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Uncertainty on the  
Tragedy of the Commons

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# The Effect of Environmental Uncertainty on the Tragedy of the Commons

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We model a common pool resource game under environmental uncertainty. A symmetric group of individuals face the dilemma of sharing a common resource: each player chooses her consumption level and obtains a corresponding share of the common resource; if the total consumption exceeds the sustainable resource size, the resource deteriorates and all the players are worse off. We consider the effect of uncertainty about the sustainable resource size on the outcome of this game. We extend the existing model (Rapoport and Suleiman 1992, Budescu et al. 1995) in two ways: (a) we consider a general deterioration function in contrast to the existing model in which any excess consumption results in total destruction of the common resource; and (b) we consider the effect of ambiguity about the common knowledge probability distribution governing the size of the common resource. We show for ambiguity-averse agents that increasing ambiguity about the size of the common pool resource may lead to lower consumption, in contrast to existing results derived under conditions of risk.

*Key words:* Common Pool Resources, Environmental Uncertainty, The Tragedy of the Commons.

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## 1. Introduction

The problem of sharing common resources lies at the heart of many large-scale conflicts among human societies today such as global warming and international usage of the planet earth's non-renewable resources. These problems typically entail tensions between individual rationality and social efficiency. For example, a single country may benefit from not investing in clean energy technologies and using cheaper technologies, such as coal, but this leads to a pollution of the global commons through increased greenhouse gas emissions. Similarly, greater consumption of a natural resource, such as fish, is individually desirable, but leads at the aggregate level to overfishing and potentially catastrophic thinning of the overall fish population. Social dilemmas were made famous

by a paper that Hardin published in *Science* (Hardin 1968), although the earlier version of the problem was portrayed by Lloyd (see Lloyd 1977) and can be traced back to Aristotle (Barker 1948). They have been extensively studied in social psychology (Kopelman et al. 2002), political science (Ostrom et al. 1992), decision sciences and economics (see e.g. Kollock (1998) and the references therein).

This paper focuses on an important category of social dilemmas, namely the Common Pool Resource (CPR) game. A CPR game is a situation where a group of individuals face the consumption or appropriation of a common resource with an individual benefit and a social cost. The social cost in a CPR game is that when aggregate consumption exceeds a certain level, the common resource deteriorates, possibly completely. CPR games are the most common modeling form of the tragedy of the commons and have been extensively studied both theoretically and experimentally (see the references in Budescu et al. (1995), henceforth BRR). The main findings in this body of research include recognizing the effect of individual differences and social value orientations (Kuhlman and Marshello 1975), the positive effect of communication (Orbell et al. 1990) and the positive effect of the mechanisms enforcing credible agreements such as iteration and reciprocity (Axelrod and Hamilton 1981). The focus of this essay is the effect of environmental uncertainty on the outcome of the CPR game. Specifically, we address the following question: will increased uncertainty about the size of the available common resource lead to more or less consumption at the Nash equilibrium to the CPR game?

Why look at environmental uncertainty? The impact of *social* uncertainty has been well studied in the literature and findings have been robust: the mechanisms that reduce social uncertainty such as communication and formal governance are associated with more efficient outcomes (Dawes et al. 1976, Kerr and Kaufman-Gilliland 1994, Orbell et al. 1990, Axelrod and Hamilton 1981)—that is, in the presence of communication or formal governance, individuals consume less of the common resource. Binding agreements between countries, such as international protocols, proliferation treaties and coordinated usage of the international resources can therefore be expected to lead to better outcomes for all of the involved parties.

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A growing body of literature has considered the effect of *environmental* uncertainty on the equilibrium outcome of the CPR games. While in this case there have been mixed messages, most researchers suggest that under reasonable assumptions about the amount of uncertainty associated with the resource, more environmental uncertainty leads to more selfish behavior (Messick et al. 1988, Rapoport et al. 1992, Hine and Gifford 1996, Wit and Wilke 1998, Grling et al. 1998, Gustafsson et al. 1999, Biel and Grling 1995). The dominant view in the growing experimental economics literature about environmental uncertainty is also that it encourages defection, both in single-shot and repeated settings (Budescu et al. 1992, Rapoport et al. 1993, see e.g. BRR). In this paper, we further explore this notion by extending previous work just mentioned, and by examining the assumptions that lead to the conclusion that uncertainty leads to more selfish behavior. In particular, we show that individuals might *decrease* their consumption in the presence of more uncertainty depending on how uncertainty and individuals' attitude toward it are modeled.

The specific model of the CPR dilemma studied here is based on that of Rapoport and Suleiman (1992), henceforth referred to as RS. RS propose a game where individuals decide on their own appropriation levels simultaneously facing a common knowledge distribution about the available common resource size. The resource size is then realized by a risky random variable  $\tilde{X}$ . If the sum of individual appropriations (*the total consumption*) exceeds the realized value, the resource is destroyed and all players receive zero. Else, each player receives her own request. This is the only model of CPR games under uncertainty of which we are aware. BRR show that the Nash Equilibrium (abbreviated as NE) is a good predictor of the players' behavior in this game in the lab. RS and BRR both show that, under reasonable assumptions, if the risk associated with the resource size is high, increased risk about the resource size leads to more consumption.

Our model extends the RS/BRR model by relaxing two assumptions that lie at the heart of their model. First, they assume that the payoff structure is a step function. In other words, if the total consumption exceeds the realization of the resource size, the resource is destroyed and all of the players end up receiving nothing. They motivate this assumption by giving examples of biological systems that are sustainable as long as a pre-determined resource threshold is not exceeded. In

practice, however, it is possible that there is a sustainable threshold of the resource use, above which the resource deteriorates (rather than being destroyed) with consequent reductions in the players' payoffs. Considering a general deterioration model of which RS and BRR's model is a special case, our model captures this gradual erosion process. Second, RS and BRR assume that the nature of the uncertainty associated with the resource size is risk. In the decision sciences literature, risk is referred to as a situation where knowledge about the outcomes is not absolute but the probability of occurrence of each outcome is known. Hence risk might be thought of as a first level degree of uncertainty. To go beyond risk, we present a symmetric CPR game similar to the one presented in BRR, but the distribution of the resource size, represented by the random variable  $\tilde{X}$ , is ambiguous. Ambiguity is a higher degree of uncertainty than risk, and arises where outcomes are known, but the probability of each outcome is itself risky<sup>1</sup>.

Many behavioral researchers have already highlighted the importance of distinguishing between the different types of environmental uncertainty (e.g. risk, ambiguity and pure uncertainty) in the context of CPRs (Biel and Grling 1995), yet there are few game theoretical models to address these differences. Our paper intends to fill this void by explicitly modeling the imprecision of the probabilities. It should be noted, however, that although our setting is easier to motivate regarding the nature of uncertainties that arise in practice, it is still highly stylized and is intended only to demonstrate the importance of considering the nature of uncertainty associated with the resource. As such, we strive for simplicity and parsimony, in particular by relying on the basic approach proposed by RS and BRR, including symmetry of the players.

Our findings show that important insights can be obtained by considering different dynamics of resource erosion in the context of the CPR game. Moreover, our results emphasize the importance of a change in our view of the effect of uncertainty on the tragedy of the commons. We characterize sufficient conditions under which the individuals' consumption decreases when there is an increase

<sup>1</sup> If the only change in the model were to introduce a second-order probability distribution, then not much would be accomplished over the previous case of pure risk. However, as we will see, when preferences interact with the second-order probability differences differently than for risky distributions, then differences in choice and in equilibrium behavior can be expected.

in the level of ambiguity about the (sustainable) resource size. In particular, when the players are sufficiently ambiguity averse, an assumption that finds support in the literature (Ghirardato and Marinacci 2002), then increased ambiguity about the probabilities leads to decreased consumption and a more socially efficient outcome.

Our paper is not the first to note the effects of uncertainty on the efficiency of outcomes of group choice. There is already some experimental evidence in the literature of group decision making to support the idea of an efficiency enhancing effect of uncertainty on social behavior. For example Gong et al. (2009) find that while in a deterministic prisoner's dilemma game, groups tend to cooperate less than individuals, yet this effect is reversed when uncertainty is present with respect to the outcomes of the game. In an experimental setting, Apesteguia (2006) finds that in the absence of learning, 'minimal information' about the payoff increases the chance of pro-social behavior. The present paper provides a formal framework to underpin these results for the CPR setting.

The rest of the paper continues as follows. In Section 2 the CPR game with generalized payoff is defined and initial insights from this model are derived. In Section 3, we present a model of the CPR game under ambiguity and extend and contrast the results of this model to the model under risk. Concluding remarks are presented in Section 4.

## 2. The CPR Game with Generalized Payoffs under Risk

Consider the CPR game under risk  $\Gamma_R(n, u, F_{\tilde{X}}, h)$  defined as follows. A group of  $n$  individuals with homogenous risk preferences represented by the vN-M utility function  $u(\cdot)$  are provided with a resource endowment with size  $\tilde{X}$ .  $\tilde{X}$  is uncertain with a common knowledge probability distribution,  $F_{\tilde{X}}$ . Throughout the paper, we assume that the support of  $\tilde{X}$  is finite in the range  $[\alpha, \beta]$ , with  $\alpha < \beta$ . Each player  $i$  decides on a consumption level ( $r_i$ ), with the utility of  $u(r_i)$ . Player  $i$ 's payoff depends on her own consumption  $r_i$  and the total appropriation by the other players,  $r_{-i}$ , as well as the realization of the resource size. If the total consumption ( $\mathbf{r} = r_i + r_{-i}$ ) exceeds the resource size, each player's payoff is reduced by  $h(\mathbf{r}, \tilde{X}) \in [0, 1)$ , so that the risky payoff for each player is equal to:

$$\pi_i(r_i, r_{-i}) = u(r_i h(\mathbf{r}, \tilde{X})). \quad (1)$$

The deterioration function  $h$  reflects the reduction in each player's payoff due to the erosion of the resource when demands exceed the sustainable level  $\tilde{X}$ .

Throughout the paper, we make the following assumption on the deterioration function  $h(\mathbf{r}, \tilde{X})^2$ :

ASSUMPTION 1.  $h(\mathbf{r}, x)$  is decreasing in  $\mathbf{r}$ , increasing in  $x$ , and twice differentiable with respect to  $\mathbf{r}$  almost everywhere<sup>3</sup>.

Because we use the method of monotone comparative statics, which does not require the differentiability assumption, most of our results can be generalized to the case where  $h$  is not differentiable.

### 2.1. Existence and Uniqueness of the Nash Equilibrium

The players' *expected payoff* is equal to:

$$\Pi_i(r_i, r_{-i}) = \mathbb{E}_{\tilde{X}} u(r_i h(\mathbf{r}, \tilde{X})), \quad (2)$$

where the operator  $\mathbb{E}_{\tilde{X}}$  represents expectation with respect to the random variable  $\tilde{X}$ . The best-response function of player  $i$  is defined by  $r_i^*(r_{-i}) = \arg \max_{r_i \geq 0} \Pi(r_i, r_{-i})$ . When  $r_i^*(r_{-i})$  is increasing, the CPR game is a game of *strategic complements*—i.e., each player's consumption level is increasing in the consumption of the others. Similarly, if  $r_i^*(r_{-i})$  is decreasing, the CPR game is one of *strategic substitutes*, where higher consumption by others decreases a focal player's optimal consumption. Because the game is symmetric in our setting, from now on, we drop the subscript  $i$  from all of the functions related to a focal player  $i$ .

Define the marginal payoff from consumption as

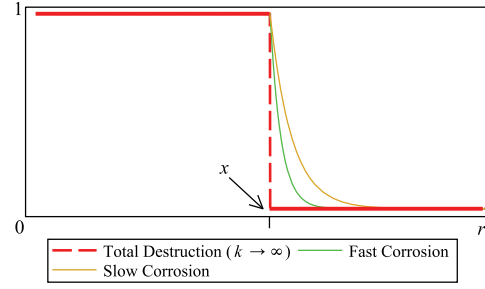
$$m(r_i, r_{-i}) = \frac{\partial}{\partial r_i} \Pi_i(r_i, r_{-i}) = \mathbb{E}_{\tilde{X}} \{u'(r_i h(\mathbf{r}, \tilde{X})) (h(\mathbf{r}, \tilde{X}) + r_i h_1(\mathbf{r}, \tilde{X}))\}. \quad (3)$$

The following proposition characterizes the strategies of the players based on the monotonicity of  $m(r_i, r_{-i})$  with respect to  $r_{-i}$ .

<sup>2</sup> The words increasing and decreasing are used in their weak sense unless it is stated otherwise.

<sup>3</sup> A sufficient condition for this condition is that the number of kinks, the points where  $h$  is continuous but not differentiable is finite.





**Figure 1** The kinked discounting function.

PROPOSITION 1. *If  $m(r_i, r_{-i})$  is increasing (decreasing) in  $r_{-i}$  then the CPR game with generalized payoff is a game of strategic complements (strategic substitutes).*

To illustrate Proposition 1, consider the following example, which we will revisit in subsequent sections.

**Example 1:**

Consider the CPR game under risk  $\Gamma_R$  where  $\tilde{X} \sim U[\alpha, \beta]$  and players have a power utility function of the form  $u(z) = z^c$ , with  $c > 0$ . Note that all such power utility functions belong to the CRRA<sup>4</sup> family. Let the deterioration function be given by:

$$h(\mathbf{r}, \tilde{X}) = \begin{cases} 1 & r \leq \tilde{X} \\ e^{-k(r-\tilde{X})} & r > \tilde{X}, \end{cases} \quad (4)$$

Where players' payoff gets discounted by an exponential term as a function of the deviation from the realized size of the resource, if total consumption exceeds  $\tilde{X}$ .  $k$  is an index of the speed of the rate of deterioration of the resource when overconsumption occurs. Figure 1 shows this function for various values of  $k$ . Clearly, when  $k$  goes to infinity, for  $r > \tilde{X}$  players receive zero, so that the RS model is a boundary case of this example. The expected payoff of player  $i$ , as a function of her own appropriation ( $r_i$ ), and the total consumption by others ( $r_{-i}$ ) is calculated as:

$$\begin{aligned} \Pi(r_i, r_{-i}) &= \mathbb{E}_{\tilde{X}} u(r_i h(r_i + r_{-i}, \tilde{X})) \\ &= \int_{r_i+r_{-i}}^{\beta} r_i^c \cdot \frac{1}{\beta - \alpha} dx + \int_{\alpha}^{r_i+r_{-i}} r_i^c \cdot e^{-k(r_i+r_{-i}-x)} \cdot \frac{1}{\beta - \alpha} dx \end{aligned}$$

<sup>4</sup> Constant Relative Risk Aversion

$$= \frac{1}{kc(\beta - \alpha)} r_i^c \left( -(\alpha - \beta) e^{-ck(r_i + r_{-i} - \alpha)} + ck(\beta - r_i - r_{-i}) + \beta - \alpha \right). \quad (5)$$

We calculate the derivative of  $m(r_i, r_{-i})$  with respect to  $r_{-i}$  to obtain:

$$\frac{\partial}{\partial r_{-i}} m(r_i, r_{-i}) = \frac{\partial^2}{\partial r_i \partial r_{-i}} \Pi(r_i, r_{-i}) = \frac{r_i^c}{(\beta - \alpha)} \left( (\beta - \alpha) e^{-ck(r_i + r_{-i} - \alpha)} - 1 \right) \quad (6)$$

From (6) it is clear that when  $k$  is “sufficiently” large—i.e. the resource deteriorates quickly,  $m(r_i, r_{-i})$  is decreasing in  $r_{-i}$ ; hence the CPR game is a game of strategic substitutes. In particular, the CPR game presented in RS, where  $k \rightarrow \infty$  is a game of strategic substitutes. On the other hand, when  $k$  is small enough, meaning that the rate of resource deterioration is slow, this is a game of strategic complements: the more other players consume, the more each individual player consumes in equilibrium.

So far, we have not imposed any assumptions on the monotonicity of  $m(r_i, r_{-i})$ . However, a typical characteristic of the tragedy of the commons is that the marginal utility of consumption for a focal player is never increasing in others’ consumption. In other words, marginal utility of consumption for each player is always higher if others consume less, so according to Proposition 1, a typical CPR game is a game of strategic substitutes. Based on this observation we make the following assumption throughout the rest of the paper. This assumption helps us obtain the existence and uniqueness results<sup>5</sup>.

ASSUMPTION 2. (a)  $m_2(r_i, r_{-i}) < 0$ . (b)  $\forall x, r_i, r_{-i}, -\frac{h_{11}(\mathbf{r}, x)}{h_1(\mathbf{r}, x)} \leq \frac{1}{r_i}$ .

Assumption 2(b) puts an upper bound on the degree of concavity of  $h$ . In other words the deterioration functions that are “too concave” do not satisfy Assumption 2(b). In particular, we have seen in Example 1 that Assumption 2 is satisfied for the deterioration function (4) if  $k$  is sufficiently large.

PROPOSITION 2. *Suppose that players are risk averse. The Nash equilibrium of the CPR game with generalized payoff under risk  $r_R^{**}$  exists and is unique.*

<sup>5</sup> The subscript number in  $h_i$  represents the derivative with respect to the  $i$ th argument.

PROPOSITION 3. *Given risk averse players, the unique NE of the CPR game under risk is characterized as the solution to*

$$\mathbb{E}_{\tilde{X}}\{u'(rh(nr, \tilde{X}))\left(h(nr, \tilde{X}) + rh_1(nr, \tilde{X})\right)\} = 0, \quad (7)$$

*truncated to<sup>6</sup> the interval  $[\frac{\alpha}{n}, \frac{\beta}{n}]$ .*

The lower *boundary solution*  $\frac{\alpha}{n}$ , arises when players agree to share the certain lower bound of the resource size evenly. In this case, the players do not risk going into the uncertain region with excessive consumption bids. This might be due to large risk aversion or due to drastic consequences of exceeding the sustainable resource size. The other boundary solution  $\frac{\beta}{n}$  arises when the players are not sufficiently risk averse, or when the consequences of exceeding the sustainable threshold of the resource size are less severe.

### Example 1: Characterization of the NE

Consider Example 1, where  $k \rightarrow \infty$ . This is the case where exceeding the sustainable resource size leads to total destruction of the resource, and the players receive nothing. This example is extensively studied in BRR. The following result is a corollary of Proposition 3.

COROLLARY 1. *The NE of the game  $\Gamma_R(h, F, n, u)$  specified in Example 1, with  $\tilde{X}$  uniformly distributed on  $[\alpha, \beta]$  when  $k \rightarrow \infty$  is characterized by:*

$$r_1^{**} = \max\left\{\frac{\alpha}{n}, \frac{c\beta}{nc+1}\right\}. \quad (8)$$

We notice in this case that due to the extreme nature of the game, namely total destruction of the resource if the total consumption exceeds the resource size, the upper bound NE never occurs. We also observe that when individuals are highly risk averse ( $c$  is very small), then the lower bound NE is the outcome of the game. This is also true when the uncertainty range about the resource size is small, namely  $\beta - \alpha \leq \frac{1}{nc}$ . If none of these is the case, the players go to the uncertain region, so that the total consumption exceeds the lower bound of the distribution and there is some probability of excess exploitation of the CPR.

<sup>6</sup> The truncation operator  $\tau(\cdot; a, b)$  on the interval  $[a, b]$  is defined by  $\tau(z; a, b) = \max(a, \min(z, b))$ .

## 2.2. The Effect of Increased Risk

In this section, we consider the effect of increased risk associated with the resource size. We start by defining what we mean by an increase in the risk. Recall that for any two distributions  $F$  and  $G$  with the same mean,  $F$  second-order stochastically dominates  $G$  ( $F \succeq_{SSD} G$ ), if for every nondecreasing concave function  $u: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  we have:  $\int_0^{+\infty} u(z)dF(z) \geq \int_0^{+\infty} u(z)dG(z)$ . Of interest here is the fact that  $F \succeq_{SSD} G$  if  $F$  is obtained from  $G$  through a mean-preserving spread (see Rothschild and Stiglitz 1970).

Let  $\tilde{X}(\sigma)$  denote the resource size, parameterized by an index  $\sigma$  which represents the amount of risk associated with  $\tilde{X}$ , with a distribution function  $F_{\tilde{X}}(x, \sigma)$ . We will consider only parameter changes that induce mean-preserving spreads. Thus, for  $\sigma_2 \geq \sigma_1$ , we will say that there is an increase in environmental risk in replacing  $\tilde{X}(\sigma_1)$  by  $\tilde{X}(\sigma_2)$  if  $F_{\tilde{X}(\sigma_2)}$  is a mean preserving spread of  $F_{\tilde{X}(\sigma_1)}$  (so that also  $F_{\tilde{X}(\sigma_1)} \succeq_{SSD} F_{\tilde{X}(\sigma_2)}$ ).

Let  $r^{**}(\sigma)$  be the set of the Nash Equilibria of the CPR game with generalized payoff as a function of the uncertainty associated with the resource size. Recall that a nonnegative function  $h(r, x)$  is log-supermodular (abbreviated log-spm) if for every  $r^H \geq r^L$ , the ratio  $h(r^H, x)/h(r^L, x)$  is nondecreasing in  $x$ . Proposition 4 provides sufficient conditions for the *interior* NE of the CPR game to be increasing in parameter  $\sigma$ .

**PROPOSITION 4.** *Suppose that players are risk averse and the NE  $r_R^{**}$  for  $\Gamma_R(h, f, n, u)$  is an interior solution (i.e.  $r_R^{**} \in (\frac{\alpha}{n}, \frac{\beta}{n})$ ). The following conditions are sufficient for  $r_R^{**}$  to be increasing in  $\sigma$ :*

1.  $f_{\tilde{X}}(x; \sigma)$  is log-spm in  $(x; \sigma)$ ;
2.  $\forall r_i, r_{-i}, x, u(r_i h(r_i + r_{-i}, x))$  is log-spm in  $(r_i, r_{-i})$ .

Before we examine the conditions of the above proposition, we note that Proposition 4 distinguishes boundary and interior NE solutions with respect to the effect of uncertainty. In particular, when the parameter of interest  $\sigma$  affects the lower bound, then relaxing the assumption of interior solution as  $\sigma$  increases could lead to a potential decrease in the NE, as we show below for the

uniform distribution.

A parameterized density function  $f_{\bar{X}}(\sigma)$  for the distribution  $F_{\bar{X}}(\sigma)$  is log-spm if the parameter  $\sigma$  shifts the distribution according to the monotone likelihood ratio (Atthey 2002). In our context, if the increase in risk maintains a monotone likelihood ratio for the distribution, then the condition of Proposition 4 is satisfied. For a simple distribution function such as the uniform distribution, the parameter  $\sigma = \beta - \alpha$  represents the risk associated with the distribution. It is straightforward to verify that the uniform distribution,  $f_{\bar{X}}(x) = 1/(\beta - \alpha)$  is log-spm in  $\sigma = \beta - \alpha$ . This also holds for the Gamma distribution family, with respect to each of the two parameters<sup>7</sup>.

Consider Condition 2 in Proposition 4. To illustrate this condition, consider the family of HARA<sup>8</sup> utility functions  $u(z) = \zeta \left( \eta + \frac{z}{\gamma} \right)^{1-\gamma}$  for the players. This large family of utility functions includes CARA<sup>9</sup>, CRRA and power utility functions among others. Assuming this, Condition 2 of Proposition 4 can be reduced to

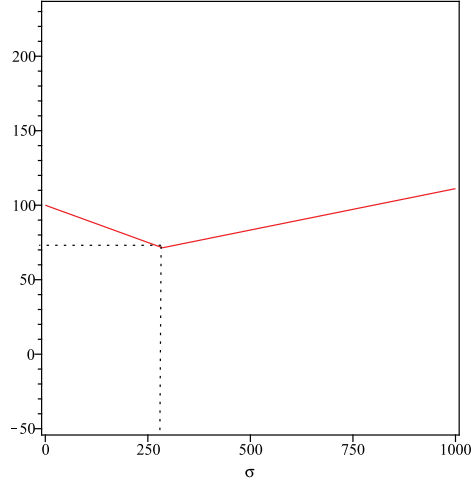
$$(1 - \gamma) \left( \gamma \eta (h_1(\mathbf{r}, x) + r_i h_{12}(\mathbf{r}, x)) + r_i^2 (h_{12}(\mathbf{r}, x) h(\mathbf{r}, x) - h_1(\mathbf{r}, x) h_2(\mathbf{r}, x)) \right) \geq 0. \quad (9)$$

Based on (9), we can identify several sets of conditions, such that Condition 2 in Proposition 4 is satisfied. For example, a CRRA utility function is obtained when we have  $\eta = 0$ . For CRRA utility functions, risk aversion for players is characterized as  $\gamma \leq 0$ . So one set of sufficient conditions to satisfy Condition 2 in Proposition 4 is (i) risk averse players with CRRA utility function and (ii)  $\forall \mathbf{r}, x, h_{12}(\mathbf{r}, x) h(\mathbf{r}, x) - h_1(\mathbf{r}, x) h_2(\mathbf{r}, x) \geq 0$  which is less restrictive than demanding  $h$  to be supermodular. Another set of conditions can be obtained by considering CARA utility functions which are obtained from the HARA family by letting  $\gamma \rightarrow +\infty$  and  $\eta > 0$ . Hence the following set of conditions is also sufficient for satisfying Condition 2 in Proposition 4: (i) players with CARA utility functions and (ii)  $\forall r_i, r_{-i}, x$  we have  $h_1(r_i + r_{-i}, x) + r_i h_{12}(r_i + r_{-i}, x) < 0$ .

<sup>7</sup> The Gamma distribution function with its two parameters  $(k, \theta)$  is represented by the following pdf:  $f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma(k)}$ , for  $k, \theta \geq 0$ . Note that a change in only one of the parameters never leads to mean preserving spreads.

<sup>8</sup> Hyperbolic Absolute Risk Aversion.

<sup>9</sup> Constant Absolute Risk Aversion



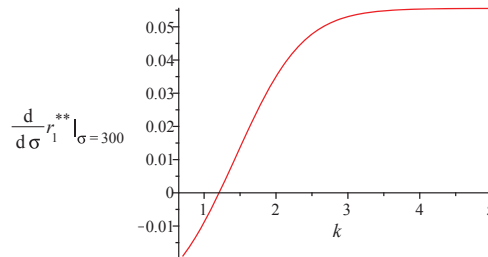
**Figure 2**  $r_1^{**}$  as a function of  $\sigma$ .  $c = 0.25$ ;  $n = 5$ ;  $\frac{\alpha + \beta}{2} = 500$ .

Interestingly, under certain sets of conditions, Proposition 4 holds regardless of individuals' attitude toward risk—that is *interior* NE of the CPR game is increasing in the parameter of interest (in our case we are focusing on uncertainty), regardless of the players' attitude toward risk. In other words, in the context of Proposition 4, there exist sufficient conditions under which the attitude toward risk is immaterial to the increased consumption. Risk averse players, as well as risk seeking players may increase their consumption when uncertainty increases.

### Example 1: Discussion of The Effect of Risk

We show in Figure 2 the Nash equilibrium of Example 1 for the case where  $k \rightarrow \infty$ . It is a kinked function of  $\sigma = \beta - \alpha$  consisting of two segments, the certain and the uncertain regions. In the certain region (the boundary NE), the NE is decreasing in  $\sigma = \beta - \alpha$ , as is also pointed out in BRR. This is because by keeping the mean of the distribution fixed, increasing the variance results in a reduction in  $\alpha$ , which in turn leads to the reduction in the boundary NE. In the uncertain region, as the range of the resource size distribution increases, the NE of the game also increases. Note that the mean of the resource size is fixed (500), therefore this result is not due to a change in the players' belief about the expected value of  $\tilde{X}$ .

The deterioration function  $h$  has indeed an important role in determining whether or not the NE is increasing or decreasing in  $\sigma$ . To show this, in Figure 3 we plot the slope of the NE as a function of  $\sigma$  at a fixed  $\sigma = 300$ —i.e.,  $\frac{d}{d\sigma} r^{**}(\sigma)|_{\sigma=300}$  as a function of  $k$ . Observe in Figure 2 that at  $\sigma = 300$ ,



**Figure 3** The slope of the NE of the basic game under risk as a function of  $\sigma$ .  $c = 0.25$ ;  $n = 5$ ;  $\frac{\alpha + \beta}{2} = 500$ ;  
 $\sigma = 300$

the NE is increasing in  $\sigma$  so the slope of  $r^{**}(\sigma)$  is positive. We can see that this is not the case for all values of  $k$  (Figure 2 is for  $k \rightarrow \infty$ ). In particular when  $k$  is low, the slope of the *interior* NE is decreasing. In other words, when the deterioration of the environment is slow, the players *decrease* their consumption when uncertainty increases. Note that this is *not* due to the fact that for a low  $k$  the players share the lower bound of the resource size; this is an interior NE. For example, at  $k = 1$  and for the parameters given in Figure 3, the NE is equal to  $r^{**} = 73.2$ , which is significantly higher than the amount from sharing the lower bound of the resource, i.e.  $(500 - 300)/5 = 40$ . So unlike BRR, we argue that depending on the shape of the deterioration function, the interior NE (i.e. the NE in the uncertain region) may be increasing or decreasing.

### 3. The CPR Dilemma Game under Ambiguity

In this section, we consider the effects of uncertainty about the probabilities (ambiguity) and show that depending on the attitude of individuals toward ambiguity, the NE consumption level may increase or decrease.

In the decision sciences literature, it is now well established that in certain situations (in particular in the gain domain) the decision makers (DMs) sometimes exhibit an aversion toward ambiguity associated with the probabilities. In his classic paper, Ellsberg (1961) shows that individuals prefer to bet on prospects with known probabilities rather than on “equivalent” prospects with ambiguous probabilities, that is, individual DMs are ambiguity averse. Besides the wide evidence for individual ambiguity aversion in experimental and empirical studies (for extensive reviews see Fox and Tversky (1995), Ghirardato and Marinacci (2002), Maccheroni et al. (2006)), this effect has also

been evidenced in strategic environments such as stock markets (Hirshleifer 2001), although the latter is still subject to ongoing discussion (see e.g. Du and Budescu 2005).

Define the CPR game under ambiguity  $\Gamma_A(n, u, F_{\tilde{X}(\bar{\sigma})}, \phi, F_{\bar{\sigma}(\theta)}, h)$  as follows. The resource size ( $\tilde{X}$ ) is distributed with the probability function  $F_{\tilde{X}}(\tilde{\sigma})$  where  $\tilde{\sigma}$  is itself a random variable with the common knowledge distribution  $F_{\bar{\sigma}}(\theta)$ , where  $\theta$  is an index of ambiguity described bellow. To capture the DM's preferences in the presence of ambiguity, Klibanoff et al. (2005) propose a smooth model of ambiguity aversion where for each plausible distribution function  $F_{\bar{\sigma}(\theta)}$ , players' expected profit takes the following double expectation form:

$$\pi(r_i, r_{-i}) = \mathbb{E}_{\bar{\sigma}} \phi \left( \mathbb{E}_{\tilde{X}} u \left( r_i h(r_i + r_{-i}, \tilde{X}(\tilde{\sigma})) \right) \right). \quad (10)$$

Here  $\phi(\cdot)$  is an increasing function representing players' attitudes toward ambiguity, similar to the way that utility function  $u(\cdot)$  characterizes the players' attitude toward risk. A linear  $\phi$  represents the preference of ambiguity neutral DMs, implying that they are indifferent between betting on a risky prospect with known probabilities and an ambiguous prospect with risky probabilities.

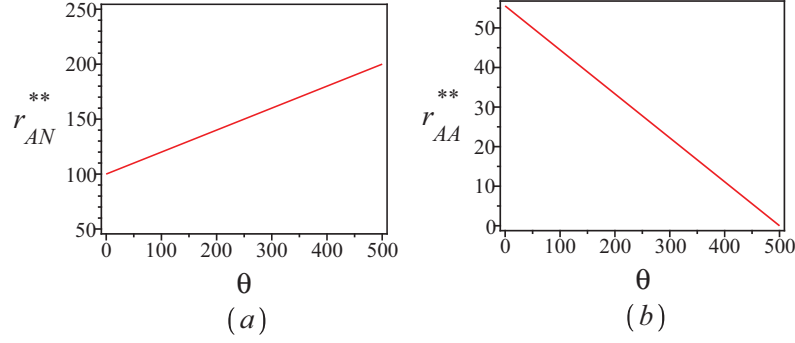
### Example 2: Ambiguity Neutral Players

Consider  $\Gamma_A$  where  $\tilde{X} \sim U[0, \bar{\sigma}]$  and  $\tilde{\sigma} \sim [\bar{\sigma} - \theta, \bar{\sigma} + \theta]$  with  $\bar{\sigma} > \theta$ ,  $u(z) = z^c$  with  $c > 0$  and  $\phi(z) = z$ . We set  $\alpha = 0$  in this example in order to rule out the boundary solution of Example 1 ( $\frac{\alpha}{n}$ ), and  $\beta = \bar{\sigma}$  to simply represent the amount of risk associated with the sustainable resource size. Here,  $\bar{\sigma}$  represents the uncertainty associated with the payoff (risk), whereas  $\theta$  is an index of the uncertainty about the distribution of resource size. Let  $k \rightarrow \infty$  (full exhaustion of the resource) and  $\phi(z) = z$ . Observe that when  $\theta = 0$ , this is exactly equivalent to Example 1. Because  $\phi(z)$  is linear, the expected profit of player  $i$  is equal to:

$$\begin{aligned} \Pi(r_i, r_{-i}) &= Pr(r_i + r_{-i} \leq X) \cdot u(r_i) \\ &= \mathbb{E}_{\bar{\sigma}} \left\{ \frac{r_i^c (\bar{\sigma} - r_i - r_{-i})}{\bar{\sigma}} \mid \bar{\sigma} \geq r_i + r_{-i} \right\}. \end{aligned}$$

Let  $r_{AN}^{**}(\bar{\sigma}, \theta)$  be the NE of this game with ambiguity-neutral players, as a function of the amount of risk ( $\bar{\sigma}$ ) and the amount of ambiguity ( $\theta$ ).





**Figure 4** (a) The NE of the CPR game under ambiguity with ambiguity neutral players. (b) The NE of the CPR game under ambiguity with extremely ambiguity averse players.  $c = 0.25$ ;  $n = 5$ ;  $\bar{\sigma} = 500$ .

REMARK 1. The NE of  $\Gamma_A$  specified in Example 2 is unique and is equal to

$$r_{AN}^{**}(\bar{\sigma}, \theta) = \frac{-\lambda(\bar{\sigma} + \theta)}{nW(-\lambda(\bar{\sigma} + \theta) \exp\{\lambda - \ln(\bar{\sigma} + \theta)\})}, \quad (11)$$

where  $\lambda = \frac{cn}{1+cn}$  and  $W(\cdot)$  is the Lambert W function.<sup>10</sup> Furthermore,  $r_{AN}^{**}(\bar{\sigma}, \theta)$  is increasing in both arguments.

Remark 1 makes the point that if the players are ambiguity neutral, increased uncertainty leads to increased consumption in agreement with the results presented in the previous section (see Figure 4(a)). This is not surprising. In an expected utility framework, when players are ambiguity neutral, one can just calculate the second-order mixture probability measure, combining the two distribution functions, to obtain a new probability distribution to calculate the expected payoff of the players, similar to what we discussed in the previous sections. Increase in the risk associated with each of the initial probability measures results in an increase in the risk (associated with the distribution resulting from the convolution of the two probability distributions), hence having a similar effect to the one we discussed in the previous section. However, as we shall see in the next section, this is not the case when the DMs are ambiguity averse.

### 3.1. Ambiguity-Averse Players

The smooth ambiguity model by Klibanoff et al. (2005) characterizes ambiguity aversion by a concave  $\phi(\cdot)$ . Let  $H(r_i, r_{-i}; \tilde{\sigma}) = \mathbb{E}_{\tilde{X}} u\left(r_i h(r_i + r_{-i}, \tilde{X}(\tilde{\sigma}))\right)$ . With a non-linear ambiguity function  $\phi(\cdot)$ , the expected payoff of each player is calculated as:

<sup>10</sup> Lambert W is the inverse function of  $f(w) = we^w$ .

$$\Phi(r_i, r_{-i}) = \mathbb{E}_{\tilde{\sigma}} \phi(H(r_i, r_{-i}; \tilde{\sigma})). \quad (12)$$

The optimal strategy  $r_i^*(r_{-i})$  is the solution of the following optimization problem:

$$\max_{r_i} \mathbb{E}_{\tilde{\sigma}} \{\phi(H(r_i, r_{-i}; \tilde{\sigma}))\}. \quad (13)$$

$$\text{Let } \psi(r_i, r_{-i}; \tilde{\sigma}) = \frac{\frac{\partial^2}{\partial r_i \partial \tilde{\sigma}} H(r_i, r_{-i}; \tilde{\sigma}) \frac{\phi'(H(r_i, r_{-i}; \tilde{\sigma}))}{\phi(H(r_i, r_{-i}; \tilde{\sigma}))}}{\frac{\partial}{\partial r_i} H(r_i, r_{-i}; \tilde{\sigma}) \frac{\partial}{\partial \tilde{\sigma}} H(r_i, r_{-i}; \tilde{\sigma})}.$$

PROPOSITION 5. *The following conditions collectively are sufficient for the NE of the CPR game  $\Gamma_A(n, u, F_{\tilde{X}(\tilde{\sigma})}, \phi, F_{\tilde{\sigma}(\theta)}, h)$  under ambiguity to be decreasing in the ambiguity index  $\theta$ :*

1.  $\mathbb{E}_{\tilde{X}} h(\cdot, \tilde{X}(\tilde{\sigma}))$  is increasing in  $\tilde{\sigma}$ ;
2.  $H(r_i, r_{-i}; \tilde{\sigma})$  is increasing in  $r_i$ .
3.  $f_{\tilde{\sigma}}(\sigma, \theta)$  is log-spm in  $(\sigma, \theta)$ ;
4.  $-\frac{\phi''(H(r_i, r_{-i}; \tilde{\sigma}))}{\phi'(H(r_i, r_{-i}; \tilde{\sigma}))} \geq \psi(r_i, r_{-i}; \tilde{\sigma}), \forall r_i, r_{-i}, \tilde{\sigma}$ .

The first condition of Proposition 5 states that the expected deterioration is increasing in the amount of risk associated with the distribution. For a step deterioration function, where the resource is destroyed completely if total consumption exceeds the sustainable resource size, The first condition of Proposition 5 amounts to the probability distribution function of the resource size being decreasing in the parameter  $\sigma$ . This holds, for example, for the uniform distribution. As the range of the distribution increases, the probability of each state strictly decreases.

The second condition of Proposition 5 is essentially equivalent to the marginal utility of consumption  $m(r_i, r_{-i})$  being increasing in  $r_i$ , which we already discussed in Section 3. It means that the expected marginal utility of one unit of consumption is higher than the negative impact of the consequences of such additional consumption for the environment, something that characterizes the tragedy of the commons. Condition 3 of Proposition 5 is similar to the condition on the distribution of the resource size discussed, in the explanation of Proposition 4.

Regarding condition 4, the ratio  $-\frac{\phi''(\cdot)}{\phi'(\cdot)}$  represents the degree of concavity of the ambiguity function, measuring the amount of ambiguity aversion in the DM's preferences, similar to the Arrow-Pratt measure of risk<sup>11</sup>. Proposition 5 states that if the players are sufficiently ambiguity averse,

<sup>11</sup> The Arrow-Pratt measure of risk aversion for a utility function  $U(\cdot)$  is represented by  $-U''(\cdot)/U'(\cdot)$ .

then the amount of each player's consumption declines, as the ambiguity about the probabilities increases. Once again, to understand this idea better, we go back to our Example 1.

**Example 2: Extremely Ambiguity Averse Players**

Consider Example 2, but this time with a concave ambiguity function with the CARA form  $\phi(z) = -\exp(-\eta z)$ . This is a “constant absolute ambiguity aversion function” because for all  $z$ ,  $-\phi''(z)/\phi'(z) = \eta$ . This functional form is particularly interesting, because Klibanoff et al. (2005) show that as  $\eta \rightarrow \infty$ , players risk-ambiguity preferences converge to the maxmin expected utility problem originally set out by (Gilboa and Schmeidler 1989):

$$\Pi(r_i, r_{-i}) = \min_{\tilde{\sigma}} \frac{r_i^c(\tilde{\sigma} - r_i - r_{-i})}{\tilde{\sigma}} = \frac{r_i^c((\bar{\sigma} - \theta) - r_i - r_{-i})}{\bar{\sigma} - \theta}. \quad (14)$$

Clearly this problem is equivalent to the basic CPR game under risk specified in Example 1 where  $\alpha = 0$  and  $\beta = \bar{\sigma} - \theta$  for which the NE is characterized in Proposition 1. Therefore the NE of this problem is equal to:

$$r_{AA}^{**}(\bar{\sigma}, \theta) = \frac{c(\bar{\sigma} - \theta)}{nc + 1} \quad (15)$$

As we can see in Figure 4(b), the NE in this case is a decreasing function of  $\theta$ .

When the players are extremely ambiguity averse, they assume the most pessimistic probability distribution for the resource size. When  $\theta$  increases, the lower bound on resource size gets smaller, which causes players to lower their consumption. Going from ambiguity neutral players to extremely ambiguity averse players, we observe that their reaction regarding the uncertainty with respect to probabilities is reversed. So we conclude that players' attitudes about probabilities is an important driver of their response toward increased ambiguity.

The results of this section and the previous section seem to agree on one important message: high 'uncertainty aversion' leads to decreased consumption in the context of the tragedy of the commons. Note that this runs counter to the experimental results of BRR on the uncertainty effect which suggest that increased risk leads to increased consumption for real world subjects. Note, however,

that the BRR experiments were done under the condition of known probabilities. The players are not so risk averse to give rise to a boundary solution for the NE; therefore, they increase their consumption with an increase in the riskiness of resource size. It would clearly be interesting to check the robustness of the BRR experimental result under partial information about probabilities (i.e. under conditions of ambiguity). Given some behavioral evidence for high ambiguity aversion, the results of this section predict the possibility of opposite experimental results to those of BRR's wherein players would reduce their equilibrium consumption with increased ambiguity.

Recent studies using brain imaging techniques suggest that risk aversion and ambiguity aversion are not the same behavioral effects. There are significant differences between the sections of the brain that process the two. Hsu and Camerer (2004) and Hsu et al. (2005) find that ambiguity aversion is highly related to the emotional regions of the brain, such as those regions that are involved in processing pain, disgust and other negative discomforts, while risk aversion is more related to the reward/value related parts of the brain. When the uncertainty is presented in terms of known probabilities, the subjects focus on the value and then the effects such as the big pool illusion where the subjects put most of the weight on the higher bound of the distribution comes into play. On the other hand, one may expect that ambiguous information about resource size could invoke the players' pessimism, and therefore they may behave more cautiously.

#### **4. Conclusion**

In this paper, we considered the effect of environmental uncertainty on the Common Pool Resource game. Building on existing models by assuming a general deterioration function for the resource, we characterized the conditions under which rational individual consumption is monotone in the other players' strategies.

We considered the effects of uncertainty on the outcome of a CPR game, and showed that individual' attitudes toward uncertainty as well as the nature of environmental deterioration is pivotal for their response to the increased uncertainty. In contrast to the current paradigm on the effect of uncertainty on the tragedy of the commons, we showed that greater uncertainty aversion

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may lead to decreased consumption. This effect has not been observed in laboratory results thus far because experimental designs for gradual environmental deterioration as well as communicating different levels of information about the states of the environment to the subjects have not yet been examined.

Experimental economics literature has overlooked the effects of ambiguity associated with probabilities in the context of CPR games, although the importance of distinguishing between risk and ambiguity is apparent in the behavioral and social psychology literatures. Perhaps this is due to the complications of designing experiments that communicate gradual deterioration to the subjects and capture the differences between risk and ambiguity. Our paper highlights the theoretical importance of doing such experiments in order to get a better understanding of the effect of environmental uncertainty on the outcomes of social dilemma settings, such as those captured by the CPR game.

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## References

- Apestequia, J. 2006. Does information matter in the commons? experimental evidence. *Journal of Economic Behavior and Organization* **60**(1) 55–69.
- Athey, S. 2002. Monotone comparative statics under uncertainty\*. *Quarterly Journal of Economics* **117**(1) 187–223.
- Axelrod, R., WD Hamilton. 1981. The evolution of cooperation. *Science* **211**(4489) 1390–1396.
- Barker, E. 1948. *The politics of Aristotle*. Clarendon press.
- Biel, A., T. Grling. 1995. The role of uncertainty in resource dilemmas. *Journal of Environmental Psychology* **15**(3) 221–233.
- Budescu, D. V., A. Rapoport, R. Suleiman. 1995. Common pool resource dilemmas under uncertainty: Qualitative tests of equilibrium solutions. *Games and Economic Behavior* **10**(1) 171–201.

- Budescu, David V., Amnon Rapoport, Ramzi Suleiman. 1992. Simultaneous vs. sequential requests in resource dilemmas with incomplete information. *Acta Psychologica* **80**(1-3) 297–310.
- Dawes, R. M., J. McTavish, H. Shaklee, Oregon Research Institute. 1976. *Behavior, Communication, and Assumptions About Other Peoples' Behavior in a Commons Dilemma Situation..* Defense Technical Information Center.
- Du, N., D.V. Budescu. 2005. The effects of imprecise probabilities and outcomes in evaluating investment options. *Management Science* **51**(12) 1791.
- Ellsberg, D. 1961. Risk, ambiguity, and the Savage axioms. *The Quarterly Journal of Economics* 643–669.
- Fox, C. R., A. Tversky. 1995. Ambiguity aversion and comparative ignorance. *The Quarterly Journal of Economics* **110**(3) 585–603.
- Ghirardato, P., M. Marinacci. 2002. Ambiguity made precise: A comparative foundation. *Journal of Economic Theory* **102**(2) 251–289.
- Gilboa, I., D. Schmeidler. 1989. Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics* **18**(2) 141–153.
- Gong, M., J. Baron, H. Kunreuther. 2009. Group cooperation under uncertainty. *Risk Management and Decision Processes Center, The Wharton School of the University of Pennsylvania* .
- Grling, T., A. Biel, M. Gustafsson. 1998. Different kinds and roles of environmental uncertainty. *Journal of Environmental Psychology* **18**(1) 75–83.
- Gustafsson, M., A. Biel, T. Grling. 1999. Overharvesting of resources of unknown size. *Acta Psychologica* **103**(1-2) 47–64.
- Hardin, G. 1968. The tragedy of the commons. *Science* **162**(3859) 1243–1248.
- Hine, D. W., R. Gifford. 1996. Individual restraint and group efficiency in commons dilemmas: The effects of two types of environmental uncertainty 1. *Journal of Applied Social Psychology* **26**(11) 993–1009.
- Hirshleifer, D. 2001. Investor psychology and asset pricing. *The Journal of Finance* **56**(4) 1533–1597.
- Hsu, M., M. Bhatt, R. Adolphs, D. Tranel, C. F. Camerer. 2005. Neural systems responding to degrees of uncertainty in human decision-making. *Science* **310**(5754) 1680.
- Hsu, M., C. F. Camerer. 2004. Ambiguity-aversion in the brain. *Caltech Working Paper* .

- 
- Kerr, N. L., C. M. Kaufman-Gilliland. 1994. Communication, commitment, and cooperation in social dilemmas. *Journal of personality and social psychology* **66** 513–513.
- Klibanoff, P., M. Marinacci, S. Mukerji. 2005. A smooth model of decision making under uncertainty. *Econometrica* **73** 1849–1892.
- Kollock, P. 1998. Social dilemmas: The anatomy of cooperation. *Annual Reviews in Sociology* **24**(1) 183–214.
- Kopelman, S., J. M. Weber, D. M. Messick. 2002. Factors influencing cooperation in a commons dilemmas: A review of experimental psychological research. *The Drama of the Commons: Committee on the human dimensions of global change* 113.
- Kuhlman, D. M., A. F. Marshello. 1975. Individual differences in game motivation as moderators of pre-programmed strategy effects in prisoner's dilemma. *Journal of personality and social psychology* **32**(5) 922–931.
- Lloyd, W. F. 1977. On the checks to population. *Reprinted in HARDIN G.et BADEN J.(ds.), Managing the Commons, Freeman, San Francisco* 8–15.
- Maccheroni, F., M. Marinacci, A. Rustichini. 2006. Ambiguity aversion, robustness, and the variational representation of preferences. *Econometrica* **74**(6) 1447–1498.
- Messick, D. M., S. T. Allison, C. D. Samuelson. 1988. Framing and communication effects on group members' responses to environmental and social uncertainty. *Applied Behavioural Economics* **2** 677–700.
- Orbell, J., R. Dawes, A. Van de Kragt. 1990. The limits of multilateral promising. *Ethics* 616–627.
- Ostrom, E., J. Walker, R. Gardner. 1992. Covenants with and without a sword: Self-governance is possible. *The American Political Science Review* 404–417.
- Rapoport, A., D. V. Budescu, R. Suleiman. 1993. Sequential requests from randomly distributed shared resources. *Journal of mathematical psychology* **37** 241–241.
- Rapoport, A., D. V. Budescu, R. Suleiman, E. Weg. 1992. Social dilemmas with uniformly distributed resources. *Social dilemmas: Theoretical issues and research findings* 43–57.
- Rapoport, A., R. Suleiman. 1992. Equilibrium solutions for resource dilemmas. *Group Decision and Negotiation* **1**(3) 269–294.
- Rothschild, M., J.E. Stiglitz. 1970. Increasing risk: I. A definition. *Journal of Economic theory* **2**(3) 225–243.

Roy, Sunanda, Tarun Sabarwal. 2006. On the (non-)lattice structure of the equilibrium set in games with strategic substitutes. MPRA Paper 4120, University Library of Munich, Germany.

Topkis, D. M. 1998. *Supermodularity and Complementarity*. Princeton University Press.

Wit, A., H. Wilke. 1998. Public good provision under environmental and social uncertainty. *Eur.J.Soc.Psychol* **28** 249–256.

## Appendix. Proofs.

**Proof of Proposition 1.** Given player  $i$ 's payoff  $\Pi(r_i, r_{-i})$ , the best response function is equal to:

$$r_i^*(r_{-i}) = \arg \max_{r_i \geq 0} \Pi(r_i, r_{-i}), \quad (16)$$

where  $\Pi(r_i, r_{-i})$  is defined in (2). The cross partial derivative of  $\Pi(r_i, r_{-i})$  is equal to:

$$\frac{\partial^2}{\partial r_i \partial r_{-i}} \Pi_i(r_i, r_{-i}) = \frac{\partial}{\partial r_{-i}} m(r_i, r_{-i}). \quad (17)$$

Therefore, if  $m(r_i, r_{-i})$  is increasing in  $r_{-i}$ ,  $\Pi(r_i, r_{-i})$  is supermodular in  $(r_i, r_{-i})$ . By the Topkis Theorem (Topkis (1998, Theorem 2.8.2), implies that  $x^*(y) = \arg \max f(x, y)$  is increasing in  $y$  whenever  $f$  is supermodular) the BR function  $r_i(r_{-i})$  is increasing in its argument which is equivalent to strategic complementarity. The case where  $m(r_i, r_{-i})$  is decreasing is analogous.

**Proof of Proposition 2.** We first show that if Assumption 2 holds, then  $\Pi(r_i, r_{-i})$  is strictly concave in  $r_i$ , which guarantees the uniqueness of the BR function.

$$\begin{aligned} \frac{\partial^2}{\partial r_i^2} \Pi_i(r_i, r_{-i}) &= \mathbb{E}_{\tilde{X}} \left\{ \frac{\partial^2}{\partial r_i^2} u(r_i h(r_i + r_{-i}, \tilde{X})) \right\} \\ &= \mathbb{E}_{\tilde{X}} \left\{ u''(r_i h(\mathbf{r}, \tilde{X})) \left( h(\mathbf{r}, \tilde{X}) + r_i h_1(\mathbf{r}, \tilde{X}) \right) \right. \\ &\quad \left. + u'(h(\mathbf{r}, \tilde{X})) \left( 2h_1(\mathbf{r}, \tilde{X}) + r_i h_{11}(\mathbf{r}, \tilde{X}) \right) \right\} \\ &\leq \mathbb{E}_{\tilde{X}} \left\{ u''(r_i h(\mathbf{r}, \tilde{X})) \left( h(\mathbf{r}, \tilde{X}) + r_i h_1(\mathbf{r}, \tilde{X}) \right) \right. \\ &\quad \left. + u'(h(\mathbf{r}, \tilde{X})) \left( h_1(\mathbf{r}, \tilde{X}) + r_i h_{11}(\mathbf{r}, \tilde{X}) \right) \right\} < 0, \end{aligned}$$

because by Assumption 1,  $h_1(\mathbf{r}, \tilde{X}) \leq 0$  and by Assumption 2(b),  $\forall x, h_1(\mathbf{r}, \tilde{X}) + r_i h_{11}(\mathbf{r}, \tilde{X}) < 0$ . Next, we show that  $\Pi(r_i, r_{-i})$  is submodular in  $(r_i, r_{-i})$ , which will imply that the best response function is decreasing.

It then follows from the continuity of the BR function that it has a unique fixed point, or NE. To establish submodularity of  $\Pi_i(r_i, r_{-i})$ , we calculate the cross partial derivative:

$$\frac{\partial^2}{\partial r_i \partial r_{-i}} \Pi_i(r_i, r_{-i}) = \frac{\partial}{\partial r_{-i}} m(r_i, r_{-i}) < 0, \quad (18)$$



by Assumption 2(a).

**Proof of Proposition 3.** Strict concavity of  $\Pi(r_i, r_{-i})$  which we show in the proof of Proposition 2, guarantees that the interior NE solves the following first order condition:

$$\frac{\partial}{\partial r_i} \Pi(r_i, r_{-i}) = \mathbb{E}_{\tilde{X}} \left\{ u'(r_i h(r_i + r_{-i})) \left( h(\mathbf{r}, \tilde{X}) + r_i h_1(\mathbf{r}, \tilde{X}) \right) \right\} = 0. \quad (19)$$

By symmetry of the game, the total consumption by other players  $r_{-i}^* = (n-1)r_i^*$ . Hence the interior NE solves (7). It is straightforward to verify that all other points outside the range  $[\frac{\alpha}{n}, \frac{\beta}{n}]$  are strategically dominated. Hence the NE is bracketed in  $[\frac{\alpha}{n}, \frac{\beta}{n}]$ .

**Proof of Proposition 4.** It follows directly from Athey (2002, Theorem 1) that  $r_i^*(\sigma)$  is increasing in  $\sigma$ . If the strategies were complements, one could directly conclude that the NE is increasing in  $\sigma$ . This is because the direct effect of the increased risk is in the same direction as the indirect effect due to others' strategies. However, this is not trivial in case where players' strategies are substitutes. This is because there are two direct and indirect effects of increased uncertainty on the NE that go in opposite directions. An increase in the uncertainty will directly shift the player  $i$ 's request upward. But this is also happening for the other players ( $-i$ ) and because the strategies are substitutes, it will push the player  $i$ 's request down. In general, this makes the comparative statics of the games with strategic substitutes less obvious. However, Roy and Sabarwal (2006) show that in the case of *symmetric* games with strategic substitutes, if the NE exists, it is unique and non-decreasing in  $\sigma$ .

**Proof of Remark 1.** The best-response function is calculated by solving:

$$\begin{aligned} & \frac{d}{dr_i} \left\{ \mathbb{E}_{\tilde{\sigma}} \left\{ \frac{r_i^c (\tilde{\sigma} - r_i - r_{-i})}{\tilde{\sigma}} \mid \tilde{\sigma} \geq r_i + r_{-i} \right\} \right\} \\ &= \int_{r_i + r_{-i}}^{\tilde{\sigma} + \theta} \frac{r_i^c (\sigma - r_i - r_{-i})}{\sigma} \cdot \frac{1}{\theta} d\sigma \\ &= \frac{d}{dr_i} \left\{ \left( (r_i + r_{-i}) \ln(r_i + r_{-i}) - (r_i + r_{-i}) \ln(\tilde{\sigma} + \theta) + \tilde{\sigma} + \theta - r_i - r_{-i} \right) \frac{r_i^c}{\theta} \right\} \\ &= ((r_i + r_{-i})c + r_i) \ln(r_i + r_{-i}) - ((r_i + r_{-i})c + r_i) \ln(\tilde{\sigma} + \theta) + c(-r_i + \tilde{\sigma} + \theta - r_{-i}) = 0 \end{aligned}$$

Having a symmetric equilibrium, we put  $r_{-i} = (n-1)r_i$  and obtain:

$$-r_i(cn+1) \ln(\tilde{\sigma} + \theta) + r_i(cn+1) \ln(nr_i) + c(\tilde{\sigma} + \theta - nr_i) = 0,$$

which solving for  $r_i$  gives the desired result.

We have:

$$\frac{d}{d\theta} r^{**}(\tilde{\sigma}, \theta) = \frac{d}{d\tilde{\sigma}} r^{**}(\tilde{\sigma}, \theta) = \frac{-\lambda}{nW(-\lambda(\tilde{\sigma} + \theta) \exp\{\lambda - \ln(\tilde{\sigma} + \theta)\})} \geq 0.$$

**Proof of Proposition 5.** The proof uses similar techniques to that of Proposition 4. Player  $i$ 's optimal strategy as a function of the other players consumption, with the presence of risk and ambiguity  $r_i^*(r_{-i}; \bar{\sigma}, \theta)$  maximizes:

$$\begin{aligned}\Pi(r_i, r_{-i}) &= \mathbb{E}_{\tilde{\sigma}} \phi(H(r_i, r_{-i}; \tilde{\sigma})) \\ &= \int_{\bar{\sigma}-\theta}^{\bar{\sigma}+\theta} \phi(H(r_i, r_{-i}; \sigma)) f_{\tilde{\sigma}}(\sigma; \theta) d\sigma.\end{aligned}\tag{20}$$

First, note that Condition 1 of Proposition 5 implies that:

$$\frac{\partial}{\partial r_i} H(r_i, r_{-i}; \tilde{\sigma}) \geq 0.$$

To be able to use Athey (2002, Theorem 1), we characterize conditions under which  $\phi(H(r_i, r_{-i}; \sigma))$  is log-submodular in  $(r_i, \sigma)$ . To do so, we calculate the cross partial derivative of  $\ln(\phi(H(r_i, r_{-i}; \sigma)))$ , where  $\phi(\cdot) > 0$ , to obtain:

$$\begin{aligned}\frac{\partial^2}{\partial r_i \partial \sigma} \ln(\phi(H(r_i, r_{-i}; \sigma))) &= -\frac{\phi'(H(r_i, r_{-i}; \sigma))}{\phi(H(r_i, r_{-i}; \sigma))} \frac{\partial H(r_i, r_{-i}; \tilde{\sigma})}{\partial r_i} \frac{\partial H(r_i, r_{-i}; \tilde{\sigma})}{\partial \tilde{\sigma}} \\ &\times \left( -\frac{\phi''(H(r_i, r_{-i}; \sigma))}{\phi'(H(r_i, r_{-i}; \sigma))} - \psi(r_i, r_{-i}; \tilde{\sigma}) \right) \leq 0.\end{aligned}$$

This concludes the proof.

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