\lambda_t) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)(\sigma_t^{Y_f}\sigma_t^\alpha)^2}{\left[\lambda_t(1-\varphi)\left(\alpha_t^H + \alpha^F - 1\right) - \left(1 - \alpha_t^H + \alpha^F\lambda_t\right)\alpha_t^H\right]^2((\sigma_t^{Y_h})^2 + (\sigma_t^{Y_f})^2) + (\sigma_t^\alpha)^2}.\end{aligned}$$

G Volatility Effects

The effects on volatility of removing one or more constraints are based on the closed-form solutions attained via the equilibrium dynamics of eqs. (21) and (22). As total volatility is the root of a sum of squared vector elements, the signs of volatility or variance changes after liberalization have been verified to a first approximation via Taylor expansion. Establishing the direction of these changes under consideration of the parameter conditions under which a particular liberalization can happen proved to be intractable without simplification. The change in volatility when the economy moves from `case (X)` to `case (Y)` (where `(X)` and `(Y)` are, e.g., `(FH)` and `(F)`, respectively) is determined in the following way:

$$d(\sigma_t^{S_h})^2 = \frac{\partial(\sigma_t^{S_h})^2}{\partial\Delta\kappa_t^h}d\Delta\kappa_{t|\mathtt{X}\to\mathtt{Y}}^h + \frac{\partial(\sigma_t^{S_h})^2}{\partial\Delta\kappa_t^f}d\Delta\kappa_{t|\mathtt{X}\to\mathtt{Y}}^f + \frac{\partial(\sigma_t^{S_h})^2}{\partial\Delta\kappa_t^\alpha}d\Delta\kappa_{t|\mathtt{X}\to\mathtt{Y}}^\alpha \tag{42}$$

where $d\Delta\kappa^{j}_{t|\mathtt{X}\to\mathtt{Y}}$ is the change in the market prices of risk reflected by the equilibrium when suddenly switching from equilibrium case (X) to case (Y), upon removal of one (or both) constraints.[48]

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[48]For space reasons, individual proofs about all possible liberalization cases ($FH \to F$, $FH \to H$, $FH \to U$, $F \to U$ and $H \to U$) are available from the author upon request.

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