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Abstract

We model a dynamic purchase context in which a consumer is uncertain about the product’s valuation. The consumer has two purchase opportunities for the product: forward purchase in period 1 or spot purchase in period 2. Two forms of regret are considered: buyer’s regret over the money paid in excess of his valuation of the product when buying forward; and hesitater’s regret for the lost opportunity of an increased surplus when not buying forward. We illustrate how regrets affect the purchase decision: a consumer is more likely to buy forward when more averse to hesitater’s regret but more likely to delay the decision when more averse to buyer’s regret. We also consider alternative consumer types to characterize how regret affects their spot purchase decisions as well as what triggers the regret. We show that type inconsistency—that is, a consumer’s incorrect anticipation of his future type—induces an inferior period-1 purchase decision and thereby reduces the consumer’s expected surplus.

Key Words: regret, consumer behavior, dynamic purchase, type inconsistency

1 Introduction

We study the impact of regret on the consumer’s decision making in a dynamic purchase context. Under regret theory (Bell 1982; Loomes and Sugden 1982; Fishburn 1991), the decision maker compares the option chosen with the forgone option, and this comparison can trigger regret (respectively, “rejoice” in the terminology of Loomes and Sugden 1982) if what the decision maker obtains is lower (respectively, higher) than what would have been obtained with a different choice. Regret theory allows explaining some of the most common behavioral regularities in economics and management and hence has become a descriptively appealing alternative to expected utility theory (von Neumann and Morgenstern 1947). A good body of literature in psychology emphasizes the importance of regret in shaping people’s preferences under risk (e.g., Larrick 1993; Zeelenberg et al. 2002). Smith (1996) and Yaniv (2000) analyze the role of regret in medical decision making. In economics and finance, Gollier and Salanié (2006) and Muermann et al. (2006) incorporate regret into models of asset pricing and portfolio choice, Braun and Muermann (2004) show that regret
can explain the commonly observed preference for low deductibles in personal insurance markets, Barberis et al. (2006) use regret theory to explain why people tend to invest too little in stocks, Michenaud and Solnik (2008) show that regret can explain empirically observations that many investors do not adopt a full hedging policy with respect to currency risk, and Muermann and Volkman (2007) show how regret can explain the disposition effect (Shefrin and Statman 1985). Regret preferences are applied by Perakis and Roels (2008) to the newsvendor model, by Nasiry and Popescu (2009) to advance selling, and by Filiz-Ozbay and Ozbay (2007) and Engelbrecht-Wiggans and Katok (2008, 2009) to auctions.

A consumer can buy early or delay the buying decision until later. Although buying late (spot purchase) is done with better information, potential reasons to buy early (forward purchase) include lower price and guaranteed availability.\(^1\) Two forms of regret naturally arise: buyer’s regret, the regret of having bought early; and hesitater’s regret, the regret of hesitating (i.e., of not having bought early). We consider the case where the consumer, when deciding whether or not to buy early, anticipates regret and takes it into consideration (Zeelenberg 1999). Following Engelbrecht-Wiggans and Katok (2008), we assume that (a) the anticipated regret enters additively into a consumer’s surplus and (b) the effect of regret on a consumer’s surplus is proportional to the amount of forgone surplus. Then buyer’s regret refers to the regret attributable to the amount of money a consumer paid in excess of the product’s valuation and hesitater’s regret refers to the regret due to the forgone opportunity of an increased surplus. These two forms of regret are equivalent to winner’s and loser’s regret in Engelbrecht-Wiggans and Katok (2008). They are also related to psychological findings on action and inaction regret (Gilovich and Medvec 1995) as well as to commission and omission regret (Kahneman et al. 1982). Winner’s, action, and commission regret correspond to our buyer’s regret; loser’s, inaction, and omission regret correspond to our hesitater’s regret. Note, however, that in this paper regret has a dynamic nature whereas in the cited papers regret is static.

Our main contribution is twofold. First, we believe that the dynamics of time is essential to the full characterization and understanding of regret. To reflect this consideration, we formulate a parsimonious two-period model that offers new insights into the dynamics of regret in consumer behavior: buying forward in period 1 may lead to buyer’s regret if the consumer discovers in period 2 that the product is of less value to him than the price paid; delaying the purchase decision to period 2 may trigger hesitater’s regret caused by missing a discount or facing limited availability. Second,

\(^1\)In this paper, we don’t consider “strategic consumers” who would forward-look, purchase from speculators, or anticipate product stock-outs before visiting the seller (cf. Su and Zhang 2008, 2009; Su 2010).
we model alternative consumer types in terms of how regret affects their spot purchase decisions and what triggers the regret, which in turn influence the consumer surplus. We also address the phenomenon of type inconsistency, in which a consumer’s anticipation of his future type differs from the actual type. As pointed out in Zeelenberg’s (1999) overview paper: “Although we have seen that people take regret into account when they know they will experience it, it is still crucial that they make correct predictions of the intensity of their possible future regret. The prediction of future emotions has not been studied extensively.” To our knowledge, we are the first to analyze how inconsistency in consumer’s anticipation of type influences decisions and emotions in a formal model under regret. We believe that this compact model constitutes a tractable basis for future theoretical and empirical analysis of and applications to consumer behavior and management.

2 Model under Regret Neutrality

In this section, we extend Gundepudi et al.’s (2001) two-period model of early-order discount to include uncertain availability. The extended model describes the behavior of a regret-neutral consumer under a dynamic purchase context, as illustrated in the following two examples.

**Example 1. Early-Order Discount**  The renowned Summer Ball of a famous multinational institution was to be held on May 15, 2010, in a prestigious location in Europe. Two types of tickets were offered by the organizers: a regular ticket for 210 euros and an “early bird” ticket, for 175 euros, that had to be purchased by April 21. Delphine, an administrative assistant in the operations department, was not sure whether to purchase early or to wait until May 15. This decision depended on the weather risk, on whether she had other opportunities (such as party invitations) on that day, on which of her friends would be going, and on whether she would still have the same boyfriend who was also planning to go.

**Example 2. Uncertain Availability**  Tickets for PJ Harvey’s concert in Paris—scheduled for May 17, 2009—were available three months in advance. Our colleague Jürgen was thinking of buying a ticket early because he feared they would be sold out quickly. However, he was not sure whether, before the concert, he could finish writing a paper important for his tenure and whether or not his friend Dana would be able to go.

These two examples illustrate that the trade-off between forward purchase and spot purchase is commonly observed in the market for a product whose valuation to the consumer is contingent
on future circumstances. Whereas buying forward often yields a discount and assured availability, buying later allows the consumer more time to establish the product’s true valuation. This dynamic purchase context can be stylized as follows. A consumer has two purchase opportunities for a product: forward purchase in period 1 and spot purchase in period 2. The consumer’s valuation $V$ for the product is a random variable realized in period 2, and the price $p$ for the product in period 2 is exogenously given. If the consumer makes the purchase in period 1—that is, when facing uncertain $V$—then the forward purchase price is $(1 - z)p$; here the discount $z \in [0, 1)$, and the product’s availability is ensured. If the consumer delays the purchase decision until period 2, then he can observe his true valuation of the product and choose whether or not to buy it at the spot purchase price $p$, although the product may no longer be available. We denote the availability of the product in period 2 as event $A$ (true with probability $\phi \in (0, 1)$), where $1_A = 1$ if the event is true (the product is available for the consumer) and $1_A = 0$ otherwise. We further assume that $V$ and $A$ are uncorrelated.

Define the consumer’s surplus as the difference between his valuation of the product and the price paid, and assume that the consumer is an expected surplus maximizer. In period 1, the consumer compares the expected surplus of buying forward, $\mathbb{E}_V[V - (1 - z)p]$, with the one derived from delaying the decision until period 2, $\mathbb{E}_{V,A}[1_A(V - p)^+]$; after making this comparison, the consumer decides whether or not to buy forward. The consumer’s period-1 purchase decision under regret neutrality is summarized in the following lemma. (All proofs are provided in Appendix B.)

**Lemma 1.** An arbitrary regret-neutral consumer buys forward if and only if

$$zp + (1 - \phi)\mathbb{E}_V(V - p)^+ \geq \mathbb{E}_V(p - V)^+. \quad (1)$$

In the case of guaranteed availability, the inequality (1) becomes $zp \geq \mathbb{E}_V(p - V)^+$; in the case of no forward purchase discount, (1) becomes $\phi \leq \mathbb{E}_V(V - p)/\mathbb{E}_V(V - p)^+.

Lemma 1 describes the consumer’s trade-off when making a forward purchase decision. The terms $zp$ and $(1 - \phi)\mathbb{E}_V(V - p)^+$ denote (respectively) the discount and ensured availability benefits from buying forward. The term $\mathbb{E}_V(p - V)^+$ denotes the expected loss of surplus incurred by not waiting until period 2 to discover the product’s true valuation.

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2We denote $a^+ = \max(0, a)$ and $a^- = -\min(0, a)$.

3If the consumer chooses not to buy forward, then he observes the realization of his valuation $V$ in period 2 and makes the spot purchase if and only if doing so yields a nonnegative surplus (i.e., iff $V - p \geq 0$).
3 Model under Regret

In this section we continue with our previous examples. Anticipating how the circumstances would turn out, Delphine decided not to purchase the early-bird ticket. Yet on May 15 the sun was shining and Delphine’s friends (and the same boyfriend) had all bought the early-bird tickets. Because Delphine had to pay 35 euros more to buy a ticket, she was so affected by hesitater’s regret that she ended up not going to the ball but rather sitting at home and contemplating her miserable situation. Jürgen, on the other hand, did buy a ticket in advance. However, Dana didn’t get a ticket (they were sold out) and Jürgen’s paper was not finished in time, so he didn’t go to the concert. Furthermore, the negative emotions of buyer’s regret upset him to such an extent that he was unable to focus on his work that day.

With these examples in mind, we shall describe the model under regret starting with period 2. All notation is as defined in Section 2, and consumer’s surplus now includes the disutility of anticipated regret. In Appendix A we offer an analysis that incorporates both “regret” and “rejoice” (cf. Loomes and Sugden 1982).

3.1 Period 2

Conditional on the consumer not having purchased in period 1, we consider his spot purchase decision in period 2. Before proceeding, we describe the consumer’s regret types according to two criteria: how regret affects the spot purchase decision; and what triggers the regret.

Spot Purchase Decision: Pragmatic versus Regret-Reflecting  We consider two possible types of consumer with respect to the spot purchase decision: pragmatic and regret-reflecting. A pragmatic consumer does not let regret affect his decision on whether or not to buy in period 2 and thus makes the purchase as long as the product’s valuation exceeds its price (i.e., when $V - p > 0$). A regret-reflecting consumer, in contrast, will buy in period 2 only if the surplus exceeds the regret of having forgone the discount $zp$ in period 1. Revisiting the example of Delphine, it seems clear that her decision not to buy on the spot was less about the monetary loss of 35 euros than about the emotional effects of regretting that loss. Had she been pragmatic, she would see that the 35 euros was not a big deal and that her valuation of going to the ball—given the perfect circumstances—clearly exceeded 210 euros.
Regret Experience: Buy versus Always  We also consider two possible consumer types in terms of how regret is triggered after the spot purchase decision is made: *buy* and *always*. A “buy” consumer experiences regret only in the case of actually making the purchase. Hence, upon purchase, his regret is proportional to the amount he overpaid relative to the case of purchasing in period 1—in other words, it is proportional to $z p$.

An “always” consumer, on the other hand, experiences regret regardless of whether the spot purchase is actually made. If he buys then he regrets the amount he overpaid, $z p$; if he does not buy then he regrets the positive amount that he could have obtained had he purchased in period 1, $(V - (1 - z) p)$. In our example, Delphine is an “always” consumer in the latter circumstance.

Define $\beta$ and $\eta$ as the intensity of buyer’s and hesitater’s regret, respectively. Let $P$ stand for the case when the consumer is of type *pragmatic*, let $R$ stand for *regret-reflecting*, $B$ for *buy*, and $W$ for *always*. A consumer in period 2 is therefore one of four possible types: $PB$, $PW$, $RB$, or $RW$. These four types completely characterize how regret affects consumers’ spot purchase decisions and what triggers the regret. In the interest of parsimony, our model focuses on the extreme possibilities; a real consumer’s behavior would, in general, lie in the interior. The following lemma describes a consumer’s surplus under these four combined types.

**Lemma 2.** The surpluses in period 2 for consumers of type $PB$, $PW$, $RB$, and $RW$ are given by:

\[
S_{2}^{PB} = 1_{V - p > 0, A} (V - p - \eta z p);
\]

\[
S_{2}^{PW} = 1_{V - p > 0, A} (V - p - \eta z p) - (1_{V - p > 0} - 1_{V - p > 0, A}) \eta (V - (1 - z) p)
- 1_{V - p < 0} \eta (V - (1 - z) p)^+;
\]

\[
S_{2}^{RB} = 1_{V - p > \eta z p, A} (V - p - \eta z p);
\]

\[
S_{2}^{RW} = 1_{V - p > \eta z p, A} (V - p - \eta z p) - (1_{V - p > \eta z p} - 1_{V - p > \eta z p, A}) \eta (V - (1 - z) p)
- 1_{V - p < \eta z p} \eta (V - (1 - z) p)^+.
\]

These four equalities can be unified and expressed as:

\[
S_{2} = 1_{V - p > 1_{R \eta z p, A}} (V - p - \eta z p) - (1_{V - p > 1_{R \eta z p}} - 1_{V - p > 1_{R \eta z p, A}}) 1_{W} \eta (V - (1 - z) p)^+
- 1_{V - p < 1_{R \eta z p}} 1_{W} \eta (V - (1 - z) p)^+,
\]

where, for $y \in \{R, W\}$, $1_{y} = 1$ if the consumer is of type $y$ and $1_{y} = 0$ if otherwise.

---

4We remark that a pragmatic consumer could make a spot purchase and be worse-off for having done so. Although this does not make much practical sense, we describe this case here for completeness.
Equation (2) is a compact form that describes the cases of interest when analyzing a consumer’s dynamic purchase under regret. The next lemma characterizes the relation between surpluses for each of these four types.

**Lemma 3.** For a given \( A \), the surpluses for consumers of type PB, PW, RB, and RW are ordered as follows for any realization of \( V \):

\[
S^R_B \geq S^F_B \geq S^P_W \geq S^R_W.
\]

Lemma 3 reveals two important points. First, an “always” consumer is never better-off than a “buy” consumer because the former experiences regret even when not buying. Second, being a regret-reflecting type entails a higher threshold for spot purchases—so as to avoid the negative surplus when buying—but also makes regret more likely when not buying.

### 3.2 Period 1

We next consider the consumer’s purchase decision in period 1. A consumer who purchases in this period obtains the valuation of the product less the price paid and the disutility of regret should the product’s valuation turn out to be lower than the price paid. Formally, his surplus is expressed as

\[
S_1 = V - (1 - z)p - \beta(V - (1 - z)p)^-.
\]  

(3)

We assume that the consumer’s valuation \( V \) of the product is drawn from a range of values whose cumulative distribution function (CDF) is \( F(\cdot) \). Also, we use \( \theta \) to denote a consumer’s belief about the distribution such that the higher is \( \theta \), the more likely is the consumer to believe that the product has a high valuation. Formally, such belief is defined as follows.

**Definition 1.** A consumer’s valuation \( V \in [l, h] \) is drawn from a family of distribution with parameter \( \theta \) and CDF \( F(\cdot|\theta) \). Without loss of generality, let \( \theta \in [0, 1] \). In addition, for all \( \theta \leq \theta' \), let \( F(u|\theta) \geq F(u|\theta') \) for any given nonnegative \( u \in (l, h) \).

From this definition it follows that a consumer’s belief about the valuation of a product is increasing in \( \theta \) in the sense of first-order stochastic dominance.

In period 1, the consumer has an anticipation of his period-2 type. Let this anticipation be \( x \in \{PB, PW, RB, RW\} \). To address the trade-off regarding whether or not to buy forward, the consumer compares the expected surplus from purchasing in period 1 with the one from waiting
until period 2. This difference is formally expressed as

$$ E_{V,A}[S_1 - S_2^x], \quad (4) $$

where $S_2^x$ is described in Lemma 2. Therefore, the consumer buys forward if (4) is positive but otherwise delays the purchase decision. The following proposition describes the consumer’s purchase decision in period 1.

**Proposition 1.** For a given probability $\phi$ and a consumer with valuation distribution $F(\cdot | \theta)$, there exist

$$ \theta_{RW} < \theta_{PW} < \theta_{PB} < \theta_{RB} $$

such that the following statements hold.

(i) Regardless of type, the consumer will wait until period 2 if $\theta \leq \theta_{RW}$ or will purchase in period 1 if $\theta \geq \theta_{RB}$.

(ii) A type-$x$ consumer will purchase in period 1 if $\theta \geq \theta_{x}$ but will otherwise wait until period 2.

Proposition 1 illustrates how a consumer’s decision in period 1 depends on his belief about the product’s valuation and his anticipation of his type in period 2. If he expects the product to have a sufficiently high valuation then he will buy forward regardless of his anticipation of his period-2 type; similarly, if he expects the product to have a sufficiently low valuation then he will delay the purchase decision. On the other hand, if he expects the product to have a “moderate” valuation then the consumer’s decision actually depends on his anticipation of his period-2 type: for each type, there exists a unique threshold level such that—once the consumer’s belief about the valuation distribution exceeds this threshold—he would purchase in period 1 but would otherwise wait until period 2. In particular, given that $\theta_{RW} \leq \theta_{PW} \leq \theta_{PB} \leq \theta_{RB}$, a consumer who sees himself as being an “always” type in period 2 is more likely to buy forward than as a “buy” type. The corollary that follows illustrates how the intensity of consumer regret affects the purchase decision.

**Corollary 1.** For a given $\theta, \phi \in (0,1)$, we have that $E_{V,A}[S_1 - S_2^x]$ is increasing in $\eta$ and decreasing in $\beta$ for $x \in \{PB, PW, RB, RW\}$.

Corollary 1 shows that if a consumer would regret more owing to buyer’s regret then he is more likely to buy on the spot, whereas if he would regret more owing to hesitater’s regret then he is more likely to buy forward. Simonson (1992) suggests that consumers who anticipate how they would feel if they made the wrong move (such as not buying forward) are more likely to purchase a currently available item on sale than to wait for a better sale. This corresponds to the first part
of our result. Furthermore, the second part of our result predicts that consumers who anticipate how they would feel if they made the wrong move (such as buying early) are more likely to wait.

### 3.3 Type Inconsistency

Zeelenberg (1999) points out that, in addition to acknowledging the existence of consumers’ anticipated regret, it is important to assess how accurate this anticipation is. To reflect this aspect, we consider the case where a consumer in period 1 has an incorrect anticipation of his actual period-2 type. Specifically, we define a *type-inconsistent* consumer as one whose anticipation of his future type differs from the actual one; in contrast, a *type-consistent* consumer is one whose anticipation of his future type matches the actual one.

Because of the potentially optimistic bias in self-prediction (Armor and Taylor 1998), consumers might suppose their future selves to be better behaved—that is, in our setting, more likely to be a pragmatic and “buy” type than a regret-reflecting and “always” type. There is a related stream of literature that studies possible sources of error in the prediction of emotions (for a review, see Wilson and Gilbert 2003). Our model attempts to formalize these issues in the context of regret, and it is the first to address the consequences of incorrect anticipation of one’s future type in a dynamic purchasing context. Consider a consumer in period 1 who anticipates his period-2 type to be $x'$ although his actual type turns out to be $x$, where $x, x' \in \{PB, PW, RB, RW\}$, and define his corresponding surplus as $S^{x', x}$. Then, given his actual type $x$, it follows that the consumer is never worse-off being time consistent. Formally, this translates into

$$E_{V,A}[S^{x, x} - S^{x', x}] \geq 0.$$

When making a dynamic purchase decision as described in this paper, a consumer is uncertain about his valuation of the product; furthermore, he might have incorrect anticipation of his future type. Proposition 1 takes both phenomena into consideration and captures consumer decision procedures. In addition, the analysis in this section zooms in on the second phenomenon and reveals the consequences of having an incorrect anticipation of the consumer’s own future type.

### 4 Conclusions

Psychologists have long recognized the importance of regret in human decision making. However, the cognitive environments that psychologists consider are not typical of consumer behavior or of
managerial settings in general. In particular, the dynamic aspects of consumer decisions under regret have been largely ignored. This paper aims to characterize such regret in a consumer dynamic purchase context.

Our paper makes two principal contributions. First, we show that the dynamics of time is essential to characterizing regret by formulating a parsimonious two-period model that captures the dynamics of regret in consumer behavior. A consumer might regret buying forward rather than waiting for better information, or he might regret delaying the purchase decision and thereby failing to ensure availability or a better price. Second, we model consumer types—in terms both of how regret affects their decisions and of what triggers the regret—which in turn influence the consumer surplus. Moreover, given that a consumer might not accurately predict his future type (Zeelenberg 1999; Wilson and Gilbert 2003), we study how type inconsistency influences decisions as well as emotions. We believe that our model provides a tractable basis for future theoretical and empirical analysis of and applications involving consumer behavior and management. Possible directions include: (i) a model that includes the use of strategic stock-outs and early-order discounts to stimulate revenues, in part by achieving segmentation of customer classes; (ii) a model that takes into consideration the supply quantity decision under regret; (iii) an experimental investigation to quantify the absolute and relative magnitude of the different types of regret; and (iv) a model using nonlinear forms of regret as in Bleichrodt et al. (2010). We leave these and further extensions to future work.

References


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Appendix A. Dynamic Purchase under Regret and Rejoice

In this section, we introduce the “rejoice” concept into the model for the sake of completeness. According to Loomes and Sugden (1982), individuals experience rejoice when they have made the best decision. Analogously to our definition of $\beta$ and $\eta$ in Section 3, we define $\rho_1$ and $\rho_2$ as the intensity of consumer’s rejoice in periods 1 and 2, respectively. We first consider the consumer’s spot purchase decision in period 2. Lemma 4 is an extension of Lemmas 2 and 3.

**Lemma 4.** In period 2, the surplus for consumers of type PB, PW, RB, and RW are as follows:

\[
\begin{align*}
S_{2}^{PB} &= 1_{V-p>0,A} (V - p - \eta z p) + \rho_2 ((1 - z) p - V)^+; \\
S_{2}^{PW} &= 1_{V-p>0,A} (V - p - \eta z p) - (1_{V-p>0} - 1_{V-p>0,A}) \eta (V - (1 - z) p) \\
&\quad - 1_{V-p<0} \eta (V - (1 - z) p)^+ + \rho_2 ((1 - z) p - V)^+; \\
S_{2}^{RB} &= 1_{V-p>\eta z p,A} (V - p - \eta z p) + \rho_2 ((1 - z) p - V)^+; \\
S_{2}^{RW} &= 1_{V-p>\eta z p,A} (V - p - \eta z p) - (1_{V-p>\eta z p} - 1_{V-p>\eta z p,A}) \eta (V - (1 - z) p) \\
&\quad - 1_{V-p<\eta z p} \eta (V - (1 - z) p)^+ + \rho_2 ((1 - z) p - V)^+. 
\end{align*}
\]

For a given $A$ and any realization of $V$,

\[
S_{2}^{RB} \leq S_{2}^{PB} \leq S_{2}^{PW} \leq S_{2}^{RW}. 
\]
Now we turn to the consumer’s forward purchase decision in period 1. If he purchases in period 1 then his corresponding surplus becomes

\[ S_{1}^P = V - (1 - z)p - \beta (V - (1 - z)p) - \rho_1 (V - (1 - z)p - 1_A (V - p))^+, \]

\[ S_{1}^R = V - (1 - z)p - \beta (V - (1 - z)p) - \rho_1 (V - (1 - z)p + 1_{V>p+\eta\geq p,A} (V - p))^+. \]

Note that, unlike surplus under regret only, a consumer’s surplus in period 1 under both regret and rejoice depends on his period-2 regret type—that is, on whether he is a pragmatic type or rather a regret-reflecting type. The next proposition is an extension of Proposition 1 that describes consumer’s forward purchase decision in period 1 under both regret and rejoice.

**Proposition 2.** For a given probability \( \phi \) and a consumer with valuation distribution \( F(\cdot | \theta) \), there exist \( \theta_{RW} \leq \theta_{PW} \leq \theta_{PB} \leq \theta_{RB} \) such that the following statements hold.

(i) Regardless of his type, the consumer will wait until period 2 if \( \theta \leq \theta_{RW} \) or will purchase in period 1 if \( \theta \geq \theta_{RB} \).

(ii) A type-\( x \) consumer will purchase in period 1 if \( \theta \geq \theta_{x} \) but will otherwise wait until period 2.

Under both regret and rejoice, a type-consistent consumer is—as in Section 3.3—no worse-off than a type-inconsistent one; therefore, (5) continues to hold. The next corollary illustrates how the intensity of a consumer’s regret and rejoice affects the purchase decision.

**Corollary 2.** For a given \( \theta, \phi \in (0, 1) \), we have that \( \mathbb{E}_{V,A}[S_1 - S_2] \) is increasing in \( \eta \) and \( \rho_1 \) as well as decreasing in \( \beta \) and \( \rho_2 \).

In short: the introduction of rejoice, though somewhat complicating consumers’ surpluses, does not qualitatively change the results obtained under regret only. In particular, the consumer’s purchase decision in period 1 remains the same (Proposition 2), a type-consistent consumer is still better-off, and the intensity of a consumer’s regret affects his purchase decision in the same direction (Corollary 2).

**Appendix B. Proofs**

**Proof of Lemma 1.** The consumer will buy forward if and only if \( \mathbb{E}_V [V - (1 - z)] \geq \mathbb{E}_{V,A} [1_A (V - p)^+] \).

Given that \( V - p = (V - p)^+ - (p - V)^+ \), (1) follows directly. The expression for guaranteed availability follows from setting \( \phi = 1 \), and the expression for no forward purchase discount follows from setting \( z = 0 \). \( \square \)
Proof of Lemma 2. Following the definition of four regret types, if $V - p > 0$ and the product is available then a consumer of type $PB$ purchases but regrets the price difference $zp$; if $V - p < 0$ or the product is not available, this consumer neither purchases nor regrets. The expression for $S_{2}^{PB}$ follows. For a consumer of type $RB$, the surplus is the same as for $PW$ except that the purchase threshold $V - p > \eta zp$. For a consumer of type $PW$, if $V - p > 0$ and the product is available, then he purchases it but regrets the price difference $zp$; if $V - p < 0$ or the product is not available, this consumer does not purchase yet still regrets the forgone surplus $(V - (1 - z)p)^{+}$. The expression for $S_{2}^{PW}$ follows. For a consumer of type $RW$, the surplus is the same as for $PW$ except that the purchase threshold $V - p > \eta zp$. This concludes the proof.

Proof of Lemma 3. Consider $1_A = 1$ and then $1_A = 0$. Lemma 3 follows directly from Lemma 2.

Proof of Proposition 1. We first need to show that $\mathbb{E}_{V,A}[S_1 - S_2^2]$ is nondecreasing in $\theta$, where $x \in \{PB, PW, RB, RW\}$. For a given $A$, it is straightforward that $S_1 - S_2^2$ is nondecreasing in $V$. Let $V$ and $V'$ be consumer’s valuations drawn from two different families of distribution with parameters $\theta$ and $\theta'$, respectively. By Definition 1, for $\theta \leq \theta'$ we have $V \leq_{st} V'$. By the characterization of first-order stochastic dominance (Müller and Stoyan 2002), $V \leq_{st} V'$ if and only if $\mathbb{E}_V[\psi(V)] \leq \mathbb{E}_{V'}[\psi(V')]$, where $\psi(\cdot)$ is a nondecreasing function. Hence, for a given $A$, $\mathbb{E}_V[S_1 - S_2^2]$ is nondecreasing in $\theta$. Since $\mathbb{E}_{V,A}[S_1^2 - S_2^2] = \phi \mathbb{E}_{V,1_A=1}[S_1^2 - S_2^2] + (1 - \phi) \mathbb{E}_{V,1_A=0}[S_1^2 - S_2^2]$, it follows that $\mathbb{E}_{V,A}[S_1^2 - S_2^2]$ is also nondecreasing in $\theta$. Then, at the boundaries, we have $\mathbb{E}_{V,A}[S_1 - S_2^2|\theta = 1] > 0$ and $\mathbb{E}_{V,A}[S_1 - S_2^2|\theta = 0] < 0$.

On the other hand, from Lemma 2 it follows that, for any realized $A$ and $V$,

$$S_1 - S_2^{RW} \leq S_1 - S_2^{PW} \leq S_1 - S_2^{PB} \leq S_1 - S_2^{RB},$$

furthermore,

$$\mathbb{E}_{V,A}[S_1 - S_2^{RW}] \leq \mathbb{E}_{V,A}[S_1 - S_2^{PW}] \leq \mathbb{E}_{V,A}[S_1 - S_2^{PB}] \leq \mathbb{E}_{V,A}[S_1 - S_2^{RB}].$$

Hence there exist $0 \leq \theta_{RW} < \theta_{PW} < \theta_{PB} < \theta_{RB} \leq 1$ such that $\mathbb{E}_{V,A}[S_1^2 - S_2^2] \leq 0$ when $\theta \leq \theta_x$ and $\mathbb{E}_{V,A}[S_1^2 - S_2^2] > 0$ otherwise. This concludes the proof.

Proof of Corollary 1. By Lemma 2, for a given $x \in \{PB, PW, RB, RW\}$ we have that $\mathbb{E}_{V,A}[S_2^2]$ is decreasing in $\eta$. On the other hand, from (3) it follows that $\mathbb{E}_{V,A}[S_1]$ is decreasing in $\beta$. Therefore, $\mathbb{E}_{V,A}[S_1 - S_2^2]$ is increasing in $\eta$ and decreasing in $\beta$. 


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