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Pricing Information Goods: A Strategic Analysis of the Selling and Pay-per-use Mechanisms

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We analyze two pricing mechanisms for information goods – selling, where an up-front payment allows unrestricted use by the consumer, and pay per-use pricing where the payments are tailored to the consumer’s usage patterns. We analytically model these pricing mechanisms in a market where consumers differ in terms of usage frequency and utility-per-use. When a monopolist employs each mechanism independently, we demonstrate that pay-per-use pricing generally yields higher profits than selling, provided the transaction cost associated with the former is not too high. We then show that pay-per-use yields higher profits than selling when usage frequency is uncertain, whereas selling yields higher profits when utility-per-use is uncertain. We then analyze a duopoly and demonstrate that, in the only non-zero pricing equilibrium, one duopolist employs selling and the other employs pay-per-use. Here, the findings from the monopoly case are reversed and selling always yields higher profits than pay-per-use. Further, we demonstrate that as the transaction cost associated with pay-per-use increases, the profits of both duopolists can increase. If an upgrade is to be offered later, we show that if consumers are myopic, the pay-per-use mechanism performs better in a monopoly, and selling performs better in a duopoly. Finally, we model the scenario where competing, vertically differentiated firms can choose endogenously between the two pricing mechanisms and demonstrate how the firms move from each offering both mechanisms when the transaction cost associated with pay-per-use is low to each offering only selling when this cost is very high.

Key words: information goods, competitive strategy, pricing mechanisms

History:

1. Introduction

Advances in network technology have enhanced the popularity of “pay-per-use” pricing for information goods (Altinkemer and Tomak 2001, Choudhary et al. 1998). With the rapid growth of Cloud-based computing and information management, the pay-per-use model is now increasingly being applied across usage settings including gaming and movies (Altinkemer and Bandyopadhyay 2000, Krause 2004, Machrone 2006), data plans (Chavez 2011), information storage (Clark 2011), monetary transactions (Reeve 2011), teleconferencing (McKnight and Boroumand 2000), billing
services, software applications such as Salesforce.com that now allow payment on a “per-login” basis, and Computer Aided Design (CAD) suites (Machine Design 2000). In this paper, we analyze the performance of conventional selling (i.e., unlimited-use pricing) and pay-per-use mechanisms for information-intensive goods in monopoly and duopoly contexts.

Pricing has been described as the moment of truth in the marketing context (Corey 1991), and is sensitive to consumer usage patterns and heterogeneity (Chen et al. 2001). Independent of how much value the company has created for customers, a bad pricing decision can lower profits either because the company tries to extract too much value back from the customer, or because the company leaves too much value on the table for the customer. In particular, when it comes to information goods, pay-per-use and other technology-intensive pricing strategies can play a key role in enhancing the value delivered to the customer, in facilitating better extraction of created value, and ultimately, in altering the competitive landscape (Nault and Dexter 1995).

To motivate the theoretical development, consider the application of pay-per-use pricing in the engineering design industry. Altair Engineering’s Hyperworks On-Demand is a computer-aided engineering (CAE) simulation software platform that is applied by clients to create virtual models that can aid structural optimization (Roush 2011), and the study of fluid-structure interaction and multi-body dynamics (e.g., collisions between “virtual” cars). Under a “pay-per-usage” license model, customers purchase HyperWorks “units” which are used to pay for the metered use of a suite of HyperWorks products that are hosted in the Cloud. Altair highlights four key benefits of the pay-per-use pricing model: (a) Flexibility – the client can avoid scaling up or scaling down any hardware capacity in response to demand peaks and valleys; (b) Efficiency – the hardware capabilities and solutions employed by Altair will handle workloads better that a typical client’s in-house infrastructure; (c) Accessibility – the client can access the platform from any physical location; and (d) Security – each client’s data is safely partitioned and stored in the Cloud.

In contrast to renting or leasing, which typically confer complete usage rights on the consumer for a limited temporal period, pay-per-use tightly links the consumer’s usage patterns to payment (Altman and Chu 2001). This makes pay-per-use more appealing than selling to consumers with low usage frequencies (Nicolle 2002). Despite this benefit, there is an ongoing debate about the relative advantages of pay-per-use pricing compared to selling (Bonasia 2007). We demonstrate how the strength of each mechanism varies as a function of consumers’ utility-per-use and usage frequency, and across monopoly and duopoly contexts. Our analysis contributes to the literature and practice by addressing the following questions, among others: How do conventional selling and pay-per-use pricing perform in markets where consumers differ in terms of (a) utility-per-use and
(b) usage frequency? How do sellers and pay-per-use providers compete for such a market? How do competing firms that offer vertically differentiated information goods endogenously choose a pricing strategy?

Our analysis delivers several findings across monopoly and duopoly contexts. Some key findings are briefly summarized as follows. First, consider a monopolist who chooses between selling and pay-per-use pricing. Here, pay-per-use yields higher profits provided the transaction cost associated with it is not too high. Further, consumer surplus is generally higher under selling; in contrast, social surplus is higher under selling only when the transaction cost associated with pay-per-use is low. We also study the impact of demand uncertainty in either the frequency of usage or the utility-per-use on both mechanisms, and find that uncertainty in usage frequency favors the pay-per-use mechanism, whereas uncertainty in utility-per-use favors the selling mechanism. If the firm can offer upgrades later, we find that in a monopoly, the pay-per-use mechanism performs better than the selling mechanism.

Second, we consider the case where duopolists that offer identical products can choose any one, or both, of the pricing mechanisms. Here, we demonstrate that, in equilibrium, one firm sells the good, and the other employs pay-per-use. Further, in contrast to the monopoly, selling strongly outperforms pay-per-use pricing in a duopoly. An interesting finding here is that the profits of the pay-per-use provider are inverted-U shaped with respect to the transaction costs associated with that mechanism. We demonstrate that a higher transaction cost can reduce competitive pressure and allow the profits of both firms to increase. If the firm can offer upgrades later, we find that in a duopoly, selling performs better than the pay-per-use mechanism.

Following that, we consider the case where duopolists with vertically differentiated information goods can employ either one or both mechanisms. Here, we show that the pay-per-use mechanism makes a minor contribution to the profits of each firm but plays an important role in enhancing the profits from selling. We also demonstrate how the equilibrium choices of pricing mechanisms by the firms vary as a function of the transaction cost associated with the pay-per-use mechanism.

We also consider how the relative attractiveness of the pricing mechanisms is affected by other factors, including: (a) uncertainty about consumer utility-per-use and/or usage frequency; (b) the possibility of upgrades being offered by the firm; (c) alternative (non-uniform) distributions of these utility dimensions; and (d) implementation and service costs encountered in enterprise contexts.

1.1. Extant Literature and Contribution
Research on renting versus selling in economics and marketing has primarily focused on the durable goods context. The key theme here is the seller must coordinate primary and secondary markets
to maximize profits (Bulow 1982, Desai and Purohit 1998, Shulman and Coughlan 2007). An advantage of selling is that it can help “lock up” the market when there is a threat of competitive entry (Bucovetsky and Chilton 1986). Similarly, in a competitive environment in the auto industry, Desai and Purohit (1999) find that competitors do not lease all their units, but they either use a combination of both leasing and selling, or only selling. They also find that the fraction of leasing decreases as the manufacturer’s products become more similar, and the competition between them increases. Finally, Purohit (1997) and Bhaskaran and Gilbert (2009) compare leasing and selling when manufacturers have multiple competing intermediaries. We make a number of contributions to this stream of literature. First, whereas the existing literature has considered competition between a designated seller and a designated pay-per-use provider (Fishburn and Odlyzko 1999), we analyze the equilibrium that involves the endogenous choice of the selling and/or pay-per-use by a pair of competing firms. In this context, we derive a “defensive positioning” outcome – specifically, the firm may choose pay-per-use in a monopoly, but if it expects competition, it can protect profits by selling the good. Additionally, we demonstrate that as the transaction cost associated with pay-per-use increases, the profits of both duopolists can increase. Second, we move beyond the existing literature by analyzing the more general equilibrium where duopolists who offer vertically differentiated goods can choose either or both of the pricing mechanisms. Here, we demonstrate how the market is divided among the competing mechanisms, and show that when the transaction cost associated with pay-per-use is: (a) at low to moderate levels, both firms employ both mechanisms; (b) high, then the firm that offers higher quality employs both mechanisms, but the competitor only engages in selling; and (c) very high, then both firms only sell the good.

Second, in the context of durable goods, the impact of demand uncertainty on the leasing and selling mechanisms has been studied by Desai et al. (2007), who find that demand uncertainty causes the producer’s inventory level to be U-shaped in the durability of the product, and leasing causes a larger loss due to uncertainty than selling the product. We contribute to this stream of literature by showing that demand uncertainty has different effects if the uncertainty is in the usage frequency (in which case, the pay-per-use mechanism performs better), or in the utility-per-use (in which case, selling performs better). The impact of upgrades has been studied by Yin et al. (2010) in the durable goods sector, and Sankaranarayan (2007) for information goods. Yin et al. (2010) find that frequent product upgrades are caused by the existence of electronic peer-to-peer markets, and the interaction of these markets with the markets for retail goods. Sankaranarayan (2007) finds that frequent new version releases of software result in lowering the developer’s profits due to the strategic delaying of purchasing the software by consumers, and an offer by the developer of a
warranty for new versions solves this commitment problem. Providing software as a service can also lead to lower investments in quality in the long run compared to the case where the software is sold under a perpetual license (Choudhary 2007). We contribute to this stream of literature by showing that for information goods, the introduction of upgrades favors the pay-per-use mechanism in a monopoly, and selling in a competitive environment.

A number of studies in the durable goods context has examined the choice of leasing or selling for a monopolist. Desai and Purohit (1998) find that the relative profitability of leasing and selling hinges on the rates at which leased and sold units depreciate, and if sold units depreciate significantly faster than leased units, then selling is the better option. If both units depreciate at different rates, then the firm should use a combination of leasing and selling. Bhaskaran and Gilbert (2005) find that in durable goods, a manufacturer that leases its products charges a higher price by limiting the availability of the product in response to the availability of a complement. Finally, Chien and Chu (2008) show that profits from selling durable goods is higher than from leasing durable goods, as selling durable goods creates a stronger customer base using penetration pricing.

In the study of information goods as well, selling and renting may not be mutually exclusionary – a monopolist can optimally employ both mechanisms when consumer preferences are heterogeneous and future upgrades are of uncertain quality (Zhang and Seidmann 2003). Similarly, Zhang and Seidmann (2010) demonstrate that, in the presence of quality uncertainty and network effects, the firm should offer both the selling and pay-per-use mechanisms as this increases both profits and consumer welfare. Other studies of electronic goods pricing include pricing of online services (Essegaier et al. 2002, Gurnani and Karlapalem 2001, Jain and Kannan 2002), and the interaction of pricing of online goods and product customization (Dewan et al. 2003). Postmus et al. (2009) compare selling and the pay-per-use mechanism with software development costs, and find that pay-per-use dominates if software development is expensive, while selling dominates if development costs are low. Sundararajan (2004) compares the selling and pay-per-use (usage based) mechanisms in a monopoly setting and derives conditions where a combination of usage based pricing and selling should be used. In contrast, our analyses throughout correspond to the case where consumers are jointly differentiated in terms of both usage utility and usage frequency for information goods. Jiang et al. (2007a) consider a similar differentiation and show that selling is optimal when consumers have homogeneous valuations, but pay-per-use is more profitable than selling in markets with heterogeneous consumers and low user inconvenience costs, when there is the potential of piracy in the market. The presence of network effect also favors pay-per-use over selling. We show that if
a monopolist wants to choose one mechanism, it should prefer pay-per-use if transaction costs are low, and selling if transaction costs are high. If both mechanisms can be used, then pay-per-use is used to cover a larger share of the market if transaction costs are low, and a lower share of the market if transaction costs are high.

Overall, our research contributes to the literature in a number of ways. Our analyses throughout correspond to the case where consumers are jointly differentiated in terms of both usage utility and usage frequency. This approach reveals some interesting insights that are unavailable in the literature. In a monopolistic environment, the pay-per-use mechanism performs better in a wider variety of cases if transaction costs are low, while selling dominates in a competitive environment even when transaction costs are low. When transaction costs are high, selling performs better than the pay-per-use mechanism. On balance, much remains to be studied about how usage-sensitive pricing mechanisms can create a strategic advantage (Ba and Johansson 2006). Our work takes a step towards filling in some of the knowledge gaps.

The base model is introduced in Section 2. In Section 3, the pricing mechanisms are analyzed in both monopoly and duopoly contexts. In Section 4, we analyze competition with differentiated goods. Model extensions and tests for robustness are presented in Section 5. We conclude with Section 6.

2. The model

2.1. Assumptions and notation

Assumption 1: Consumers purchase the information good, pay for it per use, or abstain from the market after considering the benefits over a $N$-period horizon.

Assumption 2: Each consumer $i$ is characterized by the coordinate pair $(\theta_i, \phi_i)$, where $\theta_i$ is the expected usage frequency of consumer $i$ in any period and $\phi_i$ is the associated utility-per-use.

Assumption 3: Usage utility $\phi$ is distributed $U[0, \phi_H]$ and usage frequency $\theta$ is distributed $U[0, \theta_H]$. The upper limits of the distributions are elastic – this captures a range of market structures and consumer behaviors. For example, if $\theta_H > 1$, consumers can use the good multiple times per period. The situation where $\theta$ is distributed $U[0, 1]$ is a special case where $\theta_i$ represents the probability of use in a given period.

Assumption 4: Consumers using pay-per-use incur a transaction cost $T$ per usage occasion. This cost reflects the disutility from the repeated administrative and transaction costs associated with pay-per-use pricing (Varian 2000, Cheng et al. 2003). For example, to play a game from an online site, the consumer may need to first establish some authentication. The transaction cost could
also capture the “ticking meter” effect that results when payment is tightly linked to consumption (Train 1991). When usage experiences in the pay-per-use and selling mechanisms converge, \( T = 0 \).

**Assumption 5:** Reflecting a perfect capital market, the discount factor for future utility, \( \delta \), is the same for the firm and consumers.

**Assumption 6:** The information good can be reproduced and delivered at negligible marginal cost. Further, we ignore fixed costs because they do not directly impinge on optimal pricing decisions.

Further, we consider a horizon of \( N \) periods. The total consumer utility over this horizon is obtained by multiplying the expected utility per period and \( D \), the NPV factor over \( N \) periods which is defined as follows:

\[
D = 1 + \delta + \delta^2 + \delta^3 + \ldots + \delta^{N-1} = \frac{1-\delta^N}{1-\delta}
\]

We will selectively relax some assumptions later to model specific market contexts. Throughout the paper, subscript \( S \) denotes a variable pertaining to the seller and subscript \( O \) denotes a variable corresponding to the pay-per-use provider. Hence, \( p_S \) denotes the selling price associated with the selling mechanism, \( MS_S \) and \( \Pi_S \) respectively denote the market share and profits from selling, and \( U_{iS} \) denotes the consumer utility from buying the good. Similarly, \( p_O, MS_O, \) and \( \Pi_O \) denote the payment-per-use, market share and profits related to the pay-per-use mechanism, and \( U_{iO} \) denotes the consumer utility surplus from that mechanism.

### 2.2. Model setup

Consumer \( i \) will find pay-per-use pricing feasible only when \( \phi_i - T \geq p_O \). This participation constraint requires the net per-use utility to be greater than the payment per-use \( p_O \). Similarly, consumer \( i \) will find buying feasible only when \( \theta_i \phi_i D \geq p_S \). Here, the left hand side represents the (expected) discounted utility consumer \( i \) gains from using the good through \( N \) periods. This utility must be greater than the purchase price for buying to be feasible. The surplus gained by consumer \( i \) under each pricing mechanism is:

\[
\text{Pay-per-use: } U_{iO}(p_O) = \theta_i(\phi_i - T - p_O)D \quad (1)
\]

\[
\text{Selling: } U_{iS}(p_S) = \theta_i \phi_i D - p_S \quad (2)
\]

In computing the surplus under pay-per-use, payments are made with a usage frequency of \( \theta_i \) in each period because the per-use payments are tightly linked to realized usage. In contrast, under selling, the upfront price \( p_S \) is directly subtracted from the consumer’s utility with no discounting. Figure 1 displays the market shares of the mechanisms when each is independently used by the monopolist. If the monopolist only offers pay-per-use, all consumers with \( \phi_i > p_O + T \) potentially use the good. Accordingly, the market share is \( \frac{2\theta_i(p_O + T)}{\phi_H} \). The average frequency of use of the pay-per-use usage mechanism in each period is \( \frac{\theta_i}{2} \). Therefore, the expected profits under pay-per-use
Figure 1  Market shares for pay-per-use pricing and selling

\[ \Phi_H \]

Pay-per-use fraction

Uncovered market

\[ p_O + T \]

\[ p_O \]

Uncovered market

\[ \theta_H \]

Buying fraction

\[ \Phi_H \]

\[ p_S = \theta \phi D \]

\[ \theta_H \]

Uncovered market

are:

\[ \Pi_O(p_O) = \frac{\phi_H - (p_O + T)}{\eta_H} p_O D \]  \hspace{1cm} (3)

If the firm sells the good, all consumers with usage frequencies in the range \( \theta \in [\frac{p_S}{\phi D}, \theta_H] \) derive (weakly) positive utility from buying the good. The market share of the seller and the resulting profits are, respectively:

\[ MS_S(p_S) = \frac{1}{\phi_H \theta_H} \int_{\phi_H p_S}^{\theta_H} \left[ \theta_H - \frac{p_S}{\phi H} \right] d\phi = \frac{1}{\phi_H \theta_H} \left[ \theta_H \phi_H - \frac{p_S}{\phi H} + \frac{p_S}{\phi D} \log\left( \frac{p_S}{\phi_H \theta_H} \right) \right] \]

\[ \Pi_S(p_S) = MS_S(p_S) p_S = \frac{p_S}{\phi_H \theta_H} \left[ \theta_H \phi_H - \frac{p_S}{\phi H} + \frac{p_S}{\phi D} \log\left( \frac{p_S}{\phi_H \theta_H} \right) \right] \]  \hspace{1cm} (4)

The model provides a parsimonious and flexible representation of selling and pay-per-use mechanisms.

3. Pricing in monopoly and duopoly contexts

3.1. Monopoly

We first analyze the separate use of each mechanism by a monopolist. The results are tabulated in Table 1 (see Appendix A1 for proof).

<table>
<thead>
<tr>
<th>Payment per use/Selling price</th>
<th>Pay-per-use</th>
<th>Selling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market share</td>
<td>( p_O = \frac{\phi_H - T}{\phi_H} )</td>
<td>( p_S = 0.285\theta_H \phi_H D )</td>
</tr>
<tr>
<td>Profits</td>
<td>( MS_O = \frac{\phi_H - T}{2\phi_H} )</td>
<td>( MS_S = 0.358 )</td>
</tr>
<tr>
<td>Consumer Surplus</td>
<td>( \frac{1}{16} \phi_H (\phi_H - T)^2 D )</td>
<td>( 0.0769 \theta_H \phi_H D )</td>
</tr>
<tr>
<td>Social Welfare</td>
<td>( \frac{3}{16} \phi_H (\phi_H - T)^2 D )</td>
<td>( 0.1787 \theta_H \phi_H D )</td>
</tr>
</tbody>
</table>

Table 1: Optimal outcomes in a monopoly

**Proposition 1:** If the monopolist uses only pay-per-use pricing or selling, the profits from pay-per-use pricing are higher than those from selling if \( T < 0.0976 \phi_H \). The net consumer surplus is
always higher under selling, but the total social welfare under pay-per-use is higher than that under selling if $T < 0.0237\phi_H$.

A monopolist prefers pay-per-use when $T$ is low, consistent with Sundararajan (2004). Intuitively, pay-per-use perfectly discriminates among consumers in terms of usage frequency. This is because, if the total cost per use – the sum of $p_O$ and $T$ – is lower than the utility-per-use, consumers will use the product on every occasion that they need to. This total cost is low when $T$ is low, thereby increasing market share and profits. In contrast, under selling, forward-looking consumers make purchase decisions by comparing their total utility across time – this depends on both utility-per-use and usage frequency – to the selling price. Therefore, selling is relatively more attractive than pay-per-use for consumers with a low per-use utility but a high usage frequency (see Figure 1). The level of $T$ decides which of these contrasting benefits translate into higher profits. Another insight is that, because the per-use payment is set to maximize profits on a period-by-period basis, the optimal per-use payment and corresponding market share are independent of the usage frequency and the NPV factor $D$ (see Table 1).

Consumers have a higher net surplus under selling because, unlike pay-per-use, the firm that sells cannot discriminate perfectly along the usage frequency dimension. Interestingly, even when $T = 0$, the net consumer surplus under pay-per-use is lower than that under selling. However, the increased profitability from pay-per-use ensures that for very low transaction costs ($T < 0.0237\phi_H$), the social welfare under pay-per-use is higher than under selling.

We extend these findings in two directions. First, consider the case where the monopolist jointly offers both mechanisms. Here, only consumers with usage frequency higher than a certain critical frequency given by $\theta_c = \frac{p_S}{(p_O+T)D}$ will buy the good, and others will prefer pay-per-use. We demonstrate that the monopolist employs only pay-per-use when $T = 0$, and only selling as $T$ approaches $\phi_H$. For intermediate levels of $T$, the monopolist employs both mechanisms. The role of pay-per-use in generating profits decreases as $T$ increases (see Electronic Appendix 1 for proof).

Second, consider the case where the good has positive marginal costs, possibly due to some material content. We demonstrate that such costs decrease profits more sharply from pay-per-use than from selling. Intuitively, under selling, there is a dedicated unit variable cost allocated to each consumer who buys the good. In contrast, the allocation of supply to demand is more finely controlled under pay-per-use. For example, as noted by Varian (2000), the same DVD at a rental outlet can be shared by different consumers at different points of time – therefore, variable costs can be allocated across a wider consumer base. Such “resource pooling” enhances profits (see Electronic Appendix 2 for proof).
Moving forward from the “base case,” we now examine how uncertainty in usage frequency and utility-per-use affect the performance of these mechanisms.

3.2. Uncertainty in usage frequency and utility-per-use

In this section, we consider the impact of uncertainty in the usage frequency and utility-per-use separately first, and then jointly. In the case where only the usage frequency is uncertain, let the usage frequency be distributed $U[0, \theta_H + v]$ with probability 0.5, and $U[0, \theta_H - v]$ with probability 0.5 (as before, utility-per-use is distributed $U[0, \phi_H]$). Similarly, in the case where only the utility-per-use is uncertain, let the utility-per-use be distributed $U[0, \phi_H + v]$ with probability 0.5, and $U[0, \phi_H - v]$ with probability 0.5 (the usage frequency is distributed $U[0, \theta_H]$). We can show that any other binary probability combinations will not affect our findings. Here, $v$ captures the degree of uncertainty, and $v = 0$ implies the absence of uncertainty. We find that (see Electronic Appendices 3A and 3B for proof):

**Proposition 2:** (i) If the usage frequency is uncertain, the cut-off transaction cost below which pay-per-use is more profitable than selling is higher than in absence of uncertainty.

(ii) If the utility-per-use is uncertain, the cut-off transaction cost below which pay-per-use is more profitable than selling is lower than in the absence of uncertainty.

Effectively, uncertainty in usage frequency increases the relative attractiveness of the pay-per-use mechanism. Recall that, under pay-per-use, a consumer pays for the good contingent on usage. If consumers use the information good more or less frequently compared to the expected usage frequency, the (risk neutral) firm still performs as well on average with the payment-per-use set at the same level as in the case with no uncertainty. That is, there is no efficiency loss from such uncertainty. In contrast, under selling, if the firm persists with same selling price as in the case without uncertainty, then it loses market share if the actual distribution of usage frequency is at the high end, and loses margin if that distribution is at the low end. Hence, the firm prices lower under such uncertainty, reducing its profits.

In contrast, the expected profits under both mechanisms decrease when utility-per-use is uncertain, but profits under pay-per-use decrease more sharply. Therefore, such uncertainty increases the attractiveness of selling. The intuition is as follows. When the firm employs pay-per-use, its market share is driven solely by utility-per-use because pay-per-use discriminates perfectly across consumers in terms of usage frequency. Uncertainty regarding utility-per-use lowers the market share under pay-per-use sharply because the firm is forced to choose a substantially lower payment-per-use that can reasonably accommodate both the possible scenarios. On the other hand, selling is less affected because the usage frequency, which plays a role in the construction of consumer utility.
when evaluating the pricing mechanism, is known to the firm. Therefore, uncertainty along the usage frequency and utility-per-use dimensions have distinct implications for each pricing mechanism.

Next, consider the case where there is uncertainty in both usage frequency and utility-per-use. We find that dual uncertainty lowers profits under pay-per-use to a lesser extent than under selling (see Electronic Appendix 3C for a numerical analysis). This is because pay-per-use is negatively affected only by uncertainty in utility-per-use whereas selling is negatively affected by both sources of uncertainty.

3.3. Duopoly with undifferentiated information goods

Consider a duopoly where each firm can employ either selling or pay-per-use, or both mechanisms. We model this game in two stages, where each firm chooses their mechanisms in the first stage, and then given their choice of mechanisms, they choose their prices in the second stage (Coughlan 1985, Gupta and Loulou 1998). Each firm’s good offers utility-per-use $\phi$ to consumers, and consumers have the same usage frequency $\theta$ for both products. Result 1 summarizes the equilibrium outcome for the channel choice game (see Electronic Appendix 4 for proof):

**Result 1:** Consider a duopoly with undifferentiated information goods. In the corresponding Nash equilibrium, one firm employs selling and the other employs pay-per-use. There exists no Nash equilibrium with positive firm profits where either firm employs both mechanisms, or both firms employ the same mechanism.

If both firms employ the same mechanism, they compete themselves down to zero profits. Each firm can lower its selling price or the payment-per-use to undercut the other – this cycle of undercutting will lead to a zero-profit outcome.

Recall that the utility surpluses related to pay-per-use pricing and buying are described in eqns (1) and (2) respectively. Consumer $i$ will prefer buying to pay-per-use only if $U_{iS}(p_S) > U_{iO}(p_O)$. Therefore, consumers will buy only if their usage frequency is higher than a certain critical frequency denoted by $\theta_c = \frac{p_S}{(p_O + T)D}$. Consumers with $\theta_i < \theta_c$ either choose pay-per-use or abstain from the market. The market shares of the mechanisms are described in Figure 2.

In the corresponding Nash equilibrium, the payment-per-use is $p_O = \frac{T(\phi_H - T)}{\phi_H + T}$, and the selling price satisfies the implicit equation: $\theta_H\phi_H - \frac{2p_S\phi_H}{(p_O + T)D} + \frac{2p_S}{D}\log\left[\frac{p_S}{(p_O + T)\theta_H D}\right] + \frac{p_S}{D} = 0$ (Proofs are in Electronic Appendix 5). Our findings are summarized in Propositions 3 and 4 (Proofs are in Electronic Appendices 6 and 7).

**Proposition 3:** The equilibrium payment per-use and the selling price in the duopoly are lower
than the corresponding optimal payment-per-use and selling price when a monopolist uses the mechanisms independently. However, the equilibrium payment-per-use of the pay-per-use provider in the duopoly first increases, and then decreases with transaction cost \((T)\).

Proposition 3 captures the price suppressing effect of competition. However, in addition, there are some subtle and interesting forces at work here (see Figures 3, 4 and 5 for a comparison of prices, profits, and market shares across the base case and the duopoly (plotted for \(\theta_H = \phi_H = 1, D = 5\)). In the duopoly, one would expect that payment-per-use and profits of the pay-per-use provider are high when the associated transaction cost \((T)\) is low. However, when \(T\) is very low, both the payment-per-use and the profits of pay-per-use provider are low. Both initially increase as \(T\) increases, leading to the increasing part of the inverted U-shaped profit function. The intuition is that, when \(T\) is very low, the seller has to sharply lower prices to attract consumers – this enhances competition and lowers profits across the board. As \(T\) increases, the competitive pressure
decreases and the profits of both firms increase. However, as $T$ increases even further, the profits of the pay-per-use provider begin to decrease because that mechanism is ultimately less attractive from the consumer’s viewpoint.

**Figure 4** Profits from selling and pay-per-use pricing in base case and duopoly

![Graphs showing profits from selling and pay-per-use pricing](image)

**Proposition 4:** In a duopoly, the profits of the seller are always higher than those of the firm that offers pay-per-use pricing.

Proposition 4 is interesting in several respects. Recall that when $T$ is low, a monopolist will prefer pay-per-use pricing over selling. However, in a duopoly, the seller’s profits are higher even when $T$ is low. The intuition is as follows. When the pay-per-use provider increases its payment-per-use, it loses substantial revenues because all consumers with a utility per-use that is lower than that payment drop out, taking the entire temporal stream of revenue associated with them. In contrast, under selling, the usage frequency contributes to the total projected utility of the consumer and ties the consumer to the selling mechanism. Further, selling captures the “best” consumers – those with high utility-per-use and high usage frequency in a duopoly – leaving the less attractive fraction of the market to the pay-per-use mechanism (see Figure 2). The ability of pay-per-use to allow consumers to flexibly tailor payments to usage is a strength in the monopoly context. The reverse side of the coin, though, is the inability of pay-per-use pricing to lock in consumers at an early stage. This inability has no adverse consequences in a monopoly, but hurts the mechanism in a duopoly.

Propositions 1 and 4 jointly offer some interesting insights on the choice of both mechanisms. When $T$ is low, a firm will adopt pay-per-use if it does not expect competition. However, if it expects
competition, it will sell the good instead. That is, the firm will adopt a “defensive positioning” strategy, where it accepts lower profits which are more robust in the face of competition, rather than higher profits that are susceptible to significant erosion on competitive entry.

The consumer surplus and social welfare generated by the selling and the pay-per-use mechanisms are derived in Electronic Appendix 8 and plotted in Figure 13 in the Electronic Appendix. When $T$ is low, the consumer surplus and social welfare generated by the pay-per-use mechanism is higher because the firm is forced to offer a low payment-per-use. When $T$ is high, the consumer surplus and social welfare generated by selling are higher.

Figure 5 Selling and pay-per-use market shares in base case and duopoly

3.4. Selling and pay-per-use pricing with upgrades

In this section, we consider the impact of a future upgrade to the information good in a monopoly (where the monopolist can choose either selling or pay-per-use pricing) and in a duopoly (where one firm chooses selling and the other chooses pay-per-use pricing), where the upgrade is a part of the sequential product introduction strategy of the firm (Padmanabhan et al. 1997). We assume that consumers are myopic, myopic consumers have no foresight regarding the introduction of the upgrade. Therefore, the upgrade will introduce no competitive effect because these consumers will initially choose between selling and pay-per-use pricing solely based on the base information good. Later on, these consumers will simply decide whether or not to purchase the upgrade.

Let the upgrade be introduced in period $k$, where $k < N$. The consumer obtains a higher per-use utility of $a\phi$ ($a > 1$) from the upgrade. Therefore, post-upgrade, the per-use utility is uniformly
distributed between 0 and \(a\phi_H\). Let \(D_{k-1}\) denote the NPV factor over \(k - 1\) periods (\(D_{k-1} = \frac{1 - \delta^{k-1}}{1 - \delta}\)), and \(D\) denote the NPV factor over \(N\) periods (as before).

3.4.1. Upgrades in a monopoly

First, consider a monopolist who adopts pay-per-use pricing in a market where consumers are myopic. The profits from the base good and the upgrade are denoted by, respectively:

\[
\Pi_{O1}(p_{O1}) = \phi_H - (p_{O1} + T) \phi_H \theta_H D_{k-1}; \quad \Pi_{O2}(p_{O2}) = a\phi_H - (p_{O2} + T) a\phi_H \theta_H D_{k-1} (D - D_{k-1}) \quad (5.1)
\]

The total profits are \(\Pi_O = \Pi_{O1} + \Pi_{O2}\) (from eqn. 5.1). Because the upgrade delivers a higher utility, the firm charges a higher per-use payment after the upgrade has been introduced (the optimal per-period payments are derived in the Electronic Appendix 9).

Second, consider the case where the monopolist employs selling in a market where consumers are myopic. Here, the base good will be priced exactly as in the monopoly without upgrades and profits are identical to that case as well (see eqn. (4) for the profit expression). Consumers only pay for the additional utility derived from the upgrade. If the upgrade is introduced in period \(k\), this additional surplus is: \((a - 1)\phi_H(D - D_{k-1}) - \delta^{k-1}p_U\). This additional surplus is set to zero to find the fraction of consumers who will buy the upgrade. This condition is structurally identical to that which applies when consumers decide whether or not to buy the base good. Therefore, all consumers who purchase the base good will also choose to upgrade. The profits from the upgrade alone (\(\Pi_{S2}\)) are denoted by:

\[
\Pi_{S2}(p_U) = \delta^{k-1}p_U MS_{S2} = \frac{\delta^{k-1}p_U}{\phi_H \theta_H} [\theta_H - \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})} + \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})} \log(\frac{\delta^{k-1}p_U}{(a-1)\phi_H \theta_H (D-D_{k-1})})] \quad (5.2)
\]

When consumers are myopic, the monopolist first solves for the optimal price of the base good. Given the optimal price \(p_{S1}\) for the base good, the firm then solves for the optimal upgrade price that maximizes the profits denoted in eqn. (5.2) above. In doing so, the monopolist must ensure that \(MS_{S2} \leq MS_{S1}\), so that only a subset of consumers who purchased the base good will purchase the upgrade. This condition is satisfied because all consumers who purchased the base good will also buy the upgrade. With this setting, the key finding here is summarized as follows (See Electronic Appendix 9 for proof):

**Proposition 5:** If a monopolist offers the information good with a future upgrade, then the cut-off transaction cost below which pay-per-use pricing yields higher profits than selling is higher compared to the case where the firm only offers the basic information good. Therefore, the presence of the future upgrade increases the relative attractiveness of pay-per-use pricing compared to selling.

In a monopoly, the cut-off transaction cost below which pay-per-use pricing is superior to selling is \(T_C = 0.0976\phi_H\) (see Proposition 1). According to the result, the cut-off transaction cost is higher
than 0.0976\(\phi_H\) if the monopolist offers a future upgrade. The intuition is as follows. Under pay-per-use, the payment-per-use can easily be shifted upward to recapture a substantial fraction of the surplus generated by the increase in utility-per-use delivered by the upgrade. In contrast, the selling mechanism cannot separately deal with this increase in utility-per-use because here consumers consider the total utility derived from a combination of usage utility and usage frequency. Therefore, selling is less efficient at recapturing the surplus generated by the upgrade.

### 3.4.2. Upgrades in a duopoly

Myopic consumers do not expect the upgrade – therefore, competition in the context of the base good is identical to competition in the case without the upgrade. However, once the consumers are locked into either mechanism for the base good, the competitors simply price the upgrade when it is introduced as if they were monopolists with captive customers. The following proposition describes how the upgrade affects the attractiveness of the two pricing mechanisms (See Electronic Appendix 10 for proof):

**Proposition 6:** Consider a duopoly where one firm sells the information good and the other adopts pay-per-use pricing. If the competitors offer a future upgrade, the profits of firm that sells the good increases to a greater extent than those of the firm that adopts pay-per-use pricing. Therefore, the presence of the future upgrade increases the relative attractiveness of selling compared to pay-per-use pricing in a duopoly.

The intuition behind Proposition 6 is as follows. In a duopoly, if the firm that offers pay-per-use pricing increases its per-use payment for the base information good and the upgrade, it loses substantial revenues. This is because all consumers with a utility-per-use that is lower than the revised per-use payment drop out, taking the entire temporal stream of revenue associated with them. In contrast, under selling, the frequency of usage contributes to the total projected utility of the consumer and performs the role of tying the consumer more tightly to the selling mechanism. Therefore, even with the upgrade, selling can address the market more efficiently in a competitive setting by jointly leveraging the utility-per-use and the usage frequency dimensions.

From Proposition 3, we know that in a competitive environment, the firm that adopts the selling mechanism has higher profits from the base information good than the firm that adopts the pay-per-use mechanism. When the two firms introduce the upgrade, they no longer compete with each other for the upgrade because consumers have chosen either the pay-per-use mechanism or the selling mechanism for the base good (under certain conditions that prevent consumers from switching to the other mechanism, which can be seen to be easily satisfied for the upgrade to yield positive utility to either set of consumers). Because the market share of the selling mechanism for the base information good is higher under competition, and because selling has already captured
the customers whose utility-per-use and usage frequency are jointly high, the firm that sells can price the upgrade to extract a higher profit than the firm that adopts pay-per-use. Therefore, the presence of the upgrade benefits the seller to a greater extent in a duopoly.

4. Duopoly with vertical differentiation

We now consider the case where duopolists who offer vertically differentiated information goods can each either or both pricing mechanisms (Soberman 2005). We assume that usage frequency \( (\theta_i) \) and base utility-per-use \( (\phi_H) \) are both distributed \( U[0,1] \) – however, firm 1’s (superior) good yields a utility-per-use of \( \phi \) (as before), whereas firm 2’s good yields a utility-per-use of \( \lambda \phi \) \( (\lambda < 1) \). Here, \( \lambda \) is the differentiation factor. Accordingly, consumer \( i \) derives the following net utilities from each firm-mechanism combination:

Firm 1: Pay-per-use: \( U_{O1}(p_{O1}) = \theta_i(\phi - T - p_{O1})D \); Buy: \( U_{S1}(p_{S1}) = \phi_iD - p_{S1} \)

Firm 2: Pay-per-use: \( U_{O2}(p_{O2}) = \theta_i(\lambda \phi - T - p_{O2})D \); Buy: \( U_{S2}(p_{S2}) = \lambda \phi_iD - p_{S2} \)

We analyze two cases, corresponding to (a) low levels of transaction cost \( T \); and (b) very high levels of \( T \).

**Low \( T \):** Each firm adopts both pricing mechanisms when \( T < 0.11 \). When \( T \) is low \( (0 < T < 0.11) \), the division of market shares is described in Figure 6.

![Figure 6 Market share division for low transaction costs](image)

As seen in Figure 6, consumers with a usage frequency lower than a certain critical frequency will prefer pay-per-use pricing. The critical frequency varies with the region that adjoins the pay-per-use market share on its right hand side in Figure 6. For example, the pay-per-use market share of firm 2 always is always adjoined to the right by the selling market share of firm 2. Therefore, the critical frequency for the pay-per-use market share of firm 2 is always \( \frac{p_{S2}}{(p_{O2}+T)D} \) — this is captured by the vertical boundary, which represents a fixed usage frequency. Likewise, for low \( T \) (Figure 6),
the pay-per-use market share of firm 1 is adjoined to the right by the selling market share of firm 1 for high levels of usage utility $\phi$—here again, the critical frequency is $p_{S1} = \frac{p_{O1}}{(p_{O1} + T)/D}$, which is again represented by a vertical boundary. However, for lower values of $\phi$ in Figure 6, the pay-per-use market share of firm 1 is adjoined by the selling market share of firm 2. To find the critical frequency in these regions, we set $U_{O1}(p_{O1}) = U_{S2}(p_{S2})$. The critical frequency here is $\theta_i = \frac{p_{S2}}{p_{O1} + T - (1 - \lambda) \phi_i}$. Intuitively, this frequency is now additionally a function of $\lambda$ and utility-per-use ($\phi$) because the competing goods yield different levels of utility.

Analytical expressions for the market shares and profits are derived in Electronic Appendix 11. We numerically characterize the Nash equilibria. The equilibrium profits are plotted for different values of $T$ ($0 < T < 0.11$, representing low transaction costs) in Figure 7 and for different values of the differentiation factor $\lambda$ in Figure 8.

Figure 7  Variation of selling, pay-per-use and total profits with $T$

The analysis yields some interesting insights. First, the profits in Figure 7 correspond to the case where the market is competitive ($\lambda = 0.9$) and $T$ is low—the configuration of market shares corresponds to Figure 6. Here, each firm uses both pricing mechanisms, but the profits from selling to each firm are much higher than those from pay-per-use. This effect is more pronounced as $T$ increases because the profits from pay-per-use decrease in $T$. In particular, the profits from the pay-per-use mechanism for firm 2, that offers lower quality, are marginal. Therefore, as $T$ increases, we can expect the firms, and firm 2 in particular, to discard the pay-per-use mechanism.

Moving to Figure 8, we observe that firm 1’s profits from both selling and pay-per-use are uniformly decreasing in $\lambda$. When $\lambda$ is low, the firms are sufficiently differentiated. Therefore, firm
1 adopts both pricing mechanisms and prices its (superior quality) good akin to a monopolist. As $\lambda$ increases, the market becomes more competitive. Here, firm 1 prefers to use the selling mechanism to a greater degree to generate profits because, as discussed earlier in this paper, that mechanism performs better than pay-per-use in a competitive setting. When $\lambda$ is low, firm 2 also uses a combination of the two mechanisms – however, given that its information good is of much lower quality, its profits are substantially lower than those of firm 1 (comparing the profits in Figure 8). As $\lambda$ increases from 0.2 to 0.5, the profits of firm 2 increase. Intuitively, even though the market becomes more competitive, firm 2 benefits from the increase in the quality of its information good. However, as $\lambda$ increases yet further ($\lambda \geq 0.6$), the profits of firm 2 decrease because the negative implications of increased competition overshadow the positive implications of better quality. Finally, as the market becomes extremely competitive, we see that firm 2 discards the pay-per-use mechanism, and relies on purely selling the information good. As discussed earlier, there is no equilibrium with non-zero profits in an undifferentiated setting ($\lambda = 1$) that involves any firm offering both mechanisms, or both firm offering the same mechanism.

**High $T$:** When $T > 0.5$, both firms drop the pay-per-use mechanism and adopt only selling. The profits of the two firms are plotted with respect to the differentiation factor $\lambda$ in Figure 9 (see Electronic Appendix 11 for derivations of equilibrium outcomes and Figure 14 in Electronic Appendix 11 for market shares). We see that firm 1’s profits are decreasing throughout in $\lambda$. In contrast, firm 2’s profits increase with $\lambda$ when $\lambda$ is low. However, for $\lambda > 0.6$, the negative implications of increased competition are stronger than the positive implications of higher quality, and firm 2’s profits decrease with $\lambda$. 
Overall, our findings in this section highlight the importance of thinking about how the pay-per-use and selling mechanisms can work in cooperation with each other to enhance profits, and competitively against each other to reduce profits. The optimal go-to-market strategy should balance these forces with the objective of maximizing firm-level profits.

5. Model extensions and tests for robustness

We extended the model in multiple directions and also tested our findings for robustness:

**Variability in usage frequency and utility-per-use:** We find that variability in usage frequency does not affect the relative performance of the two mechanisms (Electronic Appendix 12). If the utility-per-use is higher later in the horizon, the profits under pay-per-use rise more sharply as the pay-per-use mechanism differentiates perfectly between consumers on the utility-per-use dimension. Therefore, such an increase in the utility-per-use increases the attractiveness of the pay-per-use mechanism. However, when the utility-per-use decreases later in the horizon, the market share under pay-per-use decreases sharply because the firm is forced to choose a substantially lower payment-per-use later in the horizon.

**Inclusion of implementation and service costs:** In an enterprise software context, we assume that when a software application is purchased outright, an enterprise client incurs a one-time implementation cost \( C \) and a service (or maintenance) cost each time the application is used \( c_s \) (Electronic Appendix 13). Under pay-per-use, the service provider incurs a one-time implementation cost \( S \) to move the enterprise client’s database to the service provider’s server,
and a service cost \( (c_o) \) each time the client uses the application. As might be expected, an increase in \( C \) and \( c_o \) favors pay-per-use whereas an increase in \( S \) and \( c_o \) favors selling, which is similar to Ma and Seidmann (2007). Effectively, implementation and service costs shift the boundary that demarcates the parametric regions where one pricing mechanism yields higher profits than the other.

**Correlated utility dimensions:** Modifying the assumption that utility-per-use and usage frequency are independently distributed, we analyzed the case where these dimensions are perfectly correlated, so that consumers with high utility-per-use also have a high usage frequency (Electronic Appendix 14). We find that such correlation further enhances the relative advantage of the pay-per-use mechanism in a monopoly context. This is because the market share of the pay-per-use mechanism generally consists of consumers with a high utility-per-use. If these consumers also have a high usage frequency, the revenue streams that accrue to this mechanism are of even greater magnitude. The reverse is true in a duopoly with undifferentiated information goods. Intuitively, under competition, the market share carved out by the selling mechanism consists of consumers whose utility-per-use and usage utility are both relatively high even when these utility dimensions are independently distributed. However, when these dimensions are perfectly correlated, selling efficiently captures the most valuable part of the market. Therefore, such correlation enhances the relative advantage of selling over pay-per-use in the competitive context.

**Non-uniform distributions of utility-per-use and usage frequency:** We modified the assumption that utility-per-use and usage frequency were uniformly distributed (Electronic Appendix 15). Consider the case where each pricing mechanism is independently used by the monopolist. First, if utility-per-use has an upper triangular distribution (skewed to the right), we find that the pay-per-use mechanism does better than in the uniform distribution case. Intuitively, the market share of the pay-per-use mechanism is graduated solely on the utility-per-use dimension – therefore, this mechanism can efficiently capture consumers with a high utility-per-use. Selling, on the other hand, does not benefit as strongly under this distribution because it does not serve consumers with a low usage frequency, even if their utility-per-use is high. Conversely, if utility-per-use has a lower triangular distribution (skewed to the left), then the selling mechanism does better than in the uniform distribution case. This is because the scarcity of consumers with high utility-per-use hurts the pay-per-use mechanism.

Next, consider the case where usage frequency has an upper triangular distribution (skewed to the right). Here, we find that, while both mechanisms yield a higher profit than in the uniform distribution case, selling benefits to a greater extent. This is because selling captures more consumers
with both a higher utility-per-use and a higher usage frequency. In contrast, some consumers with a high utility-per-use who are captured by pay-per-use now use the information good less frequently. Finally, consider the case where usage frequency has a lower triangular distribution (skewed to the left). Here, the profits from both mechanisms decrease compared to the uniform distribution case, but the profits from selling decrease more sharply. This is because selling would ideally capture customers with both a high utility-per-use and a high frequency, but fewer customers now exhibit both these characteristics.

Endogenous usage frequency: Finally, we analyzed the case where consumers (endogenously) reduced their usage frequency if the payment-per-use was high in a setting where a monopolist chose one of the two mechanisms (Electronic Appendix 16). Here, we find that the monopolist who adopts pay-per-use lowers the payment-per-period to accommodate this externality. Therefore, ceteris paribus, selling becomes more attractive in the presence of such endogeneity.

6. Conclusion
We analyzed two pricing mechanisms for information goods – selling, where an up-front payment bestows unrestricted usage rights, and pay-per-use, where payments are closely tailored to the consumer’s usage patterns. Consumer utility was modeled as a function of the usage frequency and the utility-per-use of the good.

We first considered a monopolist who could employ either mechanism. Here, we first showed that as long as the transaction cost associated with pay-per-use is low, profits from that mechanism are higher than those from selling. Intuitively, pay-per-use pricing achieves perfect discrimination along the usage frequency dimension by allowing consumers to pay for the good only when it was used. In contrast, selling is attractive to consumers who have both a relatively high utility-per-use and a high usage frequency. We then showed that the presence of a positive unit marginal cost lowers profits from both mechanisms but hurts selling more than pay-per-use. This is because, under pay-per-use, there is superior resource pooling and a smaller quantity of goods, representing a lower total marginal cost, can be more intensively used by multiple customers across time to satisfy market demand.

Next, we considered the impact of uncertainty in consumer utility-per-use and usage frequency. We demonstrate that pay-per-use performs relatively well when usage frequency is uncertain. Going further, we demonstrate that uncertainty in utility-per-use lowers profits from both mechanisms, but lowers profits from pay-per-use to a greater. This is because the market share of the pay-per-use mechanism is determined solely based on the distribution of utility-per-use, whereas the
reduction in the profits from selling is tempered by the fact that the distribution of usage frequency is known with certainty. If both the usage frequency and the utility-per-use are uncertain, then profits under pay-per-use reduce to a lower extent than under selling compared to the case where both are known with certainty because pay-per-use is not affected by uncertainty along the usage frequency dimension.

Moving to the competitive context, we analyzed a duopoly where firms offering identical information goods could employ either or both pricing mechanisms. We find that, in equilibrium, one firm adopts selling and the competitor adopts pay-per-use. Here, in contrast to the monopoly, the seller’s profits generally dominate the pay-per-use provider’s profits. The key implication is that a monopolist who employs one pricing mechanism is better off with pay-per-use pricing provided the associated transaction costs are not too high. In contrast, a monopolist who expects future competition should pursue a “defensive” positioning strategy and choose to sell the good instead. We also find that the profits of the pay-per-use provider were inverted U-shaped with respect to the transaction cost associated with that mechanism. The intuition was traced back to the role of the transaction cost in moderating the level of competition in the market.

We then analyzed a duopoly where firms offering vertically differentiated information goods could employ either or both pricing mechanisms. When the transaction costs associated with pay-per-use are low, we find that each firm adopts both mechanisms, but the major share of the profits for each firm accrues from selling. Whereas the pay-per-use mechanisms contributes only weakly to profits, the high payments-per-use play a role in propping up the selling prices and increasing profits from selling. The profits of the firm offering the superior information good are strictly decreasing in the quality of the competitor. In contrast, the profits of the firm offering the inferior information good are inverted U-shaped in the quality of the good – the initial improvement in quality increases profits, but as the separation between the two competing goods decreases, further improvement in quality depresses profits on account of more intense competition. When the transaction cost associated with pay-per-use is high, the firm offering the inferior good dispenses with pay-per-use pricing and adopts only selling, whereas the competitor adopts both mechanisms. This reduces the intensity of competition and increases the profits of both firms. Finally, when the transaction cost is very high, both firms adopt only selling. Overall, our findings provide a range of insights into how pricing strategies for information goods must be designed in various market environments.

Our analysis has the following limitations that can be addressed by future research. First, the role of usage frequency-based discounts and other price discrimination mechanisms can be analyzed in both the monopoly and competitive contexts. Second, we have primarily focused on the case
where consumers are differentiated in terms of utility-per-use and usage frequency. Future work could incorporate the notion of horizontal differentiation where consumers have varying preferences for the offerings. This is an emerging research domain and much remains to be done. We hope our analysis and findings catalyze further inquiry in the area.

References


APPENDIX

APPENDIX A1: Proof of Proposition 1:

If the firm charges a payment-per-use of \( p_O \), all consumers with \( \phi_i \geq p_O + T \) use the information good. The market fraction using the good is, therefore: \( \frac{\phi_i - (p_O + T)}{\phi_H} \). The average frequency of use for consumers in this fraction in any given period is \( \frac{\theta_H}{\phi_H} \). The expected profits for the firm are:

\[
\Pi_O = \frac{\phi_i - (p_O + T)}{\phi_H} \frac{\theta_H}{2} p_O D
\]

The first order condition (FOC, henceforth) with respect to \( p_O \) yields \( p_O = \frac{\phi_H - T}{2} \). The second order condition (SOC, henceforth) for \( \Pi_O \) to be concave can easily be seen to be satisfied because \( \Pi_O \) is quadratic in \( p_O \) with a negative sign on the square term (the second derivative of \( \Pi_O \) with respect to \( p_O \) is \(-1\)). Substituting the optimal value for \( p_O \) in the expression for the market fraction and profits yields the optimal outcomes.

Under selling, if the firm sets a selling price of \( p_S \), then in Figure 2, at any given usage utility \( \phi \), consumers with usage frequencies in the range \( \theta \in [\frac{p_S}{\phi_H}, \theta_H] \) derive (weakly) positive utility from the purchase. From Figure 1, the market fraction that buys the good and the profits, respectively, are denoted by:

\[
MS_S = \frac{1}{\phi_H^D} \int_{\frac{p_S}{\phi_H^D}}^{\theta_H} \left( \phi_H - \frac{p_S}{\phi_H^D} \right) d\theta = \frac{1}{\phi_H^D} \left[ \theta_H \phi_H - \frac{p_S}{\phi_H^D} + \frac{p_S}{\phi_H^D} \log\left\{ \frac{p_S}{\phi_H^D} \right\} \right]
\]

\[
\Pi_S = \frac{p_S}{\phi_H^D} \left( \theta_H \phi_H - \frac{p_S}{\phi_H^D} + \frac{p_S}{\phi_H^D} \log\left\{ \frac{p_S}{\phi_H^D} \right\} \right)
\]

The FOC with respect to \( p_S \) yields \( \theta_H \phi_H - \frac{p_S}{\phi_H^D} + \frac{p_S}{\phi_H^D} \log\left\{ \frac{p_S}{\phi_H^D} \right\} = 0 \). If \( \frac{p_S}{\phi_H^D} = d \), then \( d = 0.285 \), and therefore \( p_S = 0.285\theta_H \phi_H D \). The SOC evaluated at \( p_S = 0.285\theta_H \phi_H D \) reveals that the second derivative with respect to \( p_S \) is negative. For all conditions of \( p_O \) and \( p_S \) in all the sections in this paper, the FOCs are sufficient because the optimal solutions have to be interior point solutions (except for the monopoly case, where the firm may prefer one mechanism to the other). This is because the profits go to zero if \( p_O \) and \( p_S \) are priced at the extremities (if either \( p_O \) or \( p_S \) is zero, then the margins are zero, and if they are equal to the upper bounds, then the market shares are zero). Substituting the optimal value for \( p_S \) in the expression for the market fraction and profits yields the optimal outcomes. Equating the profits from pay-per-use pricing and selling yields the cut-off transaction cost.

Social Welfare: We first integrate the cumulative surplus of all consumers who use the information good by pay-per-use (consumers with \( \phi_i > p_O^* + T \)).
Pay-per-use: \( CS(p_O) = \frac{1}{\theta_H \phi_H} \int_{\phi_H}^{\theta_H} (\phi - T - p_O) d\phi \int_0^{\phi_H} \theta d\theta D \)

\[ = \frac{1}{\phi_H} \frac{(\phi_H - T)^2}{16} \theta_H D \] since \( p_O = \frac{1}{2} \phi_H - T. \)

From Equation (2), if the monopolist uses the selling mechanism, consumer i’s cumulative surplus is given by:

Selling: \( U_iS(p_S) = \theta_i \phi_i D - p_S \)

We next integrate the cumulative surplus of all consumers who buy the information good (consumers with \( \theta_i \phi_i D > p_S = 0.285 \theta_H \phi_H \)). For consumers with a given \( \theta \), the surplus is given by \( \int_0^{\phi_H} \frac{285}{\phi_H} (\theta \phi - \theta \phi_H) D d\phi. \) The frequency of usage varies from 0.285\( \theta_H \) (when \( \phi = \phi_H \)) to \( \theta_H \).

Hence,

\[ CS(p_S) = \frac{1}{\theta_H \phi_H} \left[ \int_0^{\phi_H} \{ \int_0^{\phi_H} (\theta \phi D - p_S) d\phi \} d\theta \right] = 0.0769 \theta_H \phi_H D. \]

The social welfare is given by the sum of the profits and net consumer surplus from that mechanism.  ■

**ELECTRONIC APPENDIX FOR**

**Pricing Information Goods: A Strategic Analysis of the Selling and Pay-per-use Mechanisms**

**ELECTRONIC APPENDIX 1: Joint use of pay-per-use and selling mechanisms in a monopoly:**

Recall that the utilities related to pay-per-use pricing and buying are \( U_iO(p_O) = \theta_i(\phi - T - p_O) D \) and \( U_iS(p_S) = \theta_i \phi_i D - p_S \) respectively. Consumer i uses pay-per-use only if \( U_iO(p_O) > U_iS(p_S) \), if their usage frequency is higher than a critical frequency \( \theta_c = \frac{p_S}{(p_O + T) D}. \) Others will prefer pay-per-use, or not participate in the market. From Figure 2, the analytical expressions for the market shares of the selling and pay-per-use mechanisms are the same as in the duopoly, given by:

\[ MS_S(p_S, p_O) = \frac{1}{\phi_H \theta_H} \left[ \frac{\theta_H}{\phi_H} \left[ \phi_H - \frac{p_S}{\theta_H D} \right] d\theta - \int_{\theta_H}^{\phi_H} \frac{p_S}{\theta_H D} \left[ \phi_H - \frac{p_S}{\theta_H D} \right] d\theta \right] = \frac{1}{\phi_H \theta_H} \left[ \theta_H \phi_H - \frac{p_S \phi_H}{(p_O + T) D} + \frac{p_S}{D} \log \left( \frac{p_S}{\phi_H (p_O + T) D} \right) \right] \]

\[ MS_O(p_S, p_O) = \frac{1}{\phi_H \theta_H} \left[ \phi_H - (p_O + T) \right] \frac{p_S}{(p_O + T) D} \]

In calculating \( MS_S(p_S, p_O) \) above, we subtract out those consumers with a usage frequency lower than \( \theta_c \) from the market share of selling in the monopoly case. The profits from the selling and pay-per-use mechanisms, and total profits are, respectively:

\[ \Pi_S(p_S, p_O) = \frac{1}{\phi_H \theta_H} \left[ \theta_H \phi_H - \frac{p_S \phi_H}{(p_O + T) D} + \frac{p_S}{D} \log \left( \frac{p_S}{\phi_H (p_O + T) D} \right) \right] p_S \]

\[ \Pi_O(p_S, p_O) = \frac{1}{2\phi_H \theta_H} \phi_H - (p_O + T) \right| \frac{p_S}{(p_O + T) D} p_O D \]

\[ \Pi(p_S, p_O) = \frac{1}{\phi_H \theta_H} \left[ \theta_H \phi_H - \frac{p_S \phi_H}{(p_O + T) D} + \frac{p_S}{D} \log \left( \frac{p_S}{\phi_H (p_O + T) D} \right) \right] p_S + \frac{1}{2\phi_H \theta_H} \phi_H - (p_O + T) \right| \frac{p_S}{(p_O + T) D} p_O D \]
Here, the profits corresponding to pay-per-use are computed by multiplying the market share for pay-per-use pricing and the total payments discounted over time with the average frequency of usage, given by \( \frac{p_S}{2(p_O + T)D} \). The analytical expressions for the optimal selling price and the payment-per-use are derived from the FOCs of \( \Pi(p_S, p_O) \) with respect to \( p_S \) and \( p_O \). The optimal payment-per-period derived from the FOCs is: \( p_O = \frac{1}{4}[\phi_H - T + \sqrt{[\phi_H - T]^2 + 16\phi_H T}] - T \). Further, from the FOCs, the optimal selling price satisfies the following implicit equation: 
\[
\theta_H \phi_H - \frac{p_S(p_O + T)}{(p_O + T)^2 D} \left( \frac{1}{2} \frac{p_S}{p_O + T} \right) \left( \frac{p_O + T}{p_S} \right) D - p_S \phi_H T (p_O + T)^2 D = 0.
\]
After substituting the closed form expression for \( p_O \), this implicit equation can be solved to yield the optimal price for any given set of parameters \( \theta_H \) and \( \phi_H \).

Figure 10   Profits from joint use of selling and pay-per-use in a monopoly

Figure 10 describes the monopolist’s profits from using both mechanisms jointly (assuming \( \theta_H = \phi_H = 1, \ D = 5 \)). The optimal payment-per-period and the optimal price increase from the base case where the monopolist uses each mechanism alone. It can be verified that the monopolist obtains a lower market share from each mechanism compared to when each is used independently. However, the total market share is higher when the mechanisms are used jointly than when each is used independently. Further, even when both mechanisms are available, the monopolist uses pay-per-use pricing alone when transaction cost \( T=0 \), both mechanisms when \( T \) is at a moderate level (with a substantial fraction of the total profit accruing from pay-per-use pricing), and the selling mechanism alone when \( T \) is high.

**ELECTRONIC APPENDIX 2: Analysis with positive unit variable cost:**

As before, the market fraction that participates under pay-per-use pricing is \( \frac{\phi_H - (p_O + T)}{\phi_H} \). The average frequency with which consumers use pay-per-use pricing in any given period is \( \frac{\theta_H D}{2} \). The expected profits for the firm are \( \Pi_O = \frac{\phi_H - (p_O + T)}{\phi_H} \theta_H (p_O D - c) \). To explain this profit function, note that the market share is multiplied by the average frequency of usage of \( \frac{\theta_H}{2} \) to yield the total number of times the good is used in any given period. The resulting quantity is then multiplied
by $p_O$ to yield the total revenue per period. Finally, multiplying the revenue by the NPV discount factor $D$ yields the discounted value of total revenues. On the cost side, the firm needs to produce at the outset the quantity of goods that will be used during each period – this is denoted by $\frac{\phi_H - (p_O + T) \theta_H}{\phi_H} \theta_H$ – at the cost of $c$ per unit. For example, at a video store, a fixed number of DVDs are procured, which can be used over multiple months. This cost is subtracted from the revenues to yield the net profits in the expression above. Alternatively, if the firm sells the good outright, then from eqn (4), the profits are:

$$\Pi_S = \frac{p_S - c}{\phi_H} \left[ \theta_H \phi_H - \frac{p_S}{D} + \frac{p_S}{D} \log\left(\frac{p_S}{\theta_H \phi_H D}\right) \right]$$

The optimal payments per period and the optimal selling price are derived from the corresponding FOCs. The optimal per-period payment is $p_O = \frac{(\phi_H - T) D - c}{\phi_H}$, and the corresponding profits are $\Pi_O = \frac{(\phi_H - T) D - c}{\phi_H} \theta_H$. The optimal price is characterized by the implicit equation $\theta_H \phi_H - \frac{p_S}{D} + \frac{2 p_S}{D} \log\left(\frac{p_S}{\phi_H D}\right) - \frac{1}{D} \log\left(\frac{p_S}{\theta_H \phi_H D}\right) = 0$.

Both the optimal per-period payment and the optimal price increase if there is a positive unit variable cost. This is intuitive. The cut-off transaction cost below which pay-per-use pricing yields higher profits than selling is described in the figure in Figure 11 (for $D = 5$).

Figure 11  
Cut-off transaction cost ratio $\frac{T_C}{\phi_H}$ with unit variable cost $c$ ($D = 5$)

As observed in Figure 11, the cut-off transaction cost ($T_C$) increases in the unit variable cost. That is, the presence of a positive unit variable cost increases the relative attractiveness of pay-per-use compared to selling.
ELECTRONIC APPENDIX 3A

Proof of Proposition 2a:

If the firm adopts the selling mechanism and sets a selling price of \( p_S \), the market share of the firm is:
\[
MS_+(p_S) = \frac{1}{\phi_H(\theta_H + v)}[(\theta_H + v)\phi_H - \frac{p_S}{D} + \frac{p_S}{D} \log(\frac{p_S}{(\theta_H + v)\phi_H D})] \text{ if the actual frequency of usage } \theta^*U(0, \theta_H + v).
\]
\[
MS_-(p_S) = \frac{1}{\phi_H(\theta_H - v)}[(\theta_H - v)\phi_H - \frac{p_S}{D} + \frac{p_S}{D} \log(\frac{p_S}{(\theta_H - v)\phi_H D})] \text{ if the actual frequency of usage } \theta^*U(0, \theta_H - v).
\]

The expected profits of the firm are:
\[
E[\Pi_S(p_S)] = p_S \frac{MS_+(p_S) + MS_-(p_S)}{2}.
\]

If the firm adopts pay-per-use and sets a payment-per-use of \( p_O \), its profits are:
\[
\Pi_{O+}(p_O) = \phi_H - \phi_H(p_O + T) \frac{\phi_H + v}{2} p_O D \text{ if the actual frequency of usage } \theta^*U(0, \theta_H + v).
\]
\[
\Pi_{O-}(p_O) = \phi_H - \phi_H(p_O + T) \frac{\phi_H - v}{2} p_O D \text{ if the actual frequency of usage } \theta^*U(0, \theta_H - v).
\]

The expected profits of the firm are:
\[
E[\Pi_O(p_O)] = \frac{\Pi_{O+}(p_O) + \Pi_{O-}(p_O)}{2} \implies E[\Pi_O(p_O)] = \phi_H - \phi_H(p_O + T) \frac{\phi_H + v}{2} p_O D, \text{ which is the same as the case where the distribution of the frequency of usage is known with certainty. Therefore, the firm}
\]

sets the payment-per-use \( p_O = \phi_H - T \) and optimal profits are
\[
E[\Pi_O(p_O)] = \frac{1}{8} \phi_H(\phi_H - T)^2 D.
\]

If the firm adopts selling, its expected profits are:
\[
E[\Pi_S(p_S)] = p_S \frac{MS_+(p_S) + MS_-(p_S)}{2} \text{ where }
\]
\[
MS_+(p_S) = \frac{1}{\phi_H(\theta_H + v)}[(\theta_H + v)\phi_H - \frac{p_S}{D} + \frac{p_S}{D} \log(\frac{p_S}{(\theta_H + v)\phi_H D})] \text{ and }
\]
\[
MS_-(p_S) = \frac{1}{\phi_H(\theta_H - v)}[(\theta_H - v)\phi_H - \frac{p_S}{D} + \frac{p_S}{D} \log(\frac{p_S}{(\theta_H - v)\phi_H D})].
\]

The expected profits under selling are:
\[
E[\Pi_S(p_S)] = p_S \left\{ \left[ 1 - \frac{p_S}{\phi_H(\theta_H + v)D} \right] + \frac{p_S}{\phi_H(\theta_H + v)D} \log(\frac{p_S}{\phi_H(\theta_H + v)D}) \right\} + \left\{ 1 - \frac{p_S}{\phi_H(\theta_H - v)D} + \frac{p_S}{\phi_H(\theta_H - v)D} \log\left(\frac{p_S}{\phi_H(\theta_H - v)D}\right) \right\}
\]

In the case without uncertainty, the profits of the firm under selling mechanism are:
\[
\Pi_S = p_S \left[ 1 - \frac{p_S}{\phi_H D} \right] + \frac{p_S}{\phi_H D} \log\left(\frac{p_S}{\phi_H D}\right)
\]

We need to show that, when the firm adopts selling, expected profits under uncertainty are lower than those under no uncertainty. That is, we need to show that:
\[
\frac{p_S}{2} \left\{ \left[ 1 - \frac{p_S}{\phi_H(\theta_H + v)D} \right] + \frac{p_S}{\phi_H(\theta_H + v)D} \log(\frac{p_S}{\phi_H(\theta_H + v)D}) \right\} + \left\{ 1 - \frac{p_S}{\phi_H(\theta_H - v)D} + \frac{p_S}{\phi_H(\theta_H - v)D} \log(\frac{p_S}{\phi_H(\theta_H - v)D}) \right\}
\]<
\[
p_S \left[ 1 - \frac{p_S}{\phi_H D} \right] + \frac{p_S}{\phi_H D} \log\left(\frac{p_S}{\phi_H D}\right)
\]

This is true by Jensen’s inequality because for a concave function \( f, f(x + v) + f(x - v) < 2f(x) \).
Now, to show that \( f(\theta_H) = p_S\{1 - \frac{p_S}{\phi_H + v}\theta_H + \frac{p_S}{\phi_H + v} \log\left(\frac{p_S}{\phi_H + v}\theta_H\right)\} \) is concave in \( \theta_H \), we note that \( \frac{\partial^2 f(\theta_H)}{\partial \theta_H^2} = p_S[-1 + 2 \log\left(\frac{p_S}{\phi_H + v}\theta_H\right)] \) < 0. This is because \( p_S < \theta_H \phi_H D \) for selling to be feasible.

Therefore, profits under selling when there is uncertainty in the distribution of usage frequency are lower than in the absence of uncertainty. Because the profits of the firm under pay-per-use are identical across the cases with and without uncertainty in usage frequency, the cut-off transaction cost below which pay-use-use yields higher profits than selling is higher in the presence of such uncertainty. Stated differently, such uncertainty makes pay-per-use more attractive from the firm’s perspective. ■

**ELECTRONIC APPENDIX 3B**

**Proof of Proposition 2b:**

If the firm adopts selling and sets a selling price of \( p_S \), its market share is:

\[
MS_+(p_S) = \frac{1}{\phi_H + v}(\phi_H + v)\theta_H - \frac{p_S}{\phi_H + v} \log\left(\frac{p_S}{\phi_H + v}\theta_H\right)
\]

if the utility-per-use \( \phi^U(0, \phi_H + v) \). The expected profits are:

\[
E[\Pi_S(p_S)] = p_S\left[MS_+(p_S) + MS_-(p_S)\right] \tag{E2a.1}
\]

where \( \overline{\phi}_H = \phi_H - \frac{v^2}{\phi_H} \) and

\[
K = \frac{1}{2} \left(\frac{p_S}{\phi_H + v}\theta_H\right) \log\left(\frac{p_S}{\phi_H + v}\theta_H\right) + \frac{p_S}{\phi_H - v}\theta_H \log\left(\frac{p_S}{\phi_H - v}\theta_H\right) - \frac{p_S}{\phi_H + v}\theta_H \log\left(\frac{p_S}{\phi_H + v}\theta_H\right)
\]

**Condition A1:** We show that \( K > 0 \) because

\[
\frac{p_S}{\phi_H + v}\theta_H \log\left(\frac{p_S}{\phi_H + v}\theta_H\right) + \frac{p_S}{\phi_H - v}\theta_H \log\left(\frac{p_S}{\phi_H - v}\theta_H\right) > \frac{2p_S}{\phi_H + v}\theta_H \log\left(\frac{p_S}{\phi_H + v}\theta_H\right)
\]

That is, we prove that

\[
\frac{p_S}{\phi_H + v}\theta_H \log\left(\frac{p_S}{\phi_H + v}\theta_H\right) + \frac{p_S}{\phi_H - v}\theta_H \log\left(\frac{1}{\phi_H + v} + \frac{p_S}{\phi_H - v}\theta_H \log\left(\frac{p_S}{\phi_H - v}\theta_H\right) + \frac{p_S}{\phi_H + v}\theta_H \log\left(\frac{1}{\phi_H + v}\right) >
\]

\[
\frac{2p_s}{\phi_H \phi_D} \log \left\{ \frac{p_s}{\phi_H \phi_D} \right\} \\
\implies \text{To prove that} \]
\[
\frac{1}{\phi_H + v} \log \left( \frac{p_s}{\phi_H D} \right) + \frac{1}{\phi_H + v} \log \left( \frac{1}{\phi_H - v} \right) + \frac{1}{\phi_H} \log \left( \frac{p_s}{\phi_H D} \right) + \frac{1}{\phi_H - v} \log \left( \frac{1}{\phi_H - v} \right) > \frac{2}{\phi_H} \log \left\{ \frac{p_s}{\phi_H D} \right\} + \frac{2}{\phi_H} \log \left\{ \frac{1}{\phi_H} \right\} \\
\]
The terms corresponding to \( \log \left( \frac{p_s}{\phi_H D} \right) \) cancel out. Therefore, we need to prove that:
\[
\frac{1}{\phi_H + v} \log \left( \frac{1}{\phi_H + v} \right) + \frac{1}{\phi_H - v} \log \left( \frac{1}{\phi_H - v} \right) > \frac{2}{\phi_H} \log \left\{ \frac{1}{\phi_H} \right\} \\
\]
On simplifying, this reduces to proving that:
\[
(\phi_H + v) \log(\phi_H + v) + (\phi_H - v) \log(\phi_H - v) > 2\phi_H \log \phi_H \\
\]
which is true by Jensen’s inequality, because if \( f \) is a convex function of \( x \), \( f(x + v) + f(x - v) > 2f(x) \). \( f(\phi_H) = \phi_H \log \phi_H \) is a convex function of \( \phi_H \) as \( \frac{d^2}{d\phi_H^2} (\phi_H \log \phi_H) = \frac{1}{\phi_H} > 0 \). Therefore, Condition A1 is true.

From Equation (E2a.1),
\[
E[\Pi_S(p_S)] = p_S [1 - \frac{p_s}{\phi_H \phi_D} + \frac{p_s}{\phi_H \phi_D} \log \left\{ \frac{p_s}{\phi_H \phi_D} \right\} + K] \\
> p_S [1 - \frac{p_s}{\phi_H \phi_D} + \frac{p_s}{\phi_H \phi_D} \log \left\{ \frac{p_s}{\phi_H \phi_D} \right\}] \quad (E2a.2) \\
\]
Let \( p_S \) be the price that maximized the RHS in equation (E2a.2). \( p_S = 0.2858\phi_H D \). Substituting this back in (E2a.2) gives
\[
E[\Pi_S(p_S)] > 0.1018\phi_H D \\
\]
We have already proved that \( E[\Pi_O(p_0)] = \frac{1}{8} \frac{\phi_H^2}{\phi_D} D \). To find the cut-off transaction cost \( T_C \) at which the profits from selling and pay-per-use are equal, note that
\[
\frac{1}{8} \frac{\phi_H^2}{\phi_D} D > 0.1018\phi_H D \\
\]
Therefore, \( T_C < 0.0976\phi_H \), which is lower than the cutoff transaction cost in the case of no uncertainty \( (T_C = 0.0976 \phi_H) \).

**Electronic Appendix 3C**

Both utility-per-use and usage frequency are uncertain

In this case, we assume that the utility-per-use can be uniform either between 0 and \( \phi_H + v_\phi \) or uniform between 0 and \( \phi_H - v_\phi \), with a probability of 0.5 assigned to each of these distributions. We also assume that usage frequency can be uniform either between 0 and \( \theta_H + v_\theta \) or uniform between 0 and \( \theta_H - v_\theta \), with a probability of 0.5 assigned to each of these distributions. This case cannot be solved in closed form. We numerically simulate and compare profits under the selling and pay-per-use mechanisms for different values of \( v_\phi \) and \( v_\theta \) (we assume \( \phi_H = \theta_H = 1 \)), and . The regions of preference of the selling and pay-per-use mechanisms are described in Figure 12.

**Electronic Appendix 4**

Proof of Result 1:

Table T1 below summarizes the possible strategies used by both parties in equilibrium.
Figure 12: Regions of preference (compared to certainty case) of selling and pay-per-use mechanisms when both are uncertain

<table>
<thead>
<tr>
<th>Firm</th>
<th>Selling</th>
<th>Pay-per-use</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1</td>
<td>No equilibrium</td>
<td>Equilibrium exists</td>
<td>No equilibrium</td>
</tr>
<tr>
<td>Pay-per-use</td>
<td>Equilibrium exists</td>
<td>No equilibrium</td>
<td>No equilibrium</td>
</tr>
<tr>
<td>Both</td>
<td>No equilibrium</td>
<td>No equilibrium</td>
<td>No equilibrium</td>
</tr>
</tbody>
</table>

Table T1: Nash equilibrium outcomes when firms can use either or both mechanisms

**Case 1: Firm 1 uses only selling, Firm 2 uses only selling**

If the firms each use the selling mechanism only, let the selling price charged by firm 1 be denoted by \( p_{S1} \) and the selling price of firm 2 be denoted by \( p_{S2} \). Let the profits of firm 1 be denoted by \( \Pi_{S1}(p_{S1}, p_{S2}) \) and the profits of firm 2 be denoted by \( \Pi_{S2}(p_{S1}, p_{S2}) \). A necessary condition for the Nash equilibrium (with non-zero profits for both firms) to exist is (here \( p_{S1}^* \) and \( p_{S2}^* \) are the Nash equilibrium prices):

\[
\begin{align*}
\Pi_{S1}(p_{S1}^*, p_{S2}) &> \Pi_{S1}(p_{S1}^* + \epsilon, p_{S2}^*) \quad \text{and} \quad \Pi_{S1}(p_{S1}^*, p_{S2}^*) > \Pi_{S1}(p_{S1}^* - \epsilon, p_{S2}^*) \\
\Pi_{S2}(p_{S1}^*, p_{S2}) &> \Pi_{S2}(p_{S1}, p_{S2} + \epsilon) \quad \text{and} \quad \Pi_{S2}(p_{S1}^*, p_{S2}^*) > \Pi_{S2}(p_{S1}, p_{S2} - \epsilon)
\end{align*}
\]

That is, both firms should have no incentive to deviate from the Nash equilibrium prices.

The net surpluses of a consumer who buys the information from firm 1 and firm 2 are, respectively:

\[
\begin{align*}
U_{iS1}(p_{S1}) &= \theta_i \phi_i D - p_{S1} \\
U_{iS2}(p_{S2}) &= \theta_i \phi_i D - p_{S2}
\end{align*}
\]

If the information good is not vertically differentiated, it is easy to see that if \( p_{S1} \neq p_{S2} \), the entire market share goes to the firm with the lower selling price, and the other firm is left with zero profits. This leads to price undercutting, which can be formally proven as follows.

Without loss of generality, assume that the selling price duo \((p_{S1}^*, p_{S2}^*)\) is a Nash equilibrium and
let \( p^*_S < p^*_2 \), where \( p^*_S > 0 \) and \( p^*_2 > 0 \). Then,
\[
\Pi_1(p^*_S, p^*_2) = p^*_S [1 - \frac{p^*_S}{\theta_H \phi_H D} + \frac{p^*_S}{\theta_H \phi_H D} \log(\frac{p^*_S}{\theta_H \phi_H D})]
\]
\[
\Pi_2(p^*_S, p^*_2) = 0
\]
However, for firm 2, \( \Pi_2(p^*_S, p^*_2) > \Pi_2(p^*_S, p^*_2 - \varepsilon) \) is not satisfied, because if \( \varepsilon = \lim p^*_S - p^*_S - h \), then:
\[
\Pi_2(p^*_S, p^*_2 - \varepsilon) = \lim_{h \to 0} \Pi_2(p^*_S - h) [1 - \frac{p^*_S - h}{\theta_H \phi_H D} + \frac{p^*_S - h}{\theta_H \phi_H D} \log(\frac{p^*_S - h}{\theta_H \phi_H D})] > \Pi_2(p^*_S, p^*_2) = 0 \text{ for all } p^*_S > 0.
\]
Therefore, there is a contradiction, and a non-zero price Nash equilibrium cannot exist if \( p^*_S \neq p^*_2 \). Since profits are zero if the prices are zero, a non-zero price Nash equilibrium cannot exist if \( p^*_S \neq p^*_2 \).

Now, let us assume that \((p^*_S, p^*_2)\) is a Nash equilibrium and let \( p^*_S = p^*_2 = p^*_S \), where \( p^*_S = p^*_2 > 0 \). Because the net consumer surplus for both firms is equal, we can assume that the market shares for both firms are equal (half of that of the monopolist who prices at \( p^*_S \)). Then:
\[
\Pi_1(p^*_S, p^*_2) = \frac{p^*_S}{2} [1 - \frac{p^*_S}{\theta_H \phi_H D} + \frac{p^*_S}{\theta_H \phi_H D} \log(\frac{p^*_S}{\theta_H \phi_H D})]
\]
\[
\Pi_2(p^*_S, p^*_2) = \frac{p^*_S}{2} [1 - \frac{p^*_S}{\theta_H \phi_H D} + \frac{p^*_S}{\theta_H \phi_H D} \log(\frac{p^*_S}{\theta_H \phi_H D})]
\]
In this case, the conditions \( \Pi_1(p^*_S, p^*_2) > \Pi_1(p^*_S - \varepsilon, p^*_2) \) and \( \Pi_2(p^*_S, p^*_2) > \Pi_2(p^*_S - \varepsilon, p^*_2) \) are not satisfied for all \( p^*_S > 0 \). If \( \varepsilon = \lim h \), for firm 1, then:
\[
\Pi_1(p^*_S - h, p^*_2) = \lim_{h \to 0} (p^*_S - h) [1 - \frac{p^*_S - h}{\theta_H \phi_H D} + \frac{p^*_S - h}{\theta_H \phi_H D} \log(\frac{p^*_S - h}{\theta_H \phi_H D})]
\]
\[
> \frac{p^*_S}{2} [1 - \frac{p^*_S}{\theta_H \phi_H D} + \frac{p^*_S}{\theta_H \phi_H D} \log(\frac{p^*_S}{\theta_H \phi_H D})].
\]
A symmetric condition holds for firm 2. Therefore, there is a contradiction and Nash equilibrium with non-zero prices and profits exists when each firm employs only selling.

**Case 2:** Firm 1 uses only the pay-per-use mechanism and Firm 2 uses only the pay-per-use mechanism only

Similar to case 1, each firm can undercut the (positive) payment-per-use of other to obtain the full market share, and therefore, there is no equilibrium where either firm charges a positive payment-per-use and makes a positive profit in this case. The detailed proof is very similar to Case 1 and can be obtained from the authors.

**Case 3:** Firm 1 uses the selling mechanism only, Firm 2 uses the pay-per-use mechanism only

This case is analyzed in Electronic Appendix 6 (see below), and it yields an equilibrium with a positive payment-per-use and a positive selling price, and positive profits for each firm. The case with firm 1 using pay-per-use only and firm 2 using selling only is symmetric.
Case 4: Firm 1 uses the selling and pay-per-use mechanisms together, Firm 2 uses the pay-per-use mechanism only

Let the payment-per-use charged by firm 1 be denoted by \( p_{O1} \), its selling price by \( p_{S1} \) and the payment-per-use of firm 2 by \( p_{O2} \). Let the profits of firm 1 be denoted by \( \Pi_{B1}(p_{S1}, p_{O1}, p_{O2}) \) and the profits of firm 2 be denoted by \( \Pi_{O2}(p_{S1}, p_{O1}, p_{O2}) \). The following conditions should hold for a Nash equilibrium (with non-zero profits to each firm) to exist (here, \( p_{S1}^*, p_{O1}^* \) and \( p_{O2}^* \) are the Nash equilibrium prices):

\[
\Pi_{B1}(p_{S1}^*, p_{O1}^*, p_{O2}^*) > \Pi_{B1}(p_{S1}^*, p_{O1}^* + \epsilon, p_{O2}^*) \quad \text{and} \quad \Pi_{B1}(p_{S1}^*, p_{O1}^*, p_{O2}^*) > \Pi_{B1}(p_{S1}^*, p_{O1}^* - \epsilon, p_{O2}^*)
\]
\[
\Pi_{O2}(p_{S1}^*, p_{O1}^*, p_{O2}^*) > \Pi_{O2}(p_{S1}^*, p_{O1}^* + \epsilon, p_{O2}^*) \quad \text{and} \quad \Pi_{O2}(p_{S1}^*, p_{O1}^*, p_{O2}^*) > \Pi_{O2}(p_{S1}^*, p_{O1}^* - \epsilon, p_{O2}^*)
\]

That is, both firms should have no incentive to deviate from the Nash equilibrium prices. The net consumer surpluses derived from buying from firm 1 or from adopting the pay-per-use mechanism from firms 1 and 2 are, respectively:

\[
U_{iS1}(p_{S1}) = \theta_i\phi_i D - p_{S1}
\]
\[
U_{iO1}(p_{O1}) = \theta_i D[\phi_i - (p_{O1} + T)]
\]
\[
U_{iO1}(p_{O2}) = \theta_i D[\phi_i - (p_{O2} + T)]
\]

Similar to the previous case, we can show that if \( p_{O1}^* \neq p_{O2}^* \), then the firm with the higher payment-per-use obtains zero market share and can simply discard that pricing mechanism. If that firm is firm 2, then firm 2 obtains zero profits overall. Therefore, firm 2 has the incentive to reduce its payment-per-use to a level below that of firm 1. Therefore, no equilibrium is possible with \( p_{O1}^* \neq p_{O2}^* \), if firm 2 charges the lower payment-per-use. On the other hand, if the firm that with the higher payment-per use is firm 1, than that mechanism obtains zero market share for firm 1.

In this case, that mechanism can be discarded by firm 1, and we arrive at a situation where firm 1 adopts only selling and firm 2 adopts only pay-per-use (which does constitute an equilibrium).

Now, let us assume that \( (p_{S1}^*, p_{O1}^*, p_{O2}^*) \) is a Nash equilibrium and let \( p_{O1}^* = p_{O2}^* = p_{O}^* \), where \( p_{O1}^* = p_{O2}^* > 0 \) and \( p_{S1}^* > 0 \). Because the net consumer surplus for both firms from the pay-per-use mechanism is equal, we can assume that the market share from the pay-per-use mechanism for both firms is equal. Then,

\[
\Pi_{B1}(p_{S1}^*, p_{O1}^*, p_{O2}^*) = p_{S1}^*[1 - \frac{p_{S1}^*}{\mu + (p_{O}^* + T)}] + \frac{p_{S1}^*}{\mu} \log\left(\frac{p_{S1}^*}{\mu (p_{O}^* + T)}\right) - \frac{1}{2}\frac{p_{S1}^*}{\mu} [\phi_H - (p_{O}^* + T)][(\frac{p_{S1}^*}{\mu (p_{O}^* + T)})^2 - \frac{1}{2}]
\]
\[
\Pi_{O2}(p_{S1}^*, p_{O1}^*, p_{O2}^*) = \frac{1}{2}\frac{p_{S1}^*}{\mu} [\phi_H - (p_{O}^* + T)][(\frac{p_{S1}^*}{\mu (p_{O}^* + T)})^2 - \frac{1}{2}]
\]

In this case, the conditions \( \Pi_{B1}(p_{S1}^*, p_{O1}^*, p_{O2}^*) > \Pi_{B1}(p_{S1}^*, p_{O1}^* - \epsilon, p_{O2}^*) \) and \( \Pi_{O2}(p_{S1}^*, p_{O1}^*, p_{O2}^*) > \Pi_{O2}(p_{S1}^*, p_{O1}^* - \epsilon, p_{O2}^*) \) are not satisfied for all \( p_{O}^* > 0 \). If \( \epsilon = \lim_{h \to 0} h \), for firm 1,

\[
\Pi_{B1}(p_{S1}^*, p_{O1}^* - \lim_{h \to 0} h, p_{O2}^*) =
\]
to prove that payment-per-use charged by a monopolist who employs only the pay-per-use mechanism, we need to prove that the cells in Table T1 are proved similarly. and the proofs are available with the authors.

A symmetric argument holds for firm 2 as well. Intuitively, with equal and positive payments-per-use across the firms, one firm can always lower its payment-per-use slightly and capture all the market share associated with the pay-per-use mechanism of the competitor. Therefore, there is a contradiction and no Nash equilibrium with positive profits for each firm when the pay-per-use mechanism is offered by both firms. The no-equilibrium outcomes for the other cases (corresponding to the cells in Table T1) are proved similarly. and the proofs are available with the authors. ■

**ELECTRONIC APPENDIX 5: Proof of price and payment-per-use in a duopoly**

Consider a duopoly where one firm offers pay-per-use and the competitor sells the information good. From Figure 2, the expressions for the market shares of the selling and pay-per-use mechanisms are, respectively:

$$MS_S(p_S, p_O) = \frac{1}{\theta_H} \left[ \int_{p_S}^{p_O} (\phi_H - \frac{p_S}{D}) d\theta - \int_{p_S}^{p_O+T} (\phi_H - \frac{p_S}{D}) d\theta \right]$$

$$= \frac{1}{\theta_H} [\theta_H \phi_H - \frac{p_S \phi_H}{(p_O+T)D} + \frac{p_S}{D} \log \{\frac{p_S}{\theta_H (p_O+T)D}\}]$$

$$MS_O(p_S, p_O) = \frac{1}{\theta_H} [\phi_H - (p_O + T)] \frac{p_S}{(p_O+T)D}$$

The competing firms maximize their individual profits:

**Seller:** $Max \ \Pi_S(p_S, p_O) = \frac{1}{\theta_H} [\theta_H \phi_H - \frac{p_S \phi_H}{(p_O+T)D} + \frac{p_S}{D} \log \{\frac{p_S}{\theta_H (p_O+T)D}\}] p_S$

**Pay-per-use:** $Max \ \Pi_O(p_S, p_O) = \frac{1}{\theta_H} [\phi_H - (p_O + T)] \frac{p_S}{(p_O+T)D}^2 p_O D$

The profits of the firm that offers pay-per-use pricing are:

$$\Pi_O = \frac{1}{\theta_H} [\phi_H - (p_O + T)] \frac{p_S}{(p_O+T)D}^2 p_O D$$

The first order condition with respect to $p_O$ yields:

$$(p_O + T)^2 [\phi_H - 2p_O - T] = 2p_O [\phi_H - (p_O + T)] (p_O + T)$$

which on simplification yields $p_O = \frac{T(\phi_H - T)}{\phi_H}$.

Next, the seller’s profits in the duopoly are:

$$\Pi_S = \frac{1}{\theta_H} [\theta_H \phi_H - \frac{p_S \phi_H}{(p_O+T)D} + \frac{p_S}{D} \log \{\frac{p_S}{\theta_H (p_O+T)D}\}] p_S$$

The FOC for the seller with respect to $p_S$ yields:

$$\theta_H \phi_H - \frac{2p_S \phi_H}{(p_O+T)D} + \frac{p_S}{D} \log \{\frac{p_S}{\theta_H (p_O+T)D}\} + \frac{p_S}{D} [1 + \log \{\frac{p_S}{\theta_H (p_O+T)D}\}] = 0$$

which on simplification yields the implicit equation:

$$\theta_H \phi_H - \frac{2p_S \phi_H}{(p_O+T)D} + \frac{2p_S}{D} \log \{\frac{p_S}{\theta_H (p_O+T)D}\} + \frac{p_S}{D} = 0$$

**ELECTRONIC APPENDIX 6: Proof of Proposition 3**

(i) To prove that the equilibrium payment-per-use in the duopoly is lower than the optimal payment-per-use charged by a monopolist who employs only the pay-per-use mechanism, we need to prove that $T(\phi_H - T) \leq \frac{\phi_H - T}{2}$, which simplifies to $\phi_H \geq T$. This is true by assumption.
(ii) Next, consider the selling price. We have to show that the selling price in a duopoly (referred to in this proof as \(p_{DS}\)) as characterized by the implicit equation \(\theta_H \phi_H - \frac{2 p_{DS} \phi_H}{(p_{DO} + T) \phi_H D} + \frac{2 p_{DS}}{D} \log\left(\frac{p_{DS}}{\eta_H (p_{DO} + T) D}\right) + \frac{p_{DS}}{D} = 0\) (where \(p_{DO} = \frac{T (\phi_H - T)}{\phi_H + T}\)) is lower than the selling price in a monopoly (referred to in this proof as \(p_{PS}\)) as characterized by the implicit equation \(\theta_H \phi_H + \frac{2 p_{PS}}{D} \log p_{PS} - \frac{p_{PS}}{D} [2 \log(D \phi_H \theta_H) + 1] = 0\) (where \(p_{PS} = 0.285 \theta_H \phi_H D\)).

We rewrite the implicit equations for the selling price in the duopoly and monopoly cases as, respectively:

\[
\theta_H \phi_H + \frac{2 p_{DS}}{D} \log p_{DS} - \frac{p_{DS}}{D} [2 \log(D \theta_H) + 2 \log(p_{DO} + T) + \frac{2 \phi_H}{p_{DO} + T} - 1] = 0
\]

\[
\theta_H \phi_H + \frac{2 p_{PS}}{D} \log p_{PS} - \frac{p_{PS}}{D} [2 \log(D \phi_H \theta_H) + 1] = 0
\]

Comparing the two implicit equations, we note that if the part of the third term multiplying \(\frac{p_{DS}}{D}\) in the first equation is greater than the one multiplying \(\frac{p_{PS}}{D}\) in the second equation, then the corresponding \(p_{DS}\) must be smaller for the equations to both equal zero. Therefore, to show that \(p_{DS} < p_{PS}\), we need to show that:

\[
2 \log(D \theta_H) + 2 \log(p_{DO} + T) + \frac{2 \phi_H}{p_{DO} + T} - 1 > 2 \log(D \phi_H \theta_H) + 1
\]

which reduces to showing that \(\frac{2 \phi_H}{p_{DO} + T} - 1 > 2 \log \frac{\phi_H}{p_{DO} + T} + 1\)

which reduces to showing that \(\frac{\phi_H}{p_{DO} + T} > \log \frac{\phi_H}{p_{DO} + T} + 1\).

Now this is always true because, if \(x > 1\), then \(x > 1 + \log(x)\), as evident from the power series expansion of \(\log(x)\).

(iii) Finally, one can directly see from the expression \(p_O = \frac{T (\phi_H - T)}{\phi_H + T}\) that the equilibrium payment-per-use first increases and then decreases with \(T\).

**ELECTRONIC APPENDIX 7: Proof of Proposition 4**

To prove that the seller’s profits in the duopoly are always greater than those of the firm that offers pay-per-use pricing, we have to prove that (using expressions from above):

\[
\Pi_{DS} = \frac{1}{\phi_H D} \left[ \theta_H \phi_H - \frac{2 p_{DS} \phi_H}{(p_{DO} + T) \phi_H D} + \frac{p_{DS}}{D} \log\left(\frac{p_{DS}}{\eta_H (p_{DO} + T) D}\right) \right] p_{DS} > \Pi_{DO} = \frac{1}{2 \phi_H D} \left[ \phi_H - (p_{DO} + T) \right] \frac{p_{DS}}{p_{DO} + T} D
\]

Note that for the selling price in a duopoly, the FOC is given by:

\[
\theta_H \phi_H - \frac{p_{DS} \phi_H}{(p_{DO} + T) D} + \frac{p_{DS}}{D} \log\left(\frac{p_{DS}}{\eta_H (p_{DO} + T) D}\right) = \frac{p_{DS} \phi_H}{(p_{DO} + T) D} - \frac{p_{DS}}{D} \log\left(\frac{p_{DS}}{\eta_H (p_{DO} + T) D}\right) - \frac{p_{DS}}{D}
\]

Substituting the right hand side into the expression for \(\Pi_{DS}\), we have to prove that:

\[
p_{DS} \left[ \frac{p_{DS} \phi_H}{r D} - \frac{p_{DS}}{D} \log\left(\frac{p_{DS}}{\eta_H D}\right) - \frac{p_{DS}}{D} \right] > \frac{1}{2} (r - T) \left( \frac{\phi_H - r}{r^2} \right)^2 \left( \frac{T \phi_H}{\phi_H + T} \right).
\]

This reduces to proving that \(\phi_H > \frac{r}{r - T}\) and \(r = p_{DO} + T = \frac{2 T \phi_H}{\phi_H + T}\).

This reduces to proving that \(\phi_H > \frac{r}{r - T}\) and \(r = p_{DO} + T = \frac{2 T \phi_H}{\phi_H + T}\). After simplification, this reduces to proving that \((\phi_H - r - T) r + \phi_H T - 2 r^2 \log\left(\frac{p_{DS}}{\eta_H D}\right) > 0\) or \((\phi_H - r) (r + T) > 2 r^2 \log\left(\frac{p_{DS}}{\eta_H D}\right)\).

Note that \(\phi_H > r\) because \(r = \frac{2 T \phi_H}{\phi_H + T}\), and by assumption, \(T < \phi_H\). The term \((r + T) > \frac{2 \phi_H}{\phi_H + T}\) is positive. Therefore, the LHS is positive. Now, \(\frac{p_{DS}}{D} = \frac{p_{DS}}{(p_{DO} + T) D}\) is the cut-off frequency under which
pay-per-use yields the same utility surplus as selling (see Section 3.2 in the paper). Therefore, $\frac{p_{DS}}{r_D} < \theta_H$ and $\frac{p_{DS}}{\theta_H r_D} < 1$. The natural logarithm of a number less than 1 is always negative – therefore, the RHS is negative. Therefore, $(\phi_H - r)(r + T) > 2r^2 \log \left( \frac{p_{DS}}{\theta_H r_D} \right)$ always holds true.

**ELECTRONIC APPENDIX 8:**

**Social Welfare:**

As proved above in Electronic Appendix 6, the payment-per-use in the duopoly is $p_{DS} = \frac{T(\phi_H - T)}{\phi_H + T}$.

The total net consumer surplus from using the pay-per-use is:

$$\frac{1}{\phi_H (p_{DS} + T)^2} \int_{p_{DS} - T}^{p_{DS} + T} \phi d\phi \log \left( \frac{\phi}{\phi_H} \right) d\phi$$

Here, the maximum frequency of the consumers using the pay-per-use mechanism is given by

$$\theta_H = \frac{\phi_{DS} - \phi}{\phi_{DS} - 1}$$

Therefore, the net consumer surplus = \(\frac{\phi_{DS} (\phi_{DS} - 1)^2}{4(\phi_{DS} + T)^2} \cdot \frac{p_{DS}}{(p_{DS} + T)^2}\).

The total social welfare is the sum of the profits of the firm adopting pay-per-use and the net consumer surplus from that mechanism.

For selling, we integrate the cumulative surplus of all consumers who buy the information good (consumers with $\theta_i \phi_i D > p_S \theta_H \phi_H$ and $\theta_i > \frac{p_{DS}}{(p_{DS} + T)^2}$). We first find the cumulative surplus for consumers with a given usage frequency ($\theta$) and then integrate across $\theta$ to find the cumulative net surplus. The total social welfare is the sum of the cumulative surplus and the profits of the seller. For consumers with a given $\theta$, the net social welfare is $\int_{p_{DS} - T}^{p_{DS} + T} \phi d\phi \log \left( \frac{\phi}{\phi_H} \right) d\phi$, where $\phi_{DS} = k\theta_H \phi_H D$ satisfies $\phi_{DS} - \frac{2p_{DS} \phi_H}{\phi_H (p_{DS} + T)^2} + \frac{2p_{DS}}{D} \log \left( \frac{p_{DS}}{\phi_H (p_{DS} + T)^2} \right) = 0$, the implicit solution for the selling price in a duopoly. The frequency of usage varies from $k\theta_H$ (when $\phi = \phi_H$) to $\theta_H$.

Hence, net consumer surplus from selling = \(\frac{1}{\phi_H (p_{DS} + T)^2} \int_{p_{DS} - T}^{p_{DS} + T} \phi d\phi \log \left( \frac{\phi}{\phi_H} \right) d\phi - \int_{p_{DS} - T}^{p_{DS} + T} \phi d\phi \log \left( \frac{\phi}{\phi_H} \right) d\phi\).\]

The net social welfare, which comprises the sum of the consumer surplus and profits from selling, is plotted in Figure 13.

**ELECTRONIC APPENDIX 9:**

**Proof of Proposition 5:** We first derive the optimal payment-per-use, prices, and profits when the basic information good is introduced at the outset and an upgrade is introduced in period $k$.

**Pay-per-use pricing:** When the information good is first introduced, all consumers with $\phi_i > p_{O1} + T$ use the good, where $p_{O1}$ denotes the payment-per-period for the basic (non-upgraded) good. Therefore, the market fraction initially using the good is $\frac{\phi_{H} - (p_{O1} + T)}{\phi_H}$, the average usage frequency for consumers in this fraction in any period is $\frac{\phi_{H}}{2}$. Between the $k^{th}$ and $N^{th}$ periods (i.e., post-upgrade), if the firm charges a per-period payment of $p_{O2}$, then the market fraction using
the good on demand is \( \frac{\alpha \phi_H - (p_{O2} + T)}{\alpha \phi_H} \). The average usage frequency for consumers in this fraction in any period is again \( \frac{\theta_H}{2} \). Accordingly, the firm’s profits are \( \Pi_{O1} = \frac{\phi_H - (p_{O1} + T)}{\phi_H} p_{O1} D_{k-1} \) for the first \( k - 1 \) periods. For the periods between and including the \( k^{th} \) and \( N^{th} \) periods, its profits are \( \Pi_{O2} = \frac{\phi_H - (p_{O2} + T)}{\phi_H} \frac{\theta_H}{2} p_{O2}(D - D_{k-1}) \). It can be verified that the total profits (\( \Pi_O = \Pi_{O1} + \Pi_{O2} \)) are quadratic with respect to both \( p_{O1} \) and \( p_{O2} \). Accordingly, the FOCs with respect to each of these yield the optimal per-period payments: \( p_{O1} = \frac{\phi_H - T}{2} \) and \( p_{O2} = \frac{\phi_H - T}{2} \). Substituting them back into the profit function yields the total profits denoted by \( \Pi_O = \frac{\theta_H}{8} \left[ \frac{(\phi_H - T)^2}{\phi_H} D_{k-1} + \frac{(\phi_H - T)^2}{\phi_H} (D - D_{k-1}) \right] \).

**Selling**: This pertains to the situation where the monopolist sells the information good and consumers are myopic. Denote the selling price of the basic good by \( p_S \) and let \( p_U \) be the price of the upgrade introduced in period \( k \). There are two classes of consumers here. The first class of consumers purchase the basic information good at the outset. The second class of consumers, which is a subset of the first class, purchase the upgrade in period \( k \). Their surpluses are, respectively, \( U_1(S) = \theta_i \phi_i D - p_S \) and \( U_2(S) = \theta_i (a - 1) \phi_i (D - D_{k-1}) - \delta^{k-1} p_U \). For the basic information good in the first period, all consumers with usage probability in the range \( \theta \in [\frac{p_S}{\phi_H}, \theta_H] \) derive (weakly) positive utility from their purchase. Similar to the scenario without the upgrade, the fraction of the market purchasing the basic good and the resulting profits are, respectively:

\[
MS_{S1} = \frac{1}{\phi_H \theta_H} \int_{\phi_H \theta_H}^{\theta_H} \frac{p_S}{\phi_H D} \left[ \theta_H - \frac{p_S}{\phi_H D} \right] d\phi = \frac{1}{\phi_H \theta_H} \left[ \theta_H \phi_H - \frac{p_S}{\phi_H D} + \frac{p_S}{\phi_H D} \log \left( \frac{p_S}{\phi_H \theta_H D} \right) \right]
\]

\[
\Pi_{S1} = MS_{S1} p_S = \frac{p_S}{\phi_H \theta_H} \left[ \theta_H \phi_H - \frac{p_S}{\phi_H D} + \frac{p_S}{\phi_H D} \log \left( \frac{p_S}{\phi_H \theta_H D} \right) \right].
\]
For the upgrade in the $k^{th}$ period, all consumers with usage probability in the range $\theta \in \left(\frac{\delta^{k-1}p_U}{(a-1)\phi_H(D-D_{k-1})}, \theta_H\right)$ derive (weakly) positive utility from their purchase. The fraction of the market purchasing and the resulting profits for the upgrade are, respectively:

$$MS_{S_2} = \frac{1}{\phi_H \theta_H} \int_{\phi_H}^{\theta_H} \frac{\theta_H - \frac{\delta^{k-1}p_U}{(a-1)\phi_H(D-D_{k-1})}}{\theta_H - \frac{\delta^{k-1}p_U}{(a-1)\phi_H(D-D_{k-1})}} d\phi = \frac{1}{\phi_H \theta_H} \left[ \theta_H \phi_H - \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})} + \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})} \log\left( \frac{\theta_H \phi_H - \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})}}{\theta_H \phi_H - \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})}} \right) \right]$$

$$\Pi_{S_2} = MS_{S_2} \delta^{k-1}p_U = \frac{\delta^{k-1}p_U}{\phi_H} \left[ \theta_H \phi_H - \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})} + \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})} \log\left( \frac{\theta_H \phi_H - \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})}}{\theta_H \phi_H - \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})}} \right) \right].$$

Note that if the firm uses the selling mechanism only, its profits from selling the base information good are: $\Pi_{S_1} = MS_{S_1}p_S = \frac{p_S}{\phi_H} \theta_H \phi_H - \frac{p_S}{\phi_H} \frac{D}{D_{k-1}} + \frac{p_S}{\phi_H} \log\left( \frac{p_S}{\phi_H D} \right)$. From Proposition 4, these profits are optimized if $p_S = 0.285 \theta_H \phi_H D$ and the firm’s profits from the basic information good are: $\Pi_{S_1} = 0.1018 \theta_H \phi_H D$. From the upgrade, the firm’s profits are: $\Pi_{S_2} = MS_{S_2} \delta^{k-1}p_U = \frac{\delta^{k-1}p_U}{\phi_H} \theta_H \phi_H - \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})} + \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})} \log\left( \frac{\theta_H \phi_H - \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})}}{\theta_H \phi_H - \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})}} \right)$. The FOC with respect to $p_U$ yields:

$$\frac{\theta_H \phi_H (a-1)(D-D_{k-1})}{\phi_H} \log\left( \frac{\theta_H \phi_H - \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})}}{\theta_H \phi_H - \frac{\delta^{k-1}p_U}{(a-1)(D-D_{k-1})}} \right) = \frac{\delta^{k-1}p_U}{2(k-1)(D-D_{k-1})} + 1 = 0.$$

The solution to this implicit equation is: $\delta^{k-1}p_U = 0.285 \theta_H \phi_H (a-1)(D-D_{k-1})$. This yields the same market share for the upgrade as the basic information good, implying that all consumers who purchase the basic good also purchase the upgrade. Substituting this back into the profit function yields: $\Pi_{S_2} = 0.1018 \theta_H \phi_H (a-1)(D-D_{k-1})$. The total profits of the seller from the base information good and the upgrade are $\Pi_S = \Pi_{S_1} + \Pi_{S_2} = 0.1018 \theta_H \phi_H (D + (a-1)(D-D_{k-1}))$.

Now, if the firm offers an upgrade, then profits under pay-per-use pricing are $\Pi_O = \frac{\phi_H T}{\phi_H} \left[ \phi_H T - D_{k-1} + \frac{\phi_H T - D_{k-1}}{\phi_H} \right] (D-D_{k-1})$ and, as demonstrated above, those under selling are $\Pi_S = 0.1018 \theta_H \phi_H [D + (a-1)(D-D_{k-1})]$. To find the cut-off transaction cost in the scenario with upgrades, we equate these profits:

$$\frac{\phi_H T}{\phi_H} \left[ \phi_H T - D_{k-1} + \frac{\phi_H T - D_{k-1}}{\phi_H} \right] (D-D_{k-1}) = 0.1018 \theta_H \phi_H [D + (a-1)(D-D_{k-1})]$$

Therefore, $(1 - \frac{T_C}{\phi_H})^2 D_{k-1} + (a - \frac{T_C}{\phi_H})^2 \frac{D-D_{k-1}}{a} = 0.1018 \times 8 \left[ D + (a-1)(D-D_{k-1}) \right] + (a - \frac{T_C}{\phi_H})^2 \frac{D-D_{k-1}}{a}$. If the first term is set equal to the RHS, it yields $T_C = 0.0976 \phi_H$, because the solution to $(1 - \frac{T_C}{\phi_H})^2 D = 0.1018 \times 8 \times 8$ is $T_C = 0.0976 \phi_H$. We will first show that the sum of the other two terms on the LHS is positive. To do this, we need to show that:

$$(a - \frac{T_C}{\phi_H})^2 \frac{D-D_{k-1}}{a} > (1 - \frac{T_C}{\phi_H})^2 [D-D_{k-1} + (a-1)(D-D_{k-1})]$$

or to show that $(a - \frac{T_C}{\phi_H})^2 > a^2 (1 - \frac{T_C}{\phi_H})^2$ or to show that $a - \frac{T_C}{\phi_H} > a - \frac{aT_C}{\phi_H}$.

Now this is true by assumption because $a > 1$ for the upgrade. Now, given that the sum of the other two terms is positive, the first term on the LHS has to be lower so that the equality continues to hold. Therefore, in order to make the term $(1 - \frac{T_C}{\phi_H})^2$ smaller, the cut-off transaction cost $T_C$
below which pay-per-use is more profitable than selling is higher than it is in the absence of an upgrade.

**ELECTRONIC APPENDIX 10**

**Proof of Proposition 6:** If consumers are myopic, then when the firms introduce the base information good, as in the duopoly case in Section 3.2, the profits of the two firms from the base information good are:

\[
\Pi_S(p_S, p_O) = \frac{1}{2} \phi_H \theta_H \left[ \theta_H \phi_H - \frac{p_S \phi_H}{(p_O + T)D} + \frac{p_S}{D} \log \left( \frac{p_S}{\theta_H (p_O + T)D} \right) \right] p_S
\]

\[
\Pi_O(p_S, p_O) = \frac{1}{2} \phi_H \theta_H \left[ \phi_H - (p_O + T) \left( \frac{p_S}{(p_O + T)D} \right)^2 p_O D \right]
\]

From Proposition 3, we know that the firm that adopts the selling mechanism has higher profits from the base information good than the firm that adopts the pay-per-use mechanism. When the two firms introduce the upgrade, they no longer compete with each other for the upgrade because consumers have chosen either the pay-per-use mechanism or the selling mechanism for the base good (under certain conditions that prevent consumers from switching to the other mechanism, which can be seen to be easily satisfied for the upgrade to yield positive utility to either set of consumers). Because the market share of the selling mechanism for the base information good is higher under competition, and because selling has already captured the customers whose utility-per-use and usage frequency are jointly high, the firm that sells can price the upgrade to extract a higher profit than the firm that adopts pay-per-use. Therefore, the presence of the upgrade benefits the seller to a greater extent in a duopoly.

**ELECTRONIC APPENDIX 11:**

**Market shares and profits under competition with vertical differentiation**

We focus on deriving analytical expressions for the firms’ market shares and profits below. Equilibrium outcomes were then derived after numerically solving the first order conditions corresponding to these profit expressions.

**Case 1: Low transaction cost (Figure 6)**

When the transaction cost is low \((0 < T < 0.11)\), the equilibrium division of market shares is displayed in Figure 6 in the paper. On the lower side, the market share of the selling mechanism for firm 1 is obtained by comparing the net consumer surpluses from buying that good from firm 1 and buying the good offered by firm 2. Consumers buy the good from firm 1 iff \(U_{S1}(p_{S1}) = \phi_i \theta_i D - p_{S1} > U_{S2}(p_{S2}) = \lambda \theta_i \phi_i D - p_{S2} \Leftrightarrow \phi_i \theta_i > \frac{p_{S1} - p_{S2}}{(1 - \lambda)D}. \) On the left, the market share of the selling mechanism employed by firm 1 is characterized by \(\theta_i > \frac{p_{S1}}{(p_O + T)D}. \) Therefore, the market share of that good from selling is:
\[ MS_{S1} = \int_{\frac{p_{S1}}{(p_{O1}+T)D}}^{1} \left[ 1 - \frac{p_{S1}-p_{S2}}{p_{S1}(1-\lambda)} \right] d\theta = 1 - \frac{p_{S1}}{(p_{O1}+T)D} + \frac{p_{S1}-p_{S2}}{(1-\lambda)D} \log \left( \frac{p_{S1}}{(p_{O1}+T)D} \right) \]

To find the market share of the pay-per-use mechanism of firm 1, we note that the pay-per-use market share is to the left of the selling market share of firm 1 above and the selling market share of firm 2 below. The latter boundary is represented by \( U_{O1}(p_{O1}) = \theta_i(\phi_i - T - p_{O1})D = U_{S2}(p_{S2}) = \lambda \theta_i \phi_i D - p_{S2} \), which is characterized by \( \theta_i = \frac{p_{S1}-p_{S2}}{(p_{O1}+T)(1-\lambda)\phi_i}D \). On the right side (above), the market share is bounded by firm 1’s selling region, and this boundary is characterized by \( \theta_i = \frac{p_{S1}}{(p_{O1}+T)D} \). On the lower side, the market share of the pay-per-use mechanism of firm 1 is bounded by the market share of the pay-per-use mechanism of firm 2. This boundary is represented by \( U_{O1}(p_{O1}) = \theta_i(\phi_i - T - p_{O1})D = U_{O2}(p_{O2}) = \theta_i(\lambda \phi_i - T - p_{O2})D \), which is characterized by \( \phi_i = \frac{p_{O1}-p_{O2}}{(1-\lambda)} \). Integrating out the relevant area, the market share of the pay-per-use mechanism of firm 1 is:

\[ MS_{O1} = \left[ 1 - \frac{p_{S1}-p_{S2}}{p_{S1}(1-\lambda)} \right] \frac{p_{S1}}{(p_{O1}+T)D} + \int_{\frac{p_{S1}}{(p_{O1}+T)D}}^{\frac{p_{S1}}{(p_{O1}+T)D} + \frac{p_{S1}-p_{S2}}{(1-\lambda)}} \frac{p_{S2}}{(1-\lambda)D} \log \left( \frac{p_{S2}}{(p_{O2}+T)D} \right) d\phi \]

To find the market share of the pay-per-use mechanism of firm 2, we note that the pay-per-use market share is to the left of the selling market share of firm 2. The boundary between the two regions is represented by: \( U_{O2}(p_{O2}) = \theta_i(\lambda \phi_i - T - p_{O2})D = U_{S2}(p_{S2}) = \lambda \theta_i \phi_i D - p_{S2} \), which is characterized by \( \theta_i = \frac{p_{S2}}{(p_{O2}+T)D} \). On the lower side, the market share of the pay-per-use mechanism of firm 2 is bounded by zero, and this boundary is characterized by: \( \phi_i = \frac{p_{O2}+T}{(1-\lambda)} \). Therefore, the market share of the pay-per-use mechanism of firm 2 is:

\[ MS_{O2} = \left[ \frac{p_{O1}-p_{O2}}{(1-\lambda)} - \frac{p_{O2}+T}{(1-\lambda)} \right] \frac{p_{S2}}{(p_{O2}+T)D} \]

We next find the fraction of non-adopters. This fraction is represented by the area under the pay-per-use and selling regions of firm 2. The area under the fraction of \( \phi_i = \frac{p_{O2}+T}{(1-\lambda)} \) who buy information good from firm 2 is:

\[ \int_{\frac{p_{S2}}{\lambda D}}^{\frac{p_{S2}+T}{\lambda D}} \left[ 1 - \frac{p_{S2}}{\lambda D} \right] d\phi = \frac{p_{S2}+T}{\lambda D} - \frac{p_{S2}}{\lambda D} + \frac{p_{S2}}{\lambda D} \log \left( \frac{p_{S2}}{(p_{O2}+T)D} \right) \]

Therefore, the total fraction of non-adopters is:

\[ MS_{NA} = \frac{p_{O2}+T}{\lambda D} - \left[ \frac{p_{O2}+T}{\lambda D} - \frac{p_{S2}}{\lambda D} + \frac{p_{S2}}{\lambda D} \log \left( \frac{p_{S2}}{(p_{O2}+T)D} \right) \right] = \frac{p_{S2}}{\lambda D} - \frac{p_{S2}}{\lambda D} \log \left( \frac{p_{S2}}{(p_{O2}+T)D} \right) \]

The market share of the selling mechanism of firm 2 is: \( MS_{S2} = 1 - MS_{S1} - MS_{O1} - MS_{O2} - MS_{NA} \)

The profits from the pay-per-use mechanism for firm 1 equal the sum of the profits from two areas: the region bounded by the selling region of firm 1 to the right, and the region bounded by the selling region of firm 2 to the right. Denoting these profits by \( \Pi_{O1}(RS_1) \) and \( \Pi_{O1}(RS_2) \), we obtain:

\[ \Pi_{O1}(RS_1) = \left[ 1 - \frac{(p_{S1}-p_{S2})(p_{O1}+T)}{p_{S1}(1-\lambda)} \right] \frac{p_{S1}}{(p_{O1}+T)D}^2 p_{O1}D \]

Here \( \frac{p_{S2}}{(p_{O1}+T)(1-\lambda)\phi_i}D \) is the average frequency of usage.
The profits from the pay-per-use mechanism for firm 2 are:
\[ \Pi_{O2}(p_{S2}, p_{O2}) = \left[ \frac{p_{O1} - p_{O2}}{(1-\lambda)} - \frac{p_{O2} + T}{(1-\lambda)} \right] \frac{1}{2} \left( \frac{p_{S2}}{(p_{O2} + T)D} \right)^2 p_{O2} D \]

The total profits of the two firms are:
\[ \Pi_1(p_{S1}, p_{O1}) = p_{S1}MS_{S1} + \Pi_O(p_{S1}, p_{O1}) \]
\[ = p_{S1}MS_{S1} + p_{O1} \frac{p_{S2}^2}{2D(1-\lambda)} \left( \frac{1}{(p_{O1} + T)(1-\lambda)} - \frac{1}{p_{O2} + T} \right) \]
\[ \Pi_2(p_{S2}, p_{O2}) = p_{S2}MS_{S2} + \Pi_O(p_{S2}, p_{O2}) \]
\[ = p_{S2}MS_{S2} + \left[ \frac{p_{O1} - p_{O2}}{(1-\lambda)} - \frac{p_{O2} + T}{(1-\lambda)} \right] \frac{1}{2} \left( \frac{p_{S2}}{(p_{O2} + T)D} \right)^2 p_{O2} D \]

**Case 2: Very High transaction cost (T > 0.5)**

![Figure 14](image_url)  
Figure 14   Market shares of firm 1 and firm 2 with very high transaction cost

When the transaction cost is very high, each firm uses only the selling mechanism. The market share of firm 1 is bounded below by the market share of firm 2, and is characterized by: \( \phi_{i} \theta_{i} > \frac{p_{S1} - p_{S2}}{(1-\lambda)D} \). The market shares of firms 1 and 2 are plotted in Figure 14. Therefore, firm 1’s market share is:
\[ MS_{S1} = \int_{\phi_{i} \theta_{i} D}^{\frac{p_{S1} - p_{S2}}{(1-\lambda)D}} [1 - \frac{p_{S1} - p_{S2}}{(1-\lambda)D}] d\phi = 1 - \frac{p_{S1} - p_{S2}}{(1-\lambda)D} + \frac{p_{S1} - p_{S2}}{(1-\lambda)D} \log \left( \frac{p_{S1} - p_{S2}}{(1-\lambda)D} \right) \]

The profits of firm 1 are:
\[ \Pi_{S1} = p_{S1} \left[ 1 - \frac{p_{S1} - p_{S2}}{(1-\lambda)D} + \frac{p_{S1} - p_{S2}}{(1-\lambda)D} \log \left( \frac{p_{S1} - p_{S2}}{(1-\lambda)D} \right) \right] \]

The market share of firm 2 is bounded by non-adopters below, and buyers of information good 1 from above. Because the net utility surplus of consumers who buy from firm 2 is \( U_{S2}(p_{S2}) = \lambda \theta_{i} \phi_{i} D - p_{S2} \), the boundary with the non-adopters is represented by: \( \phi_{i} \theta_{i} > \frac{p_{S2}}{\lambda D} \). Therefore, the market share of firm 2 is:
The profits of firm 2 are:

\[
\Pi_{S2} = p_{S2} \left[ p_{S1} - p_{S2} \frac{(1 - \phi)}{\lambda D} - p_{S2} \frac{(1 - \lambda)}{\lambda D} \log \left( \frac{p_{S1} - p_{S2}}{(1 - \lambda)D} \right) + \frac{p_{S2}}{\lambda D} \log (\frac{p_{S2}}{\lambda D}) \right]
\]

ELECTRONIC APPENDIX 12

Variability in usage frequency and utility-per-use

Variability in usage frequency: In this section, we analyze the efficacy of the selling and pay-per-use mechanisms when either the frequency of usage or the utility-per-use varies across periods. For simplicity, we adopt a two-period model, the results of the model are generalizable to \(N\) periods. In the first period, let the usage frequency \(\theta\) be distributed \(U[0, \theta_H]\) as before, while in the second period, the usage frequency is modeled as \(k\theta\), if \(k > 1\), then the frequency of usage is higher in the second period, while if \(k < 1\), the frequency of usage is lower in the second period. As before, utility-per-use is distributed \(U[0, \phi_H]\).

Proposition 12a: In the presence of variability in usage frequency across periods, the cut-off transaction cost below which pay-per-use is more profitable than selling is the same as in the absence of variability.

Effectively, uncertainty in usage frequency does not change the relative attractiveness of the two mechanisms. Recall that, under pay-per-use, a consumer pays for the good contingent on usage. If consumers use the information good more or less frequently in different periods compared to the expected usage frequency, the (risk neutral) firm still performs as well on average with the payment-per-use set at the same level as in the case with no uncertainty. That is, there is no efficiency loss from such uncertainty. Under selling, rational consumers consider their total frequency of usage across periods when buying the information good. Hence, the firm sets the selling price to be consistent with the average frequency of usage by consumers (discounted appropriately). Hence, there is no difference between the two mechanisms if the usage frequency is variable across periods.

Variability in utility-per-use: Let the utility-per-use \(\phi\) be distributed \(U[0, \phi_H]\) in the first period and let the utility-per-use be \(k\phi\) in the second period, if \(k > 1\), then the consumers derive a higher utility-per-use from the information good in the second period, and a lower-utility-per-use if \(k < 1\), we also assume the frequency of usage is distributed \(U[0, \theta_H]\).

Proposition 12b: In the presence of variability in utility-per-use, the cut-off transaction cost below which pay-per-use is more profitable than selling is higher than in the absence of uncertainty if \(k > 1\) (the utility-per-use is higher in the second period), and lower if \(k < 1\).
If the utility-per-use is higher in the second period, the profits under pay-per-use rise more sharply as the pay-per-use mechanism differentiates perfectly between consumers on the utility-per-use dimension. Therefore, such an increase in the utility-per-use increases the attractiveness of the pay-per-use mechanism. However, when the utility-per-use decreases in the second period, the market share under pay-per-use decreases sharply because the firm is forced to choose a substantially lower payment-per-use in the second period. On the other hand, selling is less affected because the usage frequency, which plays a role in the construction of consumer utility when evaluating the pricing mechanism, is known to the firm and the same. Therefore, variability across periods along the usage frequency and utility-per-use dimensions have distinct implications for each pricing mechanism.

**Proof of Proposition 12a:**

If the firm adopts the selling mechanism and sets a selling price of $p_S$, the net consumer surplus in two periods is given by

$$\phi\theta + \delta\phi k\theta - p_S$$

To find the market share of the firm with the selling mechanism, we find the area for which $\phi\theta > p_S$.

This gives us:

$$MS_S(p_S) = \frac{1}{\phi_H\theta_H}[\theta_H\phi_H - \frac{p_S}{1+\delta_k} + \frac{p_S}{1+\delta_k}\log(\frac{p_S}{(H\phi_H(1+\delta_k)})].$$

The expected profits of the firm are:

$$E[\Pi_S(p_S)] = p_S MS_S(p_S)$$

If the firm adopts pay-per-use and sets a payment-per-use of $p_{O1}$ in the first period and $p_{O2}$ in the second period, its profits are:

$$\Pi_{O1}(p_{O1}) = \frac{\phi_H-(p_{O1}+T)}{\phi_H} \frac{\theta_H}{2} p_{O1}$$

$$\Pi_{O2}(p_{O2}) = \frac{\phi_H-(p_{O2}+T)}{\phi_H} k\phi_H \delta p_{O2}$$

The expected profits of the firm are $E[\Pi_O(p_O)] = \Pi_{O1}(p_{O1}) + \Pi_{O2}(p_{O2}) = \frac{\phi_H-(p_{O1}+T)}{\phi_H} \frac{\theta_H}{2} p_{O1} + \frac{\phi_H-(p_{O2}+T)}{\phi_H} k\phi_H \delta p_{O2}$.

From the first-order conditions (FOC), the firm sets the payment-per-use in the two periods to be $p_{O1} = p_{O2} = \frac{2\theta_H - T}{2}$ and optimal profits are $\Pi_O^* = \frac{1}{8} \frac{\theta_H(\phi_H-T)^2}{\phi_H^2} (1+\delta_k)$. If the firm adopts selling, its expected profits are:

$$E[\Pi_S(p_S)] = p_S [\theta_H\phi_H - \frac{p_S}{1+\delta_k} + \frac{p_S}{1+\delta_k}\log(\frac{p_S}{(H\phi_H(1+\delta_k)})].$$

The function is the same as the profit function with no variability (Proposition 1) with $D = 1+\delta_k$. Hence, the optimal selling price $p_S^* = 0.285\theta_H\phi_H(1+\delta_k)$ and the profits of the firm under the selling mechanism are given by $\Pi_S^* = 0.1018\theta_H\phi_H(1+\delta_k)$. Equating the respective profit functions from the pay-per-use mechanism and the selling mechanism ($\Pi_O^* = \frac{1}{8} \frac{\theta_H(\phi_H-T)^2}{\phi_H^2} (1+\delta_k) = \Pi_S^*$
0.1018\theta H\phi H(1 + \delta k)) gives us the cutoff transaction cost of \( T_C = 0.0976\phi H \), which is the same as before.

Therefore, profits under selling when there is variability in the distribution of usage frequency are scaled identically as the profits under the pay-per-use mechanism in the presence of such variability. Hence, the two mechanisms do not differ in efficiency if the frequency of usage varies across periods.

**Proof of Proposition 12b:**

If the firm adopts the selling mechanism and sets a selling price of \( p_S \), the net consumer surplus in two periods is given by

\[ \phi \theta + \delta k \phi \theta - p_S \]

To find the market share of the firm with the selling mechanism, we find the area for which \( \phi \theta > \frac{p_S}{1 + \delta k} \)

This gives us:

\[ MS_S(p_S) = \frac{1}{\phi H \theta H}[\theta H \phi H - \frac{p_S}{1 + \delta k} + \frac{p_S}{1 + \delta k} \log\left(\frac{p_S}{\theta H \phi H (1 + \delta k)}\right)]. \]

The expected profits of the firm are:

\[ E[\Pi_S(p_S)] = p_S MS_S(p_S) \]

If the firm adopts pay-per-use and sets a payment-per-use of \( p_O1 \) in the first period and \( p_O2 \) in the second period, its profits are:

\[ \Pi_{O1}(p_O1) = \frac{\phi H - (p_O1 + T)}{\phi H} \frac{\theta H}{2} p_O1 \text{ in the first period.} \]
\[ \Pi_{O2}(p_O2) = \frac{\delta p_O2}{k \phi H} \frac{\theta H}{2} \delta p_O2 \text{ in the second period.} \]

The expected profits of the firm are \( E[\Pi_O(p_O)] = \Pi_{O1}(p_O1) + \Pi_{O2}(p_O2) = \frac{\phi H - (p_O1 + T)}{2} p_O1 + \frac{\delta p_O2}{k \phi H} \frac{\theta H}{2} \delta p_O2. \]

From the first-order conditions (FOC), the firm sets the payment-per-use in the two periods to be \( p_O1 = \frac{\phi H - T}{2} \) and \( p_O1 = \frac{\delta p_O2}{k \phi H} \frac{\theta H}{2} \).

The optimal profits are \( \Pi_O^* = \frac{\theta H (\phi H - T)^2}{8 \phi H} + \frac{\delta \theta H (k \phi H - T)^2}{8 \phi H}. \)

If the firm adopts selling, its expected profits are:

\[ E[\Pi_S(p_S)] = p_S [\theta H \phi H - \frac{p_S}{1 + \delta k} + \frac{p_S}{1 + \delta k} \log\left(\frac{p_S}{\theta H \phi H (1 + \delta k)}\right)] \]

The function is the same as the profit function with no variability (Proposition 1) with \( D = 1 + \delta k. \)

Hence, the optimal selling price \( p_S^* = 0.285 \theta H\phi H (1 + \delta k) \) and the profits of the firm under the selling mechanism are given by \( \Pi_S^* = 0.1018 \theta H\phi H (1 + \delta k). \)

**Case 1: \( K > 1. \)** Here we show that \( T_C > 0.0976\phi H \), which is the cutoff transaction cost in the absence of variability. We begin by setting the profits from the pay-per-use and selling mechanisms to be equal to each other.
\[
\frac{1}{8} \theta_H (\phi_H - T)^2 + \frac{\delta}{8} \frac{\theta_H (k \phi_H - T)^2}{k \phi_H} = 0.1018 \theta_H \phi_H (1 + \delta k) \quad (E2b.1)
\]

Note that in the absence of variability in utility-per-use, the cutoff transaction cost is given by the solution to the equation
\[
\frac{1}{8} \theta_H (\phi_H - T)^2 + \frac{\delta}{8} \frac{\theta_H (k \phi_H - T)^2}{k \phi_H} = 0.1018 \theta_H \phi_H (1 + \delta k) \quad (E2b.2)
\]

Comparing the second term of the LHS in the two equations, we note that the term in (E2b.1) is higher as
\[
\frac{\delta}{8} \frac{\theta_H (k \phi_H - T)^2}{k \phi_H} > \frac{\delta}{8} \frac{\theta_H (k \phi_H - T)^2}{k \phi_H} \quad \text{as}
\]
\[
(\phi_H - T)^2 > k(\phi_H - T)^2 \quad \text{as}
\]
\[
k \phi_H - T > k \phi_H - T \quad \text{as} \quad k > 1.
\]

Since the LHS of (E2b.1) is higher than the LHS of (E2b.2) for the same value of the cutoff transaction cost, and the LHS is decreasing in the cutoff transaction cost, it follows that the cutoff transaction cost \(T\) is higher for (E2b.1) than it is for (E2b.2). Hence, if \(k > 1\), the cutoff transaction cost \(T\) is higher than in the case with no variability.

**Case 2:** \(K < 1\). Here we show that \(T_C < 0.0976 \phi_H\), which is the cutoff transaction cost in the absence of variability. We begin by setting the profits from the pay-per-use and selling mechanisms to be equal to each other,

\[
\frac{1}{8} \theta_H (\phi_H - T)^2 + \frac{\delta}{8} \frac{\theta_H (k \phi_H - T)^2}{k \phi_H} = 0.1018 \theta_H \phi_H (1 + \delta k) \quad (E2b.3)
\]

Note that in the absence of variability in utility-per-use, the cutoff transaction cost is given by the solution to the equation
\[
\frac{1}{8} \theta_H (\phi_H - T)^2 + \frac{\delta}{8} \frac{\theta_H (k \phi_H - T)^2}{k \phi_H} = 0.1018 \theta_H \phi_H (1 + \delta k) \quad (E2b.2)
\]

Comparing the second term of the LHS in the two equations, we note that the term in (E2b.3) is lower as
\[
\frac{\delta}{8} \frac{\theta_H (k \phi_H - T)^2}{k \phi_H} < \frac{\delta}{8} \frac{\theta_H (k \phi_H - T)^2}{k \phi_H} \quad \text{as}
\]
\[
(\phi_H - T)^2 < k(\phi_H - T)^2 \quad \text{as}
\]
\[
k \phi_H - T < k \phi_H - T \quad \text{as} \quad k < 1.
\]

Since the LHS of (E2b.3) is lower than the LHS of (E2b.2) for the same value of the cutoff transaction cost, and the LHS is decreasing in the cutoff transaction cost, it follows that the cutoff transaction cost \(T\) is lower for (E2b.3) than it is for (E2b.2). Hence, if \(k < 1\), the cutoff transaction cost \(T\) is lower than in the case with no variability. ■

**ELECTRONIC APPENDIX 13**

**Inclusion of implementation and service costs**

In this section we modify the base model to fit an enterprise software context. We assume that when a software application is purchased outright, an enterprise client incurs a one-time
implementation cost \( (C) \) and a service (or maintenance) cost each time the application is used \( (c_s) \). Under pay-per-use, the service provider incurs a one-time implementation cost \( (S) \) to move the enterprise client’s database to the service provider’s server, and a service cost \( (c_s) \) each time the client uses the application. Under these conditions, the net surplus of the client from each mechanism is:

Pay-per-use: \( U_i \) \( = \theta_i(\phi_i - T - p_o)D \)

Selling: \( U_i \) \( = \theta_i(\phi_i - c_s)D - p_s - C \)

With implementation and service costs, the net surplus decreases under selling. In contrast, under pay-per-use, these costs reduce the margins of the service provider. The implications of these costs are as follows:

**Proposition 13A** In the presence of implementation and service costs, the cut-off transaction cost below which pay-per-use yields higher profits than selling is increasing in the implementation cost \( (C) \) and the service costs \( (c_s) \) incurred by enterprise clients who buy the good outright, but is decreasing in the implementation cost \( (S) \) and the service cost \( (c_s) \) incurred by the service provider offering the pay-per-use mechanism.

**Proof of Proposition 13A:**

As before, the market share of the pay-per-use mechanism is \( MS_O(p_o) = \frac{\phi_H - (p_O + T)}{\phi_H} \). Therefore, the expected profits under pay-per-use are:

\[
\Pi_O(p_o) = \frac{\phi_H - (p_O + T)}{\phi_H} \left[ \frac{\theta_o}{2} (p_o - c_o)D - S \right]
\]

If the firm sells the enterprise software, all clients with usage frequencies in the range \( \theta \in [\frac{p_s + C}{(\phi - c_s)D}, \theta_H] \) derive (weakly) positive utility from their purchase. The fraction of the market purchasing the good, and the resulting profit expressions are, respectively:

\[
MS_S(p_s) = \frac{1}{\phi_H - \phi_H ^ {\phi_H - C} \phi_H ^ {C} \theta_H - \frac{p_s + C}{\phi_H - c_s} \theta_H ^ {D} [d \phi = (1 - \frac{c_o}{\phi_H})[1 - \frac{p_s + C}{(\phi_H - c_s)D} + \frac{p_s + C}{(\phi_H - c_s)D} \log(\frac{p_s + C}{\phi_H D})]]}
\]

\[
\Pi_S(p_s) = MS_S(p_s)p_S = p_S(1 - \frac{c_o}{\phi_H})[1 - \frac{p_s + C}{(\phi_H - c_s)D} + \frac{p_s + C}{(\phi_H - c_s)D} \log(\frac{p_s + C}{\phi_H D})]
\]

The profits of the firm from the pay-per-use mechanism in the presence of implementation and service costs are:

\[
\Pi_O(p_o) = \frac{\phi_H - (p_O + T)}{\phi_H} \left[ \frac{\theta_o}{2} (p_o - c_o)D - S \right]
\]

The FOC of these profits with respect to the payment-per-use \( p_o \) is:

\[
(1 - \frac{p_o + T}{\phi_H}) \frac{\phi_H D}{\phi_H D} - \frac{\phi_H D}{\phi_H D} p_o + \frac{\phi_H D}{\phi_H D} c_o D + \frac{S}{\phi_H} = 0
\]

This yields \( p_o = \frac{1}{\phi_H} (\phi_H - T - c_o) + \frac{2S}{\phi_H} \)

Substituting this back into the profit function yields:

\[
\Pi_O = \frac{1}{\phi_H} [\phi_H - T - c_o - \frac{2S}{\phi_H}]^2
\]
Reassuringly, this reduces to $\frac{1}{8\phi_H} [\phi_H - T]^2$ if $c_o = S = 0$.

The profits under selling are:

$$\Pi_S(p_S) = p_S (1 - \frac{c_s}{\phi_H}) \left[ 1 - \frac{p_S + C}{(\phi_H - c_s) \theta_H D} + \frac{p_S + C}{(\phi_H - c_s) \theta_H D} \log(\frac{p_S + C}{(\phi_H - c_s) \theta_H D}) \right]$$

The FOC of the selling profits with respect to $p_S$ yields:

$$1 - \frac{p_S + C}{(\phi_H - c_s) \theta_H D} + \frac{2p_S + C}{(\phi_H - c_s) \theta_H D} \log(\frac{p_S + C}{(\phi_H - c_s) \theta_H D}) = 0$$

The structure of the equation of the FOC is similar to what we had before if $C = c_s = 0$. If $C = \eta(\phi_H - c_s) \theta_H D$ and the optimal price $p_S^* = k(\phi_H - c_s) \theta_H D$, then $k$ satisfies

$$1 - (k + \eta) + (2k + \eta) \log(k + \eta) = 0$$

Note that the pay-per-use profits are decreasing in the implementation cost and the service cost. Because $\Pi_O = \frac{1}{8\phi_H} [\phi_H - T - c_o - \frac{2S}{\theta_H D}]^2$, an increase in $c_o$ and $S$ will lower the profits from the pay-per-use mechanism. Therefore, the cut-off transaction cost will have to decrease to offset this increase in the implementation and service and maintenance costs. Similarly, the selling profits are decreasing in the implementation and maintenance costs ($C$ and $c_s$). Therefore, increasing these costs would lower the profits from selling, and the cut-off transaction cost would have to increase to compensate for this reduction. ■

As might be expected, an increase in $C$ and $c_s$ favors pay-per-use whereas an increase in $S$ and $c_o$ favors selling. However, the mechanisms through which these costs affect firm profits differ. Under selling, because $C$ and $c_s$ are borne by the clients, the seller’s market share is lower than when these costs are absent. Correspondingly, the seller reduces the selling price at the margin to moderate this decrease in market share. Under pay-per-use, $S$ and $c_o$ are borne by the service provider – so the market share is undisturbed for the same payment-per-use compared to the case without these costs. However, to reduce the margin-reducing impact of these costs, the service provider increases the payment-per-use ($p_O$). Consequently, its market share decreases as well. Effectively, implementation and service costs shift the boundary that demarcates the parametric regions where one pricing mechanism yields higher profits than the other.

Electronic Appendix 14

Selling versus pay-per-use when the utility-per-use and usage frequency are correlated

In this section, we analyze the performance of the selling and pay-per-use mechanisms when utility-per-use and usage frequency are positively and perfectly correlated. For ease of exposition, we assume that each consumer has the same usage frequency as the utility-per-use – represented by the parameter $\tau$. $\tau_i$ is assumed to be uniformly distributed between 0 and $\tau_H$ (as before). From
equations (1) and (2) in the paper, the net consumer surpluses from the selling and pay-per-use mechanisms are:

Pay-per-use:  
\[
U_{iO}(p_O) = \tau_i(\tau_i - T - p_O)D 
\]
Selling:  
\[
U_{iS}(p_S) = \tau_i^2 D - p_S 
\]

We first compare the two mechanisms when each is used separately by a monopolist, and then consider the impact of competition.

E14.1 Monopoly

Result: If usage frequency and utility-per-use are perfectly correlated, the cut-off transaction cost below which pay-per-use is more profitable than selling is higher than in the case where the two utility dimensions are independent. That is, the presence of correlation increases the relative attractiveness of the pay-per-use mechanism.

Proof: First, we cover the intuition. While the profits of both the pay-per-use mechanism and the selling mechanism increase when the frequency of usage and the utility-per-use are correlated, the pay-per-use profits increase to larger extent. The consumers who adopt pay-per-use mechanism have a high utility-per-use in the first place, and under perfect correlation their usage frequency is also high. Therefore, the profits under pay-per-use increase sharply. The profits under selling also increase, but to a lower degree. This is because consumers whose utility-per-use and usage frequency were both high preferred to purchase the product even in the absence of correlation between the utility dimensions.

To prove this formally, we first note that when the firm adopts selling, then for the marginal consumer we have \( \tau_M = \sqrt{\frac{p_S}{D}} \). Therefore, the profits of the firm from selling are:

\[
\Pi_S = \frac{1}{\tau_H}[\tau_H - \sqrt{\frac{p_S}{D}}]p_S 
\]

The FOC of the profit function with respect to the selling price \( p_S \) is \( p_S = \frac{4}{9} \tau_H^2 D \). Substituting this back into the profit function yields the optimal profits:

\[
\Pi_S = \frac{4}{27} \tau_H^2 D 
\]

If the firm employs pay-per-use, the utility-per-use for the marginal consumer is obtained by setting \( \tau_M - T - p_O = 0 \), and the corresponding market share is \( [1 - \frac{p_O + T}{\tau_H}] \). The average frequency of usage of this set of consumers is:

\[
\frac{1}{\tau_M - (p_O + T)} \int_{p_O + T}^{\tau_M} \tau d\tau = \frac{\tau_M + (p_O + T)}{2}. 
\]

Therefore, the profits under pay-per-use are:

\[
\Pi_O = [1 - \frac{p_O + T}{\tau_H}] \frac{\tau_M + (p_O + T)}{2} p_O D = \frac{\tau_H^2 - (p_O + T)^2}{2\tau_H} p_O D 
\]
The FOC of the profit function under the pay-per-use with respect to the payment-per-use yields
\[ p_O = \frac{-4T + \sqrt{12\tau_H^2 + 4T^2}}{6}. \]
Substituting this back into the profit function of the firm equating the resulting expression to the profits under selling yields the cut-off transaction cost:
\[ T_C = 0.147\tau_H \]
This cut-off transaction cost is greater than that when the frequency of usage and the utility-per-use are independent (0.0976 \( \tau_H \)).

E14.2 Duopoly

Here, one firm uses selling and the other employs pay-per-use.

Result: In a duopoly where one firm employs selling and the other employs pay-per-use, the firm that employs selling always has higher profits, and these profits are higher than the corresponding profits in the case where the utility dimensions are not correlated. The firm that employs pay-per-use has lower profits, and these profits are lower than the corresponding profits in the case where the utility dimensions are not correlated. The profits of the firm that employs pay-per-use first increase and then decrease with the transaction cost \( T \).

Proof: We first discuss the intuition. In a duopoly with no correlation between the utility dimensions, selling captures the consumers who have both a high utility-per-use and a high usage frequency. The pay-per-use mechanism captures consumers with a relatively high utility-per-use, but with a lower frequency of usage (only consumers with a lower frequency of usage than \( \frac{ps}{(po+T)D} \) will choose pay-per-use over selling). When these dimensions are correlated, more consumers with a high utility-per-use now also have a high usage frequency, and selling now captures these customers as well. This hurts the pay-per-use mechanism. The inverted-U shaped profit function with the transaction cost \( T \) for the firm that employs pay-per-use is observed here as well – the intuition is similar to that presented in Section 3.2.

To prove the result, we note first that consumers who satisfy \( U_iS(p_S) = \tau_i^2D - p_S > U_iO(p_O) = \tau_i(\tau_i - T - p_O)D \) prefer selling over pay-per-use. Of the remaining consumers, those who obtain a (weakly) positive net surplus from the pay-per-use mechanism adopt that mechanism. The market shares of the pay-per-use mechanism and the selling mechanism are \( \frac{ps}{(po+T)D} \) and \( \frac{1}{\tau_H} \left[ \tau_H - \frac{ps}{(po+T)D} \right] \) respectively.

The profits of the competitors are as follows:
\[
\begin{align*}
\Pi_S &= \frac{1}{\tau_H} \left[ \tau_H - \frac{ps}{(po+T)D} \right] p_S \\
\Pi_O &= [1 - \frac{ps}{(po+T)D}] \frac{1}{2} \left[ \frac{ps}{(po+T)D} + (p_O + T) \right] p_O D = \frac{1}{2\tau_H} \left[ \left( \frac{ps}{(po+T)D} \right)^2 - (p_O + T)^2 \right] p_O D
\end{align*}
\]
Here, $\frac{1}{2}\left[\frac{p_S}{(p_O+T)}D + (p_O + T)\right]$ is the average frequency of usage of the fraction of the market using the pay-per-use mechanism.

In equilibrium, the FOC of the firm using the selling mechanism with respect to $p_S$ yields

$$p_S = \frac{\tau_H(p_O + T)D}{2}$$

The FOC of the firm using the pay-per-use mechanism yields:

$$\left\{\frac{p_S}{(p_O+T)}D\right\}^2\left[1 - 2\frac{p_O}{p_O+T}\right] = (p_O + T)(3p_O + T)$$

Substituting the equilibrium selling price in the FOC for the payment-per-use and solving the resulting (cubic) equation for different values of $T$ yields the result. Figure 15 displays the profits from the selling and pay-per-use mechanisms as a function of the transaction cost when the frequency of usage and utility-per-use are perfectly correlated. $\blacksquare$

Figure 15  Profits with perfect correlation between selling and pay-per-use ($\theta_H = 1$, $\phi_H = 1$, $D = 5$)

ELECTRONIC APPENDIX 15

Non-uniform distribution of utility-per-use and usage frequency

In this section, we examine how alternative distributions of the utility dimensions affect the attractiveness of the pricing mechanisms. We first analyze the case where utility-per-use is distributed across consumers according to an upper triangular distribution function (pdf) of $\frac{2\phi}{\phi_H}$ between 0 and $\phi_H$ (this has consumers clustering at the higher end of the distribution, skewing
the distribution to the right) and then analyze the case where utility-per-use is distributed across
consumers according to a lower triangular distribution function (pdf) of $(2\phi_H - \phi)$ between 0 and $\phi_H$
(this has consumers clustering at the lower end of the distribution). These functions are chosen to
facilitate comparison with the uniform distribution, as the net market share at zero prices will be
the same in both cases. Usage frequency is assumed to be uniformly distributed between 0 and $\theta_H$
(as before).

Following that, we analyze the reverse case where the usage frequency is distributed across con-
sumers with the upper or lower triangular distribution (pdfs), whereas utility-per-use is uniformly
distributed (as before). In general, we find that the major impact on the decision on which mech-
anism to use (selling or pay-per-use) is on account of the skew in the distribution. Symmetric
distributions like the normal distribution and the isosceles triangular distribution do not have a
significant impact. While we analyze the monopoly case here, we expect the intuition to carry
forward to the competitive cases as well.

**Case 1: Upper triangular distribution for utility-per-use**

**Result:** If utility-per-use has an upper triangular distribution and usage frequency is uniformly
distributed, then the cut-off transaction cost below which pay-per-use yields higher profits than
selling is higher than that when both utility dimensions are uniformly distributed. That is, the
relative advantage of pay-per-use is enhanced when utility-per-use exhibits an upper triangular
distribution.

**Proof:** We first discuss the intuition. If the utility-per-use has an upper triangular distribution
(skewed to the right), then consumers are clustered towards the high end in terms of utility-per-use.
This is highly advantageous to the pay-per-use mechanism because it increase the payment-per-
use and yet retain a substantial fraction of the market. Whereas selling also benefits to some
extent from the increase in average utility-per-use, it still will not serve consumers with a high
utility-per-use but a low usage frequency. Therefore, it gains less than the pay-per-use mechanism.

If the monopolist employs pay-per-use with the payment-per-use set at $p_O$, the market share is:

$$MS_O = \int_{p_O + T}^{\phi_H} 2 \frac{\phi}{\phi_H} d\phi = \frac{1}{\phi_H} [\phi_H^2 - (p_O + T)^2].$$

The profits from pay-per-use are:

$$\Pi_O = \frac{1}{\phi_H^2} [\phi_H^2 - (p_O + T)^2] \cdot p_O D$$

If the firm employs the selling mechanism with a selling price of $p_S$, the market share is:

$$MS_S = \int_{p_S}^{\phi_H} 2 \frac{\phi}{\phi_H \cdot \phi_H D} d\phi = 1 - 2 \frac{p_S}{\phi_H \cdot \phi_H D} + \frac{p_S^2}{(\phi_H \cdot \phi_H D)^2}$$

The selling profits are:

$$\Pi_S = p_S [1 - 2 \frac{p_S}{\phi_H \cdot \phi_H D} + \frac{p_S^2}{(\phi_H \cdot \phi_H D)^2}]$$
The FOC of the profit expression when using the selling mechanism is: \((\theta_H \phi_H D)^2 - 4p_S \theta_H \phi_H D + 3p_S^2 = 0\), which yields \(p_S = \frac{\theta_H \phi_H D}{3}\). Substituting this selling price into the profit function yields the optimal selling profits \(\Pi_S = \frac{4}{27} \theta_H \phi_H D\).

The FOC with respect to the payment-per-use \(p_O\) in the profit expression corresponding to the pay-per-use mechanism is: \(3p_O^2 + 4p_OT - (\phi_H^2 - T^2) = 0\). This yields \(p_O = \frac{-4T + \sqrt{12\phi_H^2 + 14T^2}}{6}\). Substituting this value into the pay-per-use profit expression and equating the resulting profit to the profits under selling yields the cut-off transaction cost of \(T_C = 0.147\phi_H\). This cut-off transaction cost is greater than that when the the utility-per-use is uniformly distributed (\(T_C = 0.0976\phi_H\)).

Case 2: Upper triangular distribution for usage frequency

**Result:** If the usage frequency has an upper triangular distribution and utility-per-use is uniformly distributed, then the cut-off transaction cost below which pay-per-use yields higher profits than selling is lower than that when both utility dimensions are uniformly distributed. That is, the relative advantage of pay-per-use is reduced when usage frequency exhibits an upper triangular distribution.

**Proof:** We first discuss the intuition. If usage frequency has an upper triangular distribution (skewed to the high end), the number of consumers who have both a high utility-per-use and a high usage frequency increases. Selling is ideally suited to appeal to these consumers, and gains substantially in market share under these conditions. In contrast, the pay-per-use mechanism does not gain as much from an upper skew in usage frequency because it can efficiently discriminate across consumers and extract surplus related to the usage frequency dimension anyway.

Formally, the profits from the pay-per-use mechanism are \(\frac{1}{\theta_H} [\phi_H - (p_O + T)]^2 \theta_H \ p_O \ D\). Here, the average usage frequency is \(\int_0^{\phi_H} \theta \ f(\theta) \ d\theta = \int_0^{\phi_H} \theta \ \frac{2\theta}{\phi_H^2} \ d\theta = \frac{2}{3} \theta_H\). The selling profits are the same as in Case 1 above (\(\Pi_S = \frac{4}{27} \theta_H \phi_H D\)) because the selling mechanism is symmetric between the utility-per-use and the usage frequency.

If the frequency of usage has an upper triangular distribution, the selling profits are given by as in the above case, while the pay-per-use profits are given by \(\frac{1}{6} (\phi_H - T)^2 \theta_H \ D\). Setting the two profits equal gives us the cut-off transaction cost to be \(T_C = 0.057\phi_H\), which is lower than the cut-off transaction cost when the utility-per-use is uniformly distributed (\(T_C = 0.0976\phi_H\)).

Case 3: Lower triangular distribution for utility-per-use

**Result:** If utility-per-use has a lower triangular distribution and usage frequency is uniformly distributed, then the cut-off transaction cost below which pay-per-use yields higher profits than
selling is lower than that when both utility dimensions are uniformly distributed. That is, the relative advantage of pay-per-use is reduced when utility-per-use exhibits a lower triangular distribution.

**Proof:** We first discuss the intuition. If the utility-per-use has a lower triangular distribution (skewed to the left), then the payment-per-use needs to be sharply lower to gain significant market share. This is because a consumer’s decision to adopt pay-per-use is contingent solely on the payment-per-use and not on usage frequency. In contrast, selling profits also decrease when utility-per-use has a lower triangular distribution, but to a lesser extent. The decrease is tempered by the fact that the attractiveness of selling depends on the distribution of the usage frequency as well – and this distribution continues to be uniform. We now formally prove the result.

If the firm employs pay-per-use mechanism with the payment-per-use set at $p_O$, its market share is:

$$M_{SO} = \int_{p_O + T}^{\phi_H} 2 \frac{\phi_H - \phi}{\phi_H^2} d\phi = \frac{1}{\phi_H} [\phi_H - (p_O + T)]^2.$$ The profits under pay-per-use are:

$$\Pi_O = \frac{1}{\phi_H} [\phi_H - (p_O + T)]^2 \frac{\theta H^2}{p_O D}.$$

In this expression, the average usage frequency of consumers who engage in pay-per-use is $\theta H^2$. If the firm sells the good and sets a selling price of $p_S$, its market share is:

$$M_{SS} = \int_{\frac{p_S}{p_H}}^{p_H} 2(\phi_H - \phi)[1 - \frac{p_S}{\theta_H^2 D}]d\phi = 1 + 2 \frac{\theta H^2}{\theta_H D^2} \log\left(\frac{p_S}{\theta_H D^2}\right) - \frac{p_S^2}{\theta_H^2 D^2}.$$ The selling profits are:

$$\Pi_S = p_S [1 + 2 \frac{\theta H^2}{\theta_H D^2} \log\left(\frac{p_S}{\theta_H D^2}\right) - \frac{p_S^2}{\theta_H^2 D^2}]$$

The FOC of the selling profits with respect to $p_S$ is:

$$\frac{d\Pi_S}{dp_S} = 1 + 4 \frac{\theta H^2}{\theta_H D^2} \log\left(\frac{p_S}{\theta_H D^2}\right) - \frac{3p_S^2}{\theta_H^2 D^2} + 2 \frac{p_S}{\theta_H D^2} = 0,$$ which yields $p_S = 0.1955\theta H\phi_H D$. Substituting this selling price into the profit function yields the optimal selling profits: $\Pi_S = 0.0632\theta_H^2 \phi_H D$.

The FOC of the profits under pay-per-use with respect to $p_O$ is: $\phi_H - 3p_O - T = 0$. This yields $p_O = \frac{\phi_H - T}{3}$. Substituting this value into the pay-per-use profit function and setting it equal to the selling profits yields the cut-off transaction cost of $T_C = 0.05\phi_H$. This cut-off transaction cost is lower than the cut-off transaction cost when the utility-per-use is uniformly distributed ($T_C = 0.0976\phi_H$).

**Case 4: Lower triangular distribution for usage frequency**

**Result:** If usage frequency has a lower triangular distribution and utility-per-use is uniformly distributed, then the cut-off transaction cost below which pay-per-use yields higher profits than selling is higher than that when both utility dimensions are uniformly distributed. That is, the relative advantage of pay-per-use is enhanced when usage frequency exhibits a lower triangular distribution.
Proof: We first discuss the intuition. Selling works best when consumers with high usage frequency and high utility-per-use are clustered together. When usage frequency has a lower triangular distribution, there are fewer consumers who have both a high utility-per-use and a high usage frequency. This hurts selling. Profits from pay-per-use also decrease because consumers use pay-per-use less often on average, but the decrease is not as substantial as in the case of selling.

To prove this, we note that when the firm uses the pay-per-use with the payment-per-use set at \( p_O \), the average usage frequency of consumers who adopt the mechanism is

\[
\int_{0}^{\theta_H} \theta f(\theta) d\theta = \int_{0}^{\theta_H} \theta \frac{2(\phi_H - \theta)}{\theta_H} d\theta = \frac{1}{3} \theta_H.
\]

The corresponding profits are

\[
\frac{1}{p_H} [\phi_H - (p_O + T)] \frac{1}{3} \theta_H p_O D.
\]

The FOC with respect to the payment-per-use \( p_O \) in the pay-per-use profits yields:

\[
\phi_H - 2 p_O - T = 0.
\]

This yields \( p_O = \frac{\phi_H - T}{2} \). Substituting this value into the pay-per-use profit function yields the firm’s profits when using that mechanism. The profits under selling are the same as in case 3 above (0.0632\( \theta_H \phi_H D \)), because the selling mechanism is symmetric between the utility-per-use and usage frequency. Equating these profits yields the cut-off transaction cost of \( T_C = 0.13 \phi_H \) which is higher than the cut-off transaction cost when both utility dimensions are uniformly distributed (\( T_C = 0.0976 \phi_H \)).

Electronic Appendix 16

Endogenous choice of usage frequency

We investigate the performance of the pricing mechanisms when the usage frequency depends on the payment-per-use charged by the pay-per-use provider. For example, in the gaming context, if the “pay-per-play” rates are high, gamers may play the game less often. This issue does not impact selling because consumers buy the information good once and then use it as often as they want for a fixed fee. To maintain consistency with the earlier analysis, we assume that the actual frequency of usage is \( \theta = \overline{\theta} - k p_O \), where \( \overline{\theta} \) is distributed uniformly between 0 and \( \theta_H \). As before, utility-per-use (\( \phi \)) is assumed to be distributed uniformly between 0 and \( \phi_H \).

The net surplus for consumer \( i \) under pay-per-use is:

\[
U_{iO}(p_O) = (\phi_i - (p_O + T)) (\overline{\theta} - k p_O) D.
\]

Accordingly, the market share of the pay-per-use mechanism is:

\[
MS_O = \frac{1}{\phi_H} [\phi - (p_O + T)] (\overline{\theta} - k p_O).
\]

The profits under pay-per-use are:

\[
\Pi_O = \frac{1}{\phi_H} [\phi_H - (p_O + T)] (\overline{\theta} - k p_O) \frac{1}{2} p_O (\overline{\theta} - k p_O) D
\]

\[
= \frac{1}{\phi_H} [\phi_H - (p_O + T)] (\overline{\theta} - k p_O) p_O D
\]

Here, \( \frac{1}{2} (\overline{\theta} - k p_O) \) is the average usage frequency of consumers who adopt pay-per-use. Differentiating these profits with respect to payment-per-use \( p_O \) and equating that differential to zero yields the relevant FOC. Solving the FOC yields the optimal payment-per-use:
\[ p^*_O = \frac{3(kH - T) - \sqrt{4kH - 4kHkT + 9k^2(\phi_H - T)^2}}{8k} \]

Setting \( \theta_H = \phi_H = 1, D = 1 \) and \( T = 0.1 \), we simulate the payment-per-use, market share and profits under the pay-per-use mechanism, and compare it to the case where \( k = 0 \) (in this case, \( p^*_O = \frac{1 - T}{2} = 0.45, MS_O = 1 - \frac{p^*_O + T}{\phi_H} = 0.45, \Pi_O = \frac{1}{8\phi_H} (\phi_H - T)^2 \theta_H D = 0.10125 \)). The profits are plotted in Figure 16, the market share is plotted in Figure 17, and the optimal payment-per-use is plotted in Figure 18. We see that if the optimal payment-per-use charged by the firm is lower when usage frequency is endogenous. The market share holds steady because the lower average frequency of usage \( \left( \frac{1}{2} [\theta_H - kp_O] \right) \) is balanced by the lower payment-per-use \( p^*_O \). The profits of the firm from the pay-per-use mechanism are strictly decreasing in the degree of endogeneity. Therefore, ceteris paribus, endogeneity of usage frequency increases the relative attractiveness of selling. ■

Figure 16  Pay-per-use profits with endogenous frequency of usage \( (\theta_H = 1, \phi_H = 1, D = 5) \)
Figure 17  Market share with endogenous frequency of usage ($\theta_H = 1, \phi_H = 1, D = 5$)
Figure 18  Payment-per-use with endogenous frequency of usage ($\theta_H = 1$, $\phi_H = 1$, $D = 5$)
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