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Abstract

A competitive stock market is embedded into a neoclassical growth economy to analyze the interplay between the acquisition of information about firms, its partial revelation through stock prices, capital allocation and income. The stock market allows investors to share their costly private signals in a cost-effective incentive-compatible way. It contributes to economic growth by raising total factor productivity, but its impact is only transitory. Several predictions on the evolution of real and financial variables are derived, including capital efficiency, total factor productivity, industrial specialization, wealth inequality, stock trading intensity, liquidity and return volatility.

JEL classification codes : O16, G11, G14
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# 1 Introduction

Economic institutions are widely believed to play a crucial role for economic growth. In particular, there is now considerable evidence that financial institutions, once considered a "sideshow" (Robinson (1952)), promote economic growth by relaxing constraints undermining the efficiency of investments. In this paper, we analyze the role of one such institution, the stock market, in alleviating one such constraint, investors' inability to perfectly communicate their private information. Economists have long argued that stock prices improve the allocation of capital by aggregating dispersed information and pointing to the most promising investment opportunities. While several aspects of the relation between the stock market and the real economy have been examined, "existing theories have not yet assembled the links in the chain from the functioning of stock markets, to information acquisition, and finally to aggregate long-run economic growth" (Levine (1997)).[1] This paper assembles these links.

We merge two standard frameworks – the neoclassical overlapping generations growth model (e.g. Diamond (1965)) and the noisy rational expectations model of the stock market (Grossman and Stiglitz (1980)) – in order to present a fully integrated model of information acquisition and dissemination through prices, capital allocation and economic growth. Its main features are the following. The economy is composed of two sectors – a final and an intermediate goods sector, and overlapping generations of agents who work in the final goods sector and invest their wage in the intermediate goods sector. Decreasing returns to intermediate goods employed as capital in the final goods sector produce the neoclassical tendency for income to grow at a decreasing rate until it reaches a steady-state in which it no longer grows.[2] The intermediate goods sector is composed of many firms which raise capital from the young through the stock market. Firms' productivity is unknown but agents can collect private signals about it at a cost. Specifically, they are endowed with one unit of free time which they can devote either to analyzing stocks or to enjoying leisure. This specification implies that the cost of learning about the stock market – foregone leisure – varies over time as a function of the relative valuation of goods and leisure. Agents' information is reflected in stock prices, but only partially so because of the presence of

[1] Page 695. More recently, Levine (2005) confirms this assessment: "While some models hint at the links between efficient markets, information and steady-state growth, existing theories do not draw the connection between market liquidity, information production and economic growth very tightly" (page 9). See Levine (1997, 2005) for reviews of the empirical and theoretical literatures on finance and growth.

[2] There is no technological progress nor population growth in the model.

noise. Prices in turn guide investors in their portfolio allocations.[3]

We focus on a situation in which sharing information is highly beneficial, for example because the intermediate firms operate complex new technologies, about which opinions diverge widely.[4] We assume that signals have errors that are independent from one another. Hence, far from being redundant, each one contributes to overall knowledge. Moreover, a signal's cost is an increasing and convex function of its precision. As a result, agents are better off collecting imprecise signals and pooling them. In fact, the first best capital allocation is reached when agents collect signals of infinitesimal precisions (arbitrarily close but not equal to zero) and combine them to discover firms' productivity thanks to the Law of Large Numbers.[5] But sharing dispersed information that is produced at a cost and privately raises complex issues about incentives and modes of communication. In particular, unless signal precisions are contractible, the first best is not a Nash equilibrium. Agents' best response is to set their precisions to zero and report noise, which results in no learning.

The stock market provides the means to share private information in a cost-effective and incentive-compatible way. For example, when agents receive optimistic signals about a firm, they buy its shares and bid up its stock price. The high stock price in turn indicates that investors collectively believe the firm to have good prospects. Thanks to stock prices, agents are better informed even though no new information is actually produced. Naturally, the effectiveness of the stock market is limited by the very existence of informative prices which undermines the incentive to collect costly information in the first place. Indeed, investors' cannot fully appropriate the benefit of their signals as they are leaked to competitors through prices (the Grossman-Stiglitz paradox). Thus, informative stock prices have an impact that is beneficial *ex post* but detrimental *ex ante* to capital efficiency. Noise trading – some trades are motivated by random shocks unrelated to fundamentals – provides the smoke screen behind which investors can conceal their informed trades and reap some benefit. The first contribution of the paper is to dissect this tradeoff, highlighting its real effects on the allocation of capital. We find that

[3] Because the standard assumptions of exponential utility and normal random variables are not compatible with this neoclassical growth economy, we solve for a rational expectations equilibrium thanks to a small risk approximation, namely assuming productivity shocks to be small.

[4] Allen (1993), Allen and Gale (2000), Subrahmanyam and Titman (1999) argue that the stock market is best suited for aggregating dispersed information. Traditional industries on the other hand call for standardized information that banks may produce in a more cost-effective way.

[5] Each generation consists of a continuum of agents.

the information sharing benefit outweighs the disincentive cost. That is, agents, though they reduce the precision of their private signals in response to a decline in the intensity of noise trading, are nevertheless better informed on the whole thanks to the increased accuracy of stock prices. The allocation of capital improves, and moreover, converges to the first best as the intensity of noise trading approaches zero.

The second contribution of the paper is to characterize the dynamics of income. Income in the stock market economy is governed by a standard neoclassical law of motion: income grows at a decreasing rate until it reaches a steady-state. The learning process thus has no bearing on long run growth – it does not counter the diminishing returns to capital, but it does influence the long run *level* of income and therefore its transitory growth rate. The dynamics of learning in turn are shaped by two competing forces. On one hand, the cost of learning – forgone leisure enjoyment – grows with income. Indeed, agents with a higher wage consume more of the final good because they invested more, which reduces its marginal utility. Hence, they would rather consume more leisure and collect less information (the substitution effect). On the other hand, information generates increasing returns to scale – its benefit, unlike its cost, rises with the amount to be invested. The substitution effect leads wealthier agents to learn less while the scale effect of information induces them to learn more.[6]

If the scale effect of information dominates the substitution effect, then investors produce more private information as their income grows, which then allows them to invest more efficiently in the intermediate firms. The resulting increased supply of intermediate goods in turn enhances the marginal product of labor and makes the next generation of workers richer. In this case, income grows at an accelerated rate compared to a standard neoclassical economy – that is, the growth rate of income falls less quickly. If instead the substitution effect dominates, wealthier investors collect less private information and invest less efficiently, so the growth rate of income declines faster. In both cases, the revelation of information through prices increases the steady-state level of income and its transitory growth rate.[7] In this way, the stock market contributes to economic growth.

Several aspects of the model are broadly consistent with the evidence, assuming that the scale effect

[6] We derive simple conditions on preferences which determine which effect dominates.

[7] Strictly speaking, this statement requires the intensity of noise trading to be weak enough. The reason is that noise trading, in addition to making investments less efficient, has a direct influence on the average income, which is unrelated to the learning process and is beneficial thanks to a Jensen inequality effect – positive noise shocks increase output more than negative shocks decrease it. This effect diminishes as the intensity of noise trading weakens.

of information dominates the substitution effect. First, the stock market develops (e.g., as measured by the time spent analyzing stocks) in tandem with income, contributes to economic growth and its effect is transitory. Empirically, Levine and Zervos (1998), Rousseau and Wachtel (2000) and Carlin and Mayer (2003) document that income grows faster in countries with better functioning stock markets. Atje and Jovanovic (1993) estimate that this growth effect is permanent, but Harris (1997) finds that it is only transitory after controlling for possible endogeneity problems. Moreover, the model shows the stock market to be particularly useful for investing in innovative technologies. Carlin and Mayer (2003) document that industries grow faster in countries with more developed stock markets, and that this relationship is stronger for industries with high R&D investments and skilled labor, such as new technologies. The model also implies that the stock market processes information only when income exceeds a threshold, again a consequence of the increasing returns to information. This is consistent with the casual observation that financial institutions only emerge once a critical level of income has been reached.

The third contribution of the paper is to derive additional observable properties of the economy during its transition to the steady-state, starting from an initial wage below its steady-state level.[8] As the economy grows, (i) capital is more efficiently allocated across firms, i.e. more (less) capital is channelled to more (less) productive firms. This superior efficiency leads to higher total factor productivity (TFP), even though there is no technological progress.[9] (ii) The economy specializes as it grows. Indeed, agents invest more selectively, leading capital and profits to become more concentrated across firms. (iii) Income inequality follows a "Kuznets curve", widening at first and then narrowing. (iv) Stock market liquidity (the inverse of the sensitivity of stock prices to uninformative noise shocks) and the share turnover (the ratio of the value of shares traded to the total capitalization of the market) increase at first and then decrease. Inequality, liquidity and turnover display similar non-monotonic behaviors because all three are driven by the extent to which investors disagree about stocks. At the early stage of development, agents follow mostly price signals since their private signals are imprecise,

[8] The opposite patterns obtain for (i) through (v) if the substitution effect dominates the scale effect.

[9] TFP, also known in the growth literature as the "Solow residual", is defined as the residual from a regression of income growth on factor growth. It encompasses any factor, beyond labor growth and the capital growth, that contributes to output growth. Empirically, most of the differences in income across countries and periods stem from differences in TFP (e.g., Harberger (1998), Prescott (1998), Hall and Jones (1999)).

so disagreement is low. As their private signals become more accurate, agents rely more on them, so disagreement, inequality, trading volume and liquidity rise with income. But they decrease beyond a level of income because private signals that are more precise are also more similar. (v) The volatility of stock prices rises with income as they track technology shocks more closely. As a result, stock returns, which absorb residual shocks, fluctuate less, as reflected in their idiosyncratic and total volatility. In contrast, the volatility of the market is constant. It follows that the cross-correlation of stock prices falls, while that of stock returns rises to offset, respectively, the rise in the volatility of individual stock prices and the reduction in the volatility of individual stock returns.

The first two predictions are, by and large, consistent with the data. (i) Wurgler (2000) documents that investments are more responsive to value added in more financially developed countries, and in particular in countries with a more informative stock market. Furthermore, Levine and Zervos (1998) show that stock markets promote TFP growth, rather than capital growth.[10] (ii) Imbs and Wacziarg (2003) report that countries go through two stages of sectoral diversification. Diversification increases at first, but beyond a certain level of income, the process is reversed and economic activity starts concentrating. The pattern of specialization among advanced countries is consistent with our model as we show that private information is collected only once a critical level of income has been reached. In a similar vein, Kalemli-Ozcan, Sørensen and Yosha (2003) report that industrial specialization in a sample of developed countries is positively related to the share of the financial sector in GDP, a proxy for financial development.

The evidence for the remaining implications is mixed. (iii) Though Kuznets (1955) found support in the data for the hypothesis that inequality widens, peaks and then narrows, more recent studies report ambiguous findings (e.g. Acemoglu and Robinson (2002) for a review of the evidence). (iv) Levine and Zervos (1998) and Rousseau and Wachtel (2000) report that the share turnover on the stock market is positively related to output growth but do not test for a non-monotonic pattern. (v) Morck, Yeung and Yu (2000) show that stock prices are less synchronous in richer economies. Campbell et al. (2001)

[10] Wurgler (2000) constructs cross-country estimates of the elasticity of investments to value added by regressing, for each country, growth in industry investment on growth in industry value added. As a proxy for stock market informativeness, he uses a measure developed by Morck, Yeung and Yu (2000) who estimate the extent to which stocks move together and argue (in line with our model) that prices move in a more unsynchronized manner when they incorporate more firm-specific information. Moreover, recent studies show that variations in the allocation of resources account for a large fraction of the cross-country differences in TFP (Restuccia and Rogerson (2008), Hsieh and Klenow (2009)).

document a strong increase in idiosyncratic return volatility in the U.S. from 1962 to 1997, while the volatility of the market remained stable.

The remaining of the paper is organized as follows. Section 2 positions the paper in the literature. Section 3 describes the economy. Section 4 studies a benchmark economy in which the first best is achieved. Section 5 characterizes the equilibrium. Section 6 discusses the role of the stock market. Section 7 examines the dynamics of income and other variables. Section 8 shows how the economy can emerge from or fall into a no-information regime. Section 9 concludes. Proofs are featured in the appendix.

## 2 Related Literature

Our work relates to three main strands of theory. First and foremost, it contributes to the theoretical literature on finance and growth.[11] Most closely related is the seminal paper by Greenwood and Jovanovic (1990). In their setup, investors choose whether to invest directly in their own project or through a financial intermediary in exchange for a fee. The intermediary pools numerous individual projects and discovers the state of the economy. Thanks to its superior information and its ability to eliminate project-specific risks, it offers a higher return and a lower risk on capital, thereby promoting growth. Greenwood and Jovanovic (1990) show that economic and financial development feed on each other, as in our model. There are several differences between the present paper and Greenwood and Jovanovic (1990). First and most importantly, Greenwood and Jovanovic do not specify where investors' private signals (projects) come from, nor how they are pooled. In particular, they do not study agents' incentives to produce and communicate private information. In contrast, we explicitly address these issues: we model how investors make their decisions to collect costly signals, and how the stock market aggregates and transmits these signals. Putting it differently, Greenwood and Jovanovic (1990) examine an economy free from contracting and communication frictions, while we consider an economy in which these frictions are so severe that eliciting effort and exchanging information between investors is impos-

[11] Many papers highlight the different functions fulfilled by financial institutions, such as monitoring managers, improving risk management, mobilizing savings and facilitating the exchange of goods and services. An important function consists in identifying the best investment opportunities, as in our paper. For example, King and Levine (1993) and Acemoglu, Aghion and Zilibotti (2006) argue that financial intermediaries such as banks promote growth by selecting the best entrepreneurs. These papers do not deal specifically with stock markets and their information processing role.

sible. Moreover, we can characterize the evolution of several observable features of the stock market as the economy grows, such as the volatility of stock returns and the trading intensity. Second, the cost of financial intermediation in Greenwood and Jovanovic (1990) is a fixed fee akin to our information cost. This fee is constant, while our cost of information grows with income. Indeed, information is produced at the expense of leisure whose value rises with income. As a result, the financial sector in Greenwood and Jovanovic (1990) always develops with income, when in our setting, it does so only if the value of information increases faster than its cost. Finally, we differ from Greenwood and Jovanovic (1990) in that they obtain a permanent growth effect while we do not. But this difference arises only because they assume that capital displays constant returns to scale while we assume that it is subject to diminishing returns.

Second, our work is connected to the endogenous growth literature (e.g. Aghion and Howitt (1998) for an overview). This literature models the discovery of technologies by profit-maximizing agents. In contrast to this literature, we *endow* the economy with technologies and focus instead on their selection by investors trading on the stock market. Similar issues arise nonetheless. In particular, technical innovations and information about stocks both give rise to increasing returns to scale, limited by the incomplete appropriability of the rents generated.[12] Whether long-run growth is possible or not depends essentially on the law of motion postulated for technological progress rather than on the structure of the models.[13] When technological progress is assumed away, we find that the information technology cannot generate any permanent growth effect.

Finally, our work belongs to the literature on trading under endogenous and asymmetric information, and in particular to the subset emphasizing the real benefits of informational efficiency. Several authors argue that stock markets are best suited for aggregating information that is dispersed and serendipitous, while banks on the other hand can more efficiently produce standardized information

[12] Increasing returns arise from the non-rivalry of information – information is costly to generate but costless to replicate. Endogenous growth models preserve incentives to do research by granting market power to innovators, while models of the stock market introduce noise into the price system. See Jones (2005) for an overview of the importance of these insights for endogenous growth theory. For applications to finance, see for example Acemoglu and Zilibotti (1999), Veldkamp (2005, 2006) and Zeira (1994).

[13] For example, if the rate of growth of technological knowledge, $dA/dt$, increases linearly with the level of technological knowledge, $A$, as in Romer (1990), then the economy grows without bound. Otherwise, growth is only transitory. As Romer (1990, page 84) puts it, "linearity in $A$ (in the equation for $dA/dt$) is what makes unbounded growth possible, and, in this sense, unbounded growth is more like an assumption than a result of the model".

and avoid duplication costs. Unlike much of this research, the real effect studied here stem from stock prices guiding investors in allocating capital across firms rather than managers in setting their firm's investment plans.[14] Our model contributes to the literature on the real effect of the stock market by developing a rational expectations framework in which learning from stock prices and income interact dynamically.

# 3 Economic Environment

We embed a competitive stock market *à la* Grossman and Stiglitz (1980) into Diamond's (1965) neoclassical growth economy. The economy is composed of two sectors – a final and an intermediate goods sector, and overlapping generations of agents. Firms in the intermediate goods sector raise capital on the stock market by issuing claims to their future profits. Young agents save by purchasing these claims.

## 3.1 Agents

The economy is populated by overlapping generations of agents who live for two periods. There is no population growth. Each generation consists of a continuum of agents with mass $L$ indexed by $l \in [0, L]$. Young agents are each endowed with one unit of labor time and one unit of free time. Utility, derived from the consumption of the final good $g$ and leisure $j$, is represented by a function $U(g, j)$, increasing and concave in each argument and with a positive cross-derivative, $\partial^2 U/\partial g \partial j$. Two aspects of preferences are of particular relevance to our analysis: risk aversion and the degree of substitutability between final goods and leisure. We define the following functions:

$$\tau(g) \equiv -\frac{\frac{\partial U}{\partial g}(g,1)}{\frac{\partial^2 U}{\partial g^2}(g,1)} \qquad \text{and} \qquad \rho(g) \equiv \frac{\frac{\partial U}{\partial j}(g,1)}{\frac{\partial U}{\partial g}(g,1)}.$$

$\tau(g)$ measures the absolute risk tolerance of an agent consuming $g$ units of the final good and one unit of leisure. $\tau$ captures attitudes toward risk because leisure consumption is deterministic in our setting. We assume that $\tau$ *is increasing in* $g$, as supported by most empirical studies. The function $\rho$ measures the marginal rate of substitution between final goods and leisure, again for an agent consuming $g$ units

[14] On the financial structure of the economy, see for example Allen (1993), Allen and Gale (2000), Dow and Gorton (1997), Boot and Thakor (1999), Subrahmanyam and Titman (1999). On the feedback effect from financial markets to the real economy, see for example Fishman and Hagerty (1992), Holmström and Tirole (1993), Dow, Goldstein, and Guembel (2010).

of the final good and one unit of leisure. Naturally, $\rho$ *is increasing in* $g$ because the marginal utility of the final good declines while that of leisure rises when more final goods are consumed.

For example, $U(g,j) \equiv (\varpi g^{\sigma} + (1-\varpi)j^{\sigma})^{1/\sigma}$, where $\varpi$ is in $(0,1)$ and $\sigma < 1$, displays a constant elasticity of substitution (CES). The case $\sigma = 0$ corresponds to Cobb-Douglas utility ($U(g,j) \equiv g^{\varpi} j^{1-\varpi}$). Under these preferences, $\tau(g) = g(\varpi g^{\sigma} + 1 - \varpi)/(1-\sigma)/(1-\varpi)$ and $\rho(g) = g^{1-\sigma}(1-\varpi)/\varpi$ – the elasticity of substitution between goods and leisure equals $1/(1-\sigma)$.

Young agents are employed in the final good sector, to which they supply their unit of labor time inelastically for a competitive wage $w_t$, so aggregate labor supply equals $L$.[15] They save their entire labor income by investing in the stock market to consume in the next period when they are old.[16] They divide their unit of free time between enjoying leisure and analyzing stocks. There are no short-sales constraints, and no riskless asset.[17]

## 3.2 Technologies

### 3.2.1 Final Good Sector

The final good is produced according to a riskless technology that employs labor and intermediate goods:

$$G_t \equiv L^{1-\beta} \sum_{m=1}^{M} (Y_t^m)^{\beta},$$

where $G_t$ is final output, $L$ is labor, $M$ is the number of types of intermediate goods, $Y_t^m$ is the employment of the $m'$th type and $0 < \beta < 1$ is the factor share of intermediate goods in the production of the final good. The production function follows Spence (1976), Dixit and Stiglitz (1977) and Romer (1990) among others. Many identical firms compete in the final good sector and aggregate to one representative firm. The final good is used as the numeraire. It can be consumed by agents or invested to produce intermediate goods in the following period.

[15] Francis and Ramey (2009) estimate that leisure per capita has remained constant in the U.S. throughout the 20th century.

[16] Thus the saving rate is exogenously set to one. This assumption simplifies the model and is consistent with evidence suggesting that financial development enhances growth through higher productivity rather than through higher saving rates (Levine and Zervos (1998), Beck, Levine and Loayza (2000)). Bonser-Neal and Dewenter (1999) find no relation between saving rates and stock market development.

[17] We assume that there is no storage technology and that final wealth is not verifiable. The latter assumption implies that a bond market cannot be set up because the probability that final wealth equals zero is strictly positive in our setting. Borrowers would simply claim that they are unable to repay their loans.

### 3.2.2 Intermediate Good Sector

$M$ firms operate in the intermediate goods sector. Firm $m$ is the exclusive producer of good $m$. Its production is determined by a risky technology that displays constant returns to capital:

$$\widetilde{Y}_{t+1}^{m} \equiv \widetilde{A}_{t}^{m} K_{t}^{m} \qquad \text{for } m = 1, ..., M$$

where $\widetilde{Y}_{t+1}^{m}$ is the quantity of intermediate goods produced in period $t+1$ by firm $m$ net of capital depreciation, $\widetilde{A}_{t}^{m}$ is its random productivity and $K_{t}^{m}$ is the amount of capital (which consists of final goods) it raises in period $t$. Tildes denote random variables not yet realized. Firms are liquidated immediately after production.[18]

The productivity shocks $\widetilde{A}_{t}^{m}$ are log-normally distributed and independent from one another and over time. Because there is no closed-form solution to investors' portfolio choice under general preferences, we resort to a small-risk expansion to solve the model. We consider small productivity shocks and log-linearize the return on investors' portfolio. Specifically, we assume that $\ln \widetilde{A}_{t}^{m} \equiv \widetilde{a}_{t}^{m} z$ where $\widetilde{a}_{t}^{m} z$ is normally distributed with mean $\widetilde{\alpha}_{t}^{m} z$ and variance $\sigma_{a}^{2} z$, $\widetilde{\alpha}_{t}^{m}$ is normally distributed with mean 0 and variance $\sigma_{\alpha}^{2}$ and $z$ is a scaling factor. The model is solved in closed-form by driving $z$ toward zero. Throughout the paper, we assume that $z$ is small enough for the approximation to be valid.[19]

Firms raise capital in the stock market. Firm $m$ issues one perfectly divisible share – a claim to its entire future profit, for a price $P_{t}^{m}$. The productivity shock $\widetilde{a}_{t}^{m}$ is not observed at the time agents invest but they can learn about its average $\widetilde{\alpha}_{t}^{m}$ as we describe next.

### 3.2.3 Information Technology

At the time they invest, agents do not observe intermediate firms' productivity. Instead, they receive private signals about its mean. The private signal $s_{l,t}^{m}$ received by agent $l$ in period $t$ about firm $m'$s

[18] Assuming firms are liquidated just after production simplifies the dynamics of the economy and allows to focus on the early stage of a firm's development. It is well known that young firms, because they have little retained earnings, are more dependent on external financing than mature firms. Several empirical studies confirm that financial development fosters growth mainly through young firms (Rajan and Zingales (1998), Beck, Demirgüç-Kunt and Maksimovic (2005), Brown, Fazzari and Petersen (2008)).

[19] Rational expectations models of competitive stock trading under asymmetric information typically conjecture that equilibrium stock prices are linear functions of random variables. This conjecture is not valid in a neoclassical framework because productivity and capital interact multiplicatively in the production of goods, and capital itself is a function of stock prices. For examples of small risk expansions applied to portfolio choice, see Campbell and Viceira (2002) and Peress (2010).

average productivity shock is given by:

$$s_{l,t}^m = \beta \widetilde{\alpha}_t^m + \widetilde{\varepsilon}_{l,t}^m,$$

where $\widetilde{\varepsilon}_{l,t}^m$ is an agent-specific disturbance independent of $\widetilde{a}_t^m$, $\widetilde{\alpha}_t^m$, across firms and time. $\widetilde{\varepsilon}_{l,t}^m$ is normally distributed with mean 0 and variance $1/x_{l,t}^m$ (precision $x_{l,t}^m$). Investors choose the precision of their signals before the stock market opens. Observing a signal of precision $x_{l,t}^m$ costs $C(x_{l,t}^m)z$ units of free time, where $C$ is continuous, increasing, convex and $C(0) = C'(0) = 0$. We emphasize that the information technology does not lead to the discovery of new physical technologies nor improve existing ones. Instead, it allows to allocate capital more efficiently to the physical technologies.

### 3.2.4 Noise Trading

Agents know that stock prices reflect other investors' private information in equilibrium, and they learn from them. Some noise is needed to blur price signals and avoid the Grossman-Stiglitz paradox, that is, preserve incentives to collect costly information. We assume that a fraction $q$ of agents form their portfolio guided by exogenous shocks. The source of these shocks is not specified. They could stem from liquidity needs, preference shifts, random stock endowments, private risky investment opportunities, or some form of irrationality. Specifically, noise traders believe that the expected return on stock $m$ equals $\widetilde{\theta}_t^m$, where $\widetilde{\theta}_t^m$ is normally distributed with mean 0 and variance $\sigma_\theta^2$, and is independent of $\widetilde{a}_t^m$, $\widetilde{\varepsilon}_{l,t}^m$, across firms and time.[20]

## 3.3 Timing

The timeline is summarized in figure 1. An agent lives one period as a young agent (as a worker, then as an investor) and one period as an old agent (as a consumer). After earning a wage and before the stock market opens, workers choose how to divide their free time between stock analysis and leisure, by setting the precision of their signals. Then, they invest their wage across the different stocks, guided

[20] Some comments on the formulation of noise traders' beliefs may be useful. First, their accuracy is arbitrary and does not affect our findings. Second, including an agent-specific component to these beliefs has no incidence on the equilibrium. Third, the intensity of noise trading remains commensurate with that of rational trading as the economy grows. As equation 9 below shows, portfolio holdings are scaled by a function of income, $\tau(\varphi(w))/\varphi(w)$. If for example this function increases with income (e.g. $\sigma > 0$ under CES utility), then trades, both rational and noise-motivated, grow with the economy. If we assumed instead that noise trades equal an exogenous constant, then they would shrink relative to rational trades. This would mechanically make stock prices more informative and the allocation of capital more efficient, and reinforce the usefulness of the stock market.

by stock prices and their private signals. In the following period, the young become old, productivity shocks are revealed, final goods are produced, and old agents consume their share of profits.

### 3.4 Notation

For any firm-specific variable $\psi_t^m$, $\overline{\psi_t}$ denotes its average across firms and $\Delta\psi_t^m$ its deviation from the average:

$$\overline{\psi}_t \equiv \frac{1}{M}\sum_{m=1}^{M}\psi_t^m \quad \text{and} \quad \Delta\psi_t^m \equiv \psi_t^m - \overline{\psi}_t.$$

The variable enclosed in brackets, $\{\psi_t^m\}$, represents the vector of stacked variables for $m = 1$ to $M$. Finally, we adopt the following notation to keep track of the quality of the approximation: $o(1)$, $o(z)$ and $o(z^2)$ capture respectively terms of an order of magnitude smaller than 1, $z$ and $z^2$.

### 3.5 Equilibrium Concept

We describe the equilibrium concept working backwards from production in period $t+1$, to capital allocation and information acquisition in period $t$. The gains from trade depend on how much information is collected in aggregate and revealed through prices. We denote $X_t^m \equiv \int_l x_{l,t}^m/L$ the average precision of private information about firm $m$. A rational expectations equilibrium satisfies the following conditions.

1. Market clearing in the intermediate goods sector

Final goods producers maximize their profit. Since labor and intermediate goods trade in competitive markets and aggregate labor supply equals $L$, the following equilibrium factor prices (denominated in units of the final good) obtain in period $t+1$ :

$$\widetilde{w}_{t+1} = (1-\beta)\sum_{m=1}^{M}(\widetilde{Y}_{t+1}^m/L)^{\beta} \quad \text{and} \quad \widetilde{\rho}_{t+1}^m = \beta(L/\widetilde{Y}_{t+1}^m)^{1-\beta}, \tag{1}$$

where $\widetilde{\rho}_{t+1}^m$ denotes the price of intermediate good $m$ in period $t+1$ and $\widetilde{\Pi}_{t+1}^m = \widetilde{\rho}_{t+1}^m\widetilde{Y}_{t+1}^m$ is firm $m$'s profit.

2. Capital allocation

Let $f_{l,t}^m$ denote the fraction of her wage that agent $l$ invests in stock $m$ in period $t$ or her 'portfolio weights'. She sets $\left\{f_{l,t}^m\right\}$ to maximize her expected utility, guided by stock prices and private signals, and taking as given her income $w_t$, her leisure time $j_t$, the precision of her signals $\left\{x_{l,t}^m\right\}$, the average

precisions $\{X_t^m\}$, share prices and capital stocks:

$$\max_{\{f_{l,t}^m\}} E\left[U(\widetilde{g}_{l,t+1}, j_t) \mid \mathcal{F}_{l,t}\right] \quad \text{subject to} \begin{cases} \widetilde{g}_{l,t+1} = w_t \widetilde{R}_{l,t+1} \\ \widetilde{R}_{l,t+1} = \sum\limits_{m=1}^{M} f_{l,t}^m \widetilde{R}_{t+1}^m \\ \sum\limits_{m=1}^{M} f_{l,t+1}^m = 1 \end{cases}, \tag{2}$$

where $\mathcal{F}_{l,t} \equiv \{s_{l,t}^m, P_t^m \text{ for } m = 1 \text{ to } M\}$, $\widetilde{g}_{l,t+1}$, $\widetilde{R}_{l,t+1}$ and $\widetilde{R}_{t+1}^m = \widetilde{\Pi}_{t+1}^m / P_t^m$ denote respectively agent $l'$s information set, her consumption of the final good, the return on her portfolio and the return on stock $m$. The time subscripts on $j_t$ and $\widetilde{g}_{l,t+1}$ make clear that leisure time is set at $t$ before private signals are observed, while the consumption of final goods is determined at $t+1$, once the return on the portfolio is realized. We call $U_0(\{x_t^m, X_t^m\}, j_t, w_t)$ the value function for this problem.

In equilibrium, prices clear the stock market. Since each firm issues one share, its capital stock coincides with its stock price: Formally,

$$\int_l w_t f_{l,t}^m = K_t^m = P_t^m \quad \text{for } m = 1, ..., M,$$

where the integral sums up the demand emanating from rational and noise traders.

3. Precision choice

An agent's optimal precisions $x_{l,t}^m = x(w_t, \{X_t^m\})$ maximize her *ex ante* expected utility subject to her free time budget constraint, taking her income $w_t$ and the average precisions $\{X_t^m\}$ as given:

$$\max_{\{j_t \geq 0,\ x_{l,t}^m \geq 0\}} E[U_0(\{x_{l,t}^m, X_t^m\}, j_t, w_t)] \quad \text{subject to} \quad \sum_{m=1}^{M} C(x_{l,t}^m) z + j_t = 1,$$

where $C(x_{l,t}^m)z$ is the time spent investigating stock $m$ and $1 - \sum_{m=1}^{M} C(x_{l,t}^m)z$ is the time left for leisure. In equilibrium, the average and optimal precisions must be consistent:

$$X_t^m = x(w_t, \{X_t^m\}) \quad \text{for } m = 1, ..., M.$$

# 4 First Best

Before we proceed to the general case, we describe a benchmark, the first best outcome, in which agents perfectly share their information. The first best is achieved when signal precisions are contractible and there is no noise trading. In that case, agents all commit to infinitesimal precisions – very close but not

equal to zero, and reveal their private signals to a central planner who invests on their behalf. The central planner can perfectly infer productivity shocks thanks to the Law of Large Numbers because there is a continuum of signals with finite variances and uncorrelated errors ($\int_l \varepsilon^m_{l,t+1} = 0$). The central planner chooses capital allocations $\left\{K_t^{mFB}\right\}$ to maximize agents' expected utility subject to an economy-wide resource constraint, taking as given their income $w_t$:

$$\max_{\{K_t^{mFB}\}} E\left[U(\widetilde{g}_{l,t+1}, 1) \mid \{\widetilde{\alpha}_t^m\}\right] \quad \text{subject to} \quad \begin{cases} \widetilde{g}_{l,t+1} = \sum\limits_{m=1}^{M} \widetilde{\Pi}_{t+1}^{mFB}/L \\ \sum\limits_{m=1}^{M} K_t^{mFB} = Lw_t \end{cases}, \tag{3}$$

where $\widetilde{\Pi}_{t+1}^{mFB} = \beta L^{1-\beta}(\widetilde{A}_t^m K_t^{mFB})^\beta$ denotes the profit generated by firm $m$, to be divided equally between agents. The following lemma describes the capital allocation in this economy.

**Lemma 1** *In the first-best outcome, firm $m$'s capital stock equals $K_t^{mFB} = \frac{Lw_t}{M}\exp(\Delta k_t^{mFB} z)$ where*

$$\Delta k_t^{mFB} = \frac{1}{1-\beta}\Delta\beta\widetilde{\alpha}_t^m + o(1). \tag{4}$$

When $z$, the factor that scales shocks, equals zero, the firms are perfectly identical so capital is equally distributed across them, each firm receiving $Lw_t/M$ units of goods.[21] When $z > 0$, the allocation depends on firms' productivity relative to one another. The more productive firms (higher $\Delta\widetilde{\alpha}_t^m \equiv \widetilde{\alpha}_t^m - \overline{\widetilde{\alpha}_t}$) receive more capital. The elasticity of investments to productivity shocks, $\partial(\ln K_t^{mFB})/\partial\widetilde{\alpha}_t^m = (1-1/M)\beta/(1-\beta)$, captures the efficiency of the capital allocation. It increases with $\beta$, the factor share of capital because a higher $\beta$ indicates that firms' marginal profits decline with their stock of capital at a slower rate, so more capital can be invested in the better firms without immediately damaging their return. Efficiency also increases with the number of stocks $M$ because there is a wider choice of uses for capital.

Given its capital stock, firm $m$ produces $\widetilde{Y}_{t+1}^m = \widetilde{A}_t^m K_t^{mFB}$ intermediate goods. As a result, the number of final goods produced is:

$$\widetilde{G}_{t+1} = Lw_t^\beta M^{1-\beta}\overline{\exp\left(\beta(\widetilde{a}_t^m z + k_t^{mFB} z)\right)},$$

and the wage equals:

$$\widetilde{w}_{t+1} = (1-\beta)\widetilde{G}_{t+1}/L = (1-\beta)w_t^\beta M^{1-\beta}\overline{\exp\left(\beta(\widetilde{a}_t^m z + k_t^{mFB} z)\right)}.$$

[21] Firm $m$'s marginal profit, $\partial\Pi_{t+1}^{mFB}/\partial K_t^{FBm} = \partial[\beta L^{1-\beta}(A_t^m K_t^{FBm})^\beta]/\partial K_t^m = \beta^2 L^{1-\beta} A_t^{m\beta} K_t^{FBm\beta-1}$, is a decreasing function of $K_t^{FBm}$. Hence, if firms are identical, the central planner distributes capital equally across the $M$ firms.

The wage is random as it depends on the realizations of the productivity shocks. The following lemma characterizes the dynamics of the economy along its average path, i.e. assuming that the wage realized in any period equals its mean. This is a good description of the economy if the number of firms is large.

**Lemma 2** *In the first-best outcome, average income evolves according to the following equation:*

$$E(\widetilde{w}_{t+1}) = \Lambda \exp\left(\lambda^{FB} z^2\right) w_t^{\beta}, \tag{5}$$

*where $\Lambda$ and $\lambda^{FB}$ are two positive constants given by:*

$$\Lambda \equiv (1-\beta) M^{1-\beta} \exp\left(\frac{1}{2}\beta^2(\sigma_a^2 z + \sigma_\alpha^2 z^2)\right), \tag{6}$$

*and*

$$\lambda^{FB} \equiv \frac{M-1}{M} \frac{\beta^3}{(1-\beta)^2} \left(1 - \frac{\beta}{2}\right) \sigma_\alpha^2 + o(1). \tag{7}$$

*Average income converges to a steady-state, $w^{FB}$, given by:*

$$w^{FB} = \Lambda^{1/(1-\beta)} \exp\left(\frac{\lambda^{FB}}{1-\beta} z^2\right). \tag{8}$$

The average wage evolves according to a standard neoclassical law of motion. The marginal product of labor increases with current income (assuming income is initially below its steady-state value) but at a decreasing rate, until it reaches a steady-state in which it no longer grows. The growth rate of income is given by $\Gamma^{FB}(w_t) \equiv E(\widetilde{w}_{t+1})/w_t = \Lambda w_t^{-(1-\beta)} \exp\left(\lambda^{FB} z^2\right)$. It declines at the rate $-(1-\beta)$, i.e. $d \ln \Gamma^{FB}(w_t)/d \ln w_t = -(1-\beta)$. The steady-state level of income $w^{FB}$ solves $w^{FB} = \Lambda w^{FB\beta} \exp\left(\lambda^{FB} z^2\right)$, which leads to equation 8. The dashed curves in figures 6 and 7 illustrate the dynamics of income in the first best. Steady-state income increases with the number of intermediate goods $M$ as the production possibility set expands, and with the variance of productivity shocks $\sigma_a^2 z + \sigma_\alpha^2 z^2$ because output is a convex function of these shocks – a positive shock increases $\widetilde{G}_{t+1}$ more than a negative shock decreases it. It decreases with the factor share of intermediate goods $\beta$ as the marginal product of labor is reduced.

The first best obtains in particular in Greenwood and Jovanovic (1990). In their model, a financial intermediary pools numerous projects (signals) supplied by individuals and discovers the state of the economy. The reason the first best is reached in their equilibrium is that agents are *endowed* with a project rather than produce it at a cost. Here in contrast, the first best is not achievable because agents cannot commit to strictly positive signal precisions. Indeed, suppose all investors do agree to

acquire some information about a stock, however imprecise, and to report it to the central planner. This will allow the planner to learn the stock's productivity shock. Given that the cost of information is not zero, the optimal strategy for an agent is to deviate from the agreement, i.e. to not collect any information and make a random announcement to the central planner. But if all agents make random announcements, then the productivity shock cannot be learned. Thus, the first-best outcome cannot be reached if signal precisions are not contractible.

# 5 Equilibrium Characterization

The remainder of the paper assumes that signal precisions are not contractible but that some trades are motivated by noise. In that case, the stock market offers a way to share information, albeit imperfectly. We characterize first investors' portfolios and the allocation of capital, then various aspects of the economy, and finally information acquisition decisions. Throughout this section, we take as given investors' income $w_t$ which we endogenize in section 7.

## 5.1 Capital Allocation

We follow the usual method for solving a noisy rational expectations equilibrium: We guess that capital is a log-linear function of shocks, solve for portfolio, derive the equilibrium capital allocation, and check that the guess is valid. The following lemma displays investors' portfolio composition for the conjectured capital allocation.

**Lemma 3** *Assume that firm $m'$s capital stock takes the form $K_t^m = \frac{Lw_t}{M}\exp(\Delta k_t^m z)$ where $k_t^m \equiv k_{\alpha t}^m(\beta\Delta\widetilde{\alpha}_t^m + \mu_t^m\Delta\widetilde{\theta}_t^m) + o(1)$ and $\mu_t^m$ is a deterministic scalar. The portfolio weights for agent $l$ are given by:*

$$f_{l,t}^m = \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2 z}E(\Delta\ln R_{t+1}^m \mid \mathcal{F}_{l,t}) + o(1), \tag{9}$$

$$\textit{where} \quad \varphi(w) \equiv \beta M^{1-\beta}w^\beta. \tag{10}$$

- *For a rational agent who receives private signals of precision $\{x_{l,t}^m\}$, weights equal:*

$$f_{l,t}^m = \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2}\left\{\frac{x_{l,t}^m}{H(\mu_t^m) + x_{l,t}^m}\Delta s_{l,t}^m + \left(\frac{1}{(H(\mu_t^m) + x_{l,t}^m)\mu_t^{m2}\sigma_\theta^2 k_{\alpha t}^m} - (1-\beta)\right)\Delta k_t^m\right\} + o(1). \tag{11}$$

$$\textit{where} \quad H(\mu) \equiv \frac{1}{\beta^2\sigma_\alpha^2} + \frac{1}{\mu^2\sigma_\theta^2}. \tag{12}$$

- *For a noise trader, weights equal:*

$$f_t^m = \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2}\Delta\widetilde{\theta}_t^m + o(1). \tag{13}$$

Stock $m$'s portfolio weight equals the weight it would receive if firms were identical, $1/M$, tilted by a measure of the stock's expected excess performance relative to the market, $E(\Delta \ln \widetilde{R}_{t+1}^m \mid \mathcal{F}_{l,t}) \equiv E(\ln \widetilde{R}_{t+1}^m - \overline{\ln R_{t+1}} \mid \mathcal{F}_{l,t})$. The deviation from equal portfolio shares is more pronounced when stocks are less risky (lower $\beta$ or $\sigma_a^2$), or when agents are relatively more risk tolerant. $\tau(\varphi(w_t))$ measures investors' absolute risk tolerance in a neighborhood of their consumption – to a first approximation (i.e., at the order 0 in $z$), they consume $\varphi(w_t)$ units of the final good. Relative risk tolerance, the ratio of absolute risk tolerance to consumption, $\tau(\varphi(w_t))/\varphi(w_t)$, determines how aggressively investors trade on their information. Though absolute risk tolerance $\tau(\varphi(w))$ rises with income by assumption, this need not be the case for relative risk tolerance, $\tau(\varphi(w))/\varphi(w)$. For example, under CES preferences $\tau(\varphi(w))/\varphi(w) = (\varpi\beta^\sigma M^{\sigma(1-\beta)}w^{\sigma\beta} + 1 - \varpi)/(1-\sigma)/(1-\varpi)$. If $\sigma > 0$ $(< 0)$, then $\tau(\varphi(w))/\varphi(w)$ increases (decreases) with income, and wealthier investors' portfolio weights deviate more (less) from equal shares. If $\sigma = 0$ (Cobb-Douglas utility), then $\tau(\varphi(w))/\varphi(w)$ is a constant, $1 - \varpi$, so portfolio weights are independent of wealth as in the case of constant relative risk aversion.

Equation 11 expresses portfolio weights as a combination of the stock price (the $\Delta k_t^m$ term) and the relative private signal (the $\Delta s_{l,t}^m$ term). In this expression, the stock price plays a dual role: it clears the stock market and provides information about the firm's productivity. Given our conjecture, observing stock prices is equivalent to observing $\beta\Delta\alpha_t^m + \mu_t^m\Delta\theta_t^m$ for each firm, a signal about $\beta\Delta\alpha_t^m$ with error $\mu_t^m\Delta\theta_t^m$. Thus, $\mu_t^m$ represents the *noisiness of stock m's price.* The function $H(\mu_t^m) + x_{l,t}^m = 1/Var(\beta\alpha_t^m \mid \mathcal{F}_{l,t})$ measures the total precision of an investor's information about a stock. She receives information from three sources: her priors (the $1/(\beta^2\sigma_\alpha^2)$ term), the price (the $1/(\mu_t^{m2}\sigma_\theta^2)$ term) and her private signal (the $x_{l,t}^m$ term), and their precisions simply add up. The next proposition describes the equilibrium allocation of capital for an arbitrary level of noisiness $\mu_t^m$. Equivalently, the equilibrium can be characterized in terms of the average precisions about stocks $X_t^m$ since $X_t^m$ and $\mu_t^m$ are connected one for one (equation 16).

**Proposition 4** *Let $\mu_t^m$ $(> \dfrac{q}{1-q})$ be the noisiness of stock m's price. There exists a log-linear rational expectations equilibrium in which firm m's capital stock and its share price equal $K_t^m = P_t^m = \frac{Lw_t}{M}\exp(\Delta k_t^m z)$ where:*

$$\Delta k_t^m \equiv k_\alpha(\mu_t^m)(\beta\Delta\widetilde{\alpha}_t^m + \mu_t^m\Delta\widetilde{\theta}_t^m) + o(1), \tag{14}$$

$$k_\alpha(\mu) \equiv \frac{1}{1-\beta}\left(1 - \frac{1}{\beta^2\sigma_\alpha^2\left(H(\mu)+X(\mu)\right)}\right) > 0, \tag{15}$$

$$\textit{and} \quad X(\mu) \equiv \frac{H(\mu)}{\dfrac{1-q}{q}\mu - 1}. \tag{16}$$

The proposition establishes that capital and stock prices are approximately log-linear functions of productivity and noise shocks. As in the first best, they equal those that would obtain if firms were identical ($Lw_t/M$), disturbed by an order-$z$ function of relative shocks. Productivity shocks appear directly in the price function though they are not known by any agent, because individual signals, $\widetilde{s}_{l,t}^m$, once aggregated, collapse to their mean, $\beta\widetilde{\alpha}_t^m$. Noise traders' introduce noise $\widetilde{\theta}_t^m$ into the price system through their trades. For simplicity, the conditions that characterize $k_\alpha$ and $X$ (equation 15 and 16) are stated under the assumption that signal precisions are identical across agents for any stock $m$ ($x_{l,t}^m = X_t^m$ for all $l$), a property which holds when signal precisions are chosen optimally (see lemma 5 below). Equation 31 in the appendix displays these conditions for arbitrary precisions. As mentioned, the average precision $X_t^m$ and stock price noisiness $\mu_t^m$ are related one for one through equation 16. A higher noisiness $\mu_t^m$ corresponds to a lower average precision $X_t^m$, as figure 2 illustrates.

Proposition 4 outlines the allocative role of the stock market. Equation 14 implies that capital and technology shocks are positively correlated. The key parameter is $k_\alpha$, which controls the elasticity of investments to productivity shocks, $\partial(\ln K_t^m)/\partial\widetilde{\alpha}_t^m = (1-1/M)\beta k_\alpha$. $k_\alpha$ is positive, meaning that funds flow to the most productive firms, and monotonically increasing with the quality of information. It starts from zero when there is no information ($\mu_t^m$ is infinite and $X_t^m = 0$), so capital is allocated independently from productivity shocks, and reaches $1/(1-\beta)$ under perfect information ($\mu_t^m = q/(1-q)$ and $X_t^m$ is infinite), so the elasticity coincides with that of the first best.

## 5.2 Impact of Noisiness on Properties of the Economy

In this section, we describe how information about firms influences real and financial aspects of the economy, holding income fixed. The following lemma characterizes the allocation of capital in terms of

efficiency and concentration.

**Lemma 5** *When information is more accurate (noisiness is lower), investments are more responsive to productivity shocks, TFP is higher and capital and profits are more concentrated across firms.*

Better-informed agents distribute capital more efficiently across firms, leading to a higher elasticity of investments to productivity shocks, $\partial \ln K_t^m / \partial \widetilde{\alpha}_t^m$. This superior efficiency translates into higher TFP, defined from the following economy-wide production function:

$$\begin{aligned} E(\widetilde{G}_{t+1}) &= ML^{1-\beta} E[(\widetilde{A}_t^m K_t^m)^\beta] \\ &= ML^{1-\beta} E(\widetilde{A}_t^{m\beta}) E(K_t^m)^\beta \exp[Cov(\beta \widetilde{a}_t^m z, \beta \Delta k_t^m z) - \beta(1-\beta) Var(\Delta k_t^m)/2]. \end{aligned}$$

We interpret the factor $\exp[Cov(\beta \widetilde{a}_t^m z, \beta \Delta k_t^m z) - \beta(1-\beta) Var(\Delta k_t^m)/2]$ as TFP. It captures the additional output obtained from distributing capital in relation to productivity shocks, in comparison to an economy in which capital is arbitrarily allocated. In the lemma, the concentration of economic activity is measured using Herfindahl indices, $Her(K_t^m) \equiv E(K_t^{m2})/[E(K_t^m)]^2$ and $Her(\widetilde{\Pi}_{t+1}^m) \equiv E(\widetilde{\Pi}_{t+1}^{m2})/[E(\widetilde{\Pi}_{t+1}^m)]^2$. When agents invest more selectively, they channel more (less) capital to the more (less) productive firms. As a result, fewer firms account for a larger fraction of the economy's stock of capital. Profits tend to be even more concentrated than capital because they compound the effect of a high productivity shock with that of a large capital stock. The next lemma presents the impact of noisiness on the next generation's expected income, $E(w_{t+1})$.

**Lemma 6** *Income is larger on average in the next period when information is more accurate (noisiness is lower), for a given level of current income.*

More accurate information leads to more efficient investments and hence to a larger supply of intermediate goods on average in the subsequent period. This in turn increases the marginal product of labor and the next generation's average income. We turn to the impact of noisiness on wealth inequality.

**Lemma 7** *Wealth inequality widens at first and then narrows as information improves (noisiness declines).*

Final wealth i.e., consumption $\widetilde{g}_{l,t+1}$, is unequal because agents, guided by their private signals, choose different portfolios. Two forces work in opposite directions when information improves. On one hand, agents put more weight on their private signals relative to public information, which tends

to increase portfolio heterogeneity. On the other hand, idiosyncratic signal errors shrink so private signals, and therefore portfolios, are less diverse across agents.[22] The first effect tends to dominate for low precisions (high noisiness) and the second for high precision (low noisiness), so inequality is non-monotonic in precision. We conclude with three financial variables, the trading intensity, stock market liquidity and the volatility of stock returns.

**Lemma 8** *Trading and liquidity on the equity market intensify at first and then weaken as information improves (noisiness declines).*

The value of shares traded equals $\sum_{m=1}^{M} \int_l |f_{l,t}^m w_t|/2$ where the factor 2 avoids double counting matching buys and sells. We measure the trading intensity as the share turnover, defined as the ratio of the value of shares traded to the total capitalization of the market, $\sum_{m=1}^{M} K_t^m$. The logic of Lemma 8 mimics that of Lemma 7 on wealth inequality. Agents trade because they disagree, and their disagreement is a source of inequality. More accurate information leads, on one hand, to more disagreement because agents use their private signals more aggressively, but on the other hand, to more consensual private signals. The resulting relation is non-monotonic.

We use the inverse of sensitivity of stock prices to (uninformative) noise shocks, $1/\left(\partial(\ln K_t^m)/\partial(\widetilde{\theta}_t^m z)\right) = 1/\left((1-1/M)k_\alpha(\mu_t^m)\mu_t^m\right)$, to capture liquidity as is common models with asymmetric information. As the formula makes clear, there are two components to liquidity. The first reflects the sensitivity to noise shocks relative to that of technology shocks (the $\mu_t^m$ term). Thanks to this factor, liquidity tends to improves when information is more accurate. The second component is the sensitivity to technology shocks (the $k_\alpha$ term), which, from Lemma 5, rises with information accuracy, thereby reducing liquidity. As a result, liquidity is non-monotonic in accuracy. The first factor (relative sensitivity) tends to dominate for low precision levels (high noisiness) and the second for high levels. The final lemma considers volatility.

**Lemma 9** *When information is more accurate (noisiness is lower), stocks' prices are more volatile, while the idiosyncratic and total volatility of their returns are lower. In contrast, the volatility of the market is unchanged.*

[22] According to equation 11 (substituting $X_t$ for $x_t^m$ to obtain equilibrium portfolio weights), an agent's portfolio weights are a function of $(X_t/h(X_t))\Delta s_{l,t}^m = (X_t/h(X_t))\Delta\widetilde{\varepsilon}_{l,t}^m$+other terms. When $X_t$ grows ($\mu_t$ falls), on one hand the ratio of the precision of private signals to the total precision, $X_t/h(X_t) = (\mu_t(1-q)/q-1)^{-1}$, rises, but on the other hand $var(\widetilde{\varepsilon}_{l,t}^m) \equiv 1/X_t$ falls. The two effects exactly cancel out when $\mu$ is such that $X(\mu) + 1/(\mu^2\sigma_\theta^2) = 1/(\beta^2\sigma_\alpha^2)$.

Stock prices fluctuate more as they incorporate technology shocks more fully. Returns, which absorb residual shocks, fluctuate less, whether fluctuations are measured as total or idiosyncratic volatility. Since the market return (price), in contrast, does not see its volatility change, a rise (decline) in the cross-correlation of stock returns (prices) offsets the reduction (rise) in individual stock volatility.[23]

### 5.3 Information Acquisition

We turn to the information acquisition decisions. The following lemma characterizes how much free time an investor devotes to learning about productivity shocks for an arbitrary level of stock price noisiness $\mu_t^m$, and given her income $w_t$.

**Lemma 10** *Let $\mu_t^m$ $(> \frac{q}{1-q})$ be the noisiness of stock $m$'s price. Investors set the precision of their private signal about stock $m$, $x_t^m$, such that*

$$\rho(\varphi(w_t))C'\left(x_t^m\right) = \tau(\varphi(w_t))\frac{M-1}{2M\beta^2\sigma_a^2}\frac{1}{\left(H(\mu_t^m)+x_t^m\right)^2} + o(1). \tag{17}$$

Investors choose a signal precision that equates the marginal benefit of information to its marginal cost, taking into account how much is revealed through stock prices. The left hand side of equation 17 represents the marginal cost and can be interpreted as follows. Increasing the precision of a signal from $x$ to $x+\delta$ requires cutting leisure time by $C'(x)\delta$ units and suffering a utility loss of $\frac{\partial U}{\partial j}C'(x)\delta$. The same loss would occur if the consumption of the final good were to fall by $\frac{\partial U}{\partial j}C'(x)\delta/\frac{\partial U}{\partial g}$ units. Thus, the left hand side of equation 17 measures the utility cost, denominated in units of the final good, of a marginal increase in the signal precision. This cost depends on income through the coefficient $\rho(\varphi(w_t))$, which measures the marginal rate of substitution between goods and leisure in a neighborhood of consumption. This coefficient, and therefore the cost of information, increase with income because of a *substitution effect*: wealthier agents invest more, hence consume more of the final good, which decreases its marginal utility and makes leisure more enjoyable.

The right hand side of equation 17 represents the utility benefit from a marginal increase in precision, again denominated in units of the final good. This benefit has the following properties. First, it rises when public information is less accurate – so private information acts as a substitute for public information. This happens when priors are less precise ($\sigma_\alpha^2$ larger) or when stock prices are less informative ($\mu_t^m$

[23] Both the market's total capitalization, $\sum_{m=1}^M K_t^m = Lw_t$, and its average return, $\frac{1}{M}\sum_{m=1}^M r_t^m z = \beta\overline{a}_t^m z$, have a constant volatility, respectively 0 and $\beta^2\sigma_a^2 z/M$.

or $\sigma_\theta^2$ larger). Indeed, stock prices reveal private signals, albeit partially, thereby limiting investors' ability to appropriate the full benefit from their information expenditures (the *ex ante* disincentive effect). Private information is more valuable when it is easier to conceal, i.e. when the price system is more noisy. Second, the benefit of private information decreases with the conditional variance of productivity shocks $\sigma_a^2$ because agents tilt less their portfolio weights away from equal shares. Last but not least, it rises with investors' income through their absolute risk tolerance, $\tau$. Indeed, discriminating across firms is more valuable when one has more to invest. Thanks to its non-rival nature, information can be applied to every dollar of investment without requiring its cost to be incurred repeatedly. Putting it differently, information generates *increasing returns with respect to the scale of investments*, captured by $\tau(\varphi(w))$.

Equation 17 admits a unique solution and implies that signal precisions are identical across agents for any stock $m$ ($x_{l,t}^m = X_t^m$ for all $l$).[24] The properties of $x_t^m$ follow from those of the marginal cost and benefit of information. $x_t^m$ rises when $\sigma_\alpha^2$, $\mu_t^m$ and $\sigma_\theta^2$ are larger, and when $\sigma_a^2$ and $C'$ are lower. Most of these properties obtain in the usual framework with exponential utility, normally distributed random variables and a riskless asset (e.g. Verrecchia (1982)).[25] The influence of income on the signal precision depends on which of the marginal rate of substitution or risk tolerance is the more sensitive to income, as outlined in Lemma 10 below.[26] The following proposition characterizes the degree of noisiness in equilibrium, $\mu_t^m$, for a given level of income $w_t$.

[24] Equation 17 admits a unique solution because its left hand side is monotonically increasing in $x_t^m$ starting from zero ($C'(0) = 0$ by assumption), while its right hand side is monotonically decreasing towards zero.

[25] In an economy similar to ours except that i) preferences display constant absolute risk aversion with a coefficient of absolute risk tolerance $\tau$, ii) stocks have normally distributed payoffs with variance $\sigma_\Pi^2$ and iii) a riskless asset with gross return $R^f$ is available, the equilibrium precision of private signals solves $2R^f C'(x_t) = \tau/(H_t + x_t)$ where $H_t \equiv 1/\sigma_\Pi^2 + 1/(\mu_t^2\sigma_\Theta^2)$ and $\sigma_\Theta^2$ is the variance of noise trading. From this equation, $x_t$ rises when $\sigma_\Pi^2$, $\tau$ or $\mu_t^2\sigma_\Theta^2$ increase or when $C$ decreases.

[26] The impact on the signal precision $x_t^m$ of the factor share of intermediate goods, $\beta$, is complex. First, a lower $\beta$ reduces investors' share of GDP and their consumption (the $\varphi(w_t)$ term), which enhances the marginal utility of final goods so both $\rho$ and $\tau$ increase. Second, a lower $\beta$ implies that stocks are less sensitive to productivity shocks. These shocks have a component that can be learnt ($\widetilde{\alpha}_t^m$) and one that cannot ($\widetilde{a}_t^m - \widetilde{\alpha}_t^m$) so the implications are twofold. On the one hand, a lower $\beta$ means that the average productivity shock $\widetilde{\alpha}_t^m$ has a smaller impact on a firm's profit so learning about it is less valuable (the term $1/\beta^2\sigma_\alpha^2$ embedded in $H(\mu_t^m)$ on the right hand side of equation 17). On the other hand, it implies that stocks are less risky so investors trade them more aggressively, which makes information more valuable (the $\beta^2\sigma_a^2$ on the right hand side of the equation). The net effect of $\beta$ depends on the relative magnitude of these effects.

**Proposition 11** *In equilibrium, the noisiness of stock prices, $\mu_t$, is the unique solution to:*

$$\rho(\varphi(w_t))C'\left(\frac{H(\mu_t)}{\frac{1-q}{q}\mu_t - 1}\right) = \tau(\varphi(w_t))\frac{M-1}{2M\beta^2\sigma_a^2}\left(\frac{1-\frac{q}{(1-q)\mu_t}}{H(\mu_t)}\right)^2 + o(1). \tag{18}$$

The noisiness of prices in equilibrium is determined by observing that the individual and average precisions, $x_t^m$ and $X_t^m$, coincide since agents all choose the same precisions, and by substituting equation 16 which relates $X_t^m$ to $\mu_t^m$ into the first-order condition 17 (this procedure amounts to searching for a fixed point to the system of equations, $X_t^m = x(w_t, \{X_t^m\})$ for $m = 1$ to $M$). The resulting noisiness and average precisions are identical across stocks so we drop the superscript $m$ from now on ($X_t^m \equiv X_t$ and $\mu_t^m = \mu_t$ for all $m$). This implies further that individual precisions are identical across stocks ($x_t^m = x_t$ for all $m$). Equation 18 admits a unique solution $\mu_t$ for any level of income $w_t$, because its left hand side is monotonically decreasing in $\mu_t$ and spans the entire positive real line, while its right hand side is monotonically increasing. It is illustrated in figure 3.

The properties of the average precision $X_t$ are identical to those of individual precisions $x_t$, discussed above. Those of the equilibrium noisiness $\mu_t$ follow. It decreases (i.e. stock prices are more informative) when priors are more accurate ($\sigma_\alpha^2$ smaller), when the variance of noise trades $\sigma_\theta^2$ is larger, when the conditional variance of productivity shocks $\sigma_a^2$ or the marginal cost of information $C'$ are lower. In contrast, $\mu_t$ increases with the fraction of noise traders $q$. This is because $q$ has a direct effect on $\mu_t$ in equilibrium that dominates its indirect effect through $X_t$. We conclude this section with an analysis of the influence of income on $X_t$.

**Lemma 12** *If $\tau/\rho$ is an increasing (decreasing) function of consumption, then the noisiness of stock prices falls (rises) with income.*

We observed in the discussion following Lemma 10 that current income increases both the marginal cost of information (through a substitution effect) and its marginal benefit (through a scale effect). The impact of income on the equilibrium precision of information depends on which of these two effects dominates. If the scale effect dominates, i.e. the marginal benefit rises with income faster than the marginal cost does ($\tau/\rho$ increasing in consumption), then agents collect more information as they grow wealthier so $d\mu_t/dw_t < 0$. If instead the substitution effect dominates ($\tau/\rho$ decreasing in consumption), then agents collect less information so $d\mu_t/dw_t > 0$. Under CES utility for example, information

improves with income if $\sigma > 0$, but deteriorates if $\sigma < 0$. The substitution and scale effects offset each other exactly under Cobb-Douglas utility ($\sigma = 0$, or constant relative risk aversion). In that case, income has no impact on the quality of information. Under constant absolute risk aversion – preferences that are usually assumed in rational expectations models of trading under asymmetric information (e.g. $U(g, j) = (-\exp(-\tau g))v(j)$ or $U(g, j) = -\exp(-\tau g) + v(j)$), there is no scale effect so the substitution effect works alone. As a result, the precision of information is a decreasing function of income in these models. Figure 4 (top left panel) illustrates lemma 12.

# 6 The Role of the Stock Market

This section delves into the information processing role of the stock market. Stock prices, by aggregating dispersed private signals about technology shocks into public signals, affect capital efficiency in two conflicting ways. On one hand, they help investors evaluate firms and deploy their capital. As such, the stock market can be viewed as a mechanism for sharing costly private information. Importantly, this mechanism is incentive-compatible and inexpensive since investors 'communicate' through their trades.[27] On the other hand, the very existence of informative prices undermines the incentive to collect costly information in the first place. Indeed, investors' cannot appropriate the full benefit of their signals as they are leaked to competitors through prices.[28] Thus, informative stock prices have an impact that is beneficial *ex post* but detrimental *ex ante* to capital efficiency. Noise trading plays a crucial part in this tradeoff as its intensity determines how much information gets revealed. By varying the fraction of noise traders $q$, one can get a sense of the net informational contribution of the stock market, as in the next lemma.

**Lemma 13** *When the fraction of noise traders $q$ decreases, less information is produced but more is shared through stock prices. The net effect is an improvement in total information, $H_t + X_t$, and in the efficiency of investments, captured by a higher elasticity, $k_{\alpha t}$.*

[27] This effect can best be understood by comparison to a fictitious economy in which agents collect the same private signals but stock prices do not reveal any of their content. In such an economy, the average precision $X(\mu_t^m)$ is the same as in the 'normal' economy, but an investor's total precision is lower because the precision of the price signal, $1/(\mu_t^{m2}\sigma_\theta^2)$, is lost – the total precision equals $1/(\beta^2\sigma_\alpha^2) + X(\mu_t^m) < H(\mu_t^m) + X(\mu_t^m)$. Accordingly, the elasticity of investments to productivity shocks falls to $(1 - 1/(1 + \beta^2\sigma_a^2 X(\mu_t^m)))/(1 - \beta)$ which is below $k_\alpha(\mu_t^m)$. The allocation of capital is not as efficient though the same private signals are produced because investors do not share them.

[28] Again, there is no such incentive problem in Greenwood and Jovanovic (1990) because agents are endowed with a private signal about the state of the economy (a project).

On the one hand, for a given precision of private signals, more information is conveyed through prices as noise trading weakens (the *ex post* information sharing effect) so capital is more efficiently deployed. Formally, $\partial\mu_t/\partial q > 0$, $\partial H(\mu_t)/\partial q < 0$ and $\partial k_\alpha(\mu_t)/\partial q < 0$ holding the average precision $X_t$ fixed, and using respectively equations 16, 12, 15 and 20. On the other hand, agents collect less private information (the *ex ante* disincentive effect). This dampens the beneficial influence that information sharing has on capital efficiency, but does not reverse it. Formally, $d\mu_t/dq > 0$, $d(H(\mu_t)+X(\mu_t))/dq < 0$ and $dk_\alpha(\mu_t)/dq < 0$. Consider for example, the net effect on investors' total precision, $H(\mu_t) + X(\mu_t)$:

$$\underset{<0}{\frac{d\left(H(\mu_t)+X(\mu_t)\right)}{dq}} = \underbrace{\underset{<0}{\frac{\partial H_t}{\partial \mu_t}_{X_t \text{ fixed}}} * \underset{>0}{\frac{\partial \mu_t}{\partial q}_{X_t \text{ fixed}}}}_{<0} + \underbrace{\underset{<0}{\frac{\partial H_t}{\partial \mu_t}_{X_t \text{ fixed}}} * \underset{<0}{\frac{\partial \mu_t}{\partial X_t}_{q \text{ fixed}}} * \underset{>0}{\frac{dX_t}{dq}} + \underset{>0}{\frac{dX_t}{dq}}}_{>0}.$$

*Ex post* information sharing  *Ex ante* disincentive

The *ex post* information sharing effect more than compensates for the *ex ante* disincentive effect.[29]

The following lemma relates the allocation of capital achieved through the stock market to the first best. Since noise trading was introduced into the stock market economy to avoid the Grossman-Stiglitz paradox, we make the comparison in the limiting situation in which noise vanishes, i.e. as the fraction of noise traders goes to zero.

**Lemma 14** *The allocation of capital achieved through the stock market converges to the first best allocation as the fraction of noise traders goes to zero:*

$$\lim_{\substack{q\to 0\\ q>0}} k_t^m = k_t^{mFB} \qquad \textit{for } m = 1, ..., M.$$

The lemma establishes that the capital allocation achieved through the stock market can be made arbitrarily close to the first best allocation by reducing the fraction of noise traders $q$.[30] It follows that the dynamics of income, as described in the next section, can also be made arbitrarily close to those obtained in the first best economy.[31] Lemmas 13 and 14 are illustrated in figure 5 which displays $\mu_t$,

[29] Only under a linear information cost do these two effects exactly balance out. In that case, the left-hand side of equation 18 is constant, so must be the right-hand side, which implies that the total precision $H(\mu_t) + X(\mu_t)$ is constant regardless of $q$.

[30] However, $q$ cannot exactly equal zero, else there is no equilibrium (the Grosssman-Stiglitz paradox).

[31] The steady-state level of income and its transitory growth rate converge to those achieved in the first best: $\lim_{\substack{q\to 0\\ q>0}} w^* = w^{FB}$ and $\lim_{\substack{q\to 0\\ q>0}} \Gamma(w_t) = \Gamma(w_t)^{FB}$.

$X_t$, $H_t + X_t$ and $k_{\alpha t}$ as a function $q$ under CES utility.

# 7 Dynamics

In this section, we tie together learning, investments and income, analyze the evolution of the economy along its average path and relate the model's predictions to the empirical evidence.

## 7.1 Observable Properties of the Growth Path

The following proposition determines the dynamics of income by combining lemmas 6 and 12.

**Proposition 15** *Average income evolves according to the following equation:*

$$E(\widetilde{w}_{t+1}) = \Lambda \exp\left(\lambda(w_t) z^2\right) w_t^{\beta}, \tag{19}$$

*where*

$$\lambda(w_t) \equiv \frac{M-1}{M}\beta^2 \left(k_\alpha(\mu_t)\beta\sigma_\alpha^2 + \frac{k_\alpha(\mu_t)^2}{2}(\beta^2\sigma_\alpha^2 + \mu_t^2\sigma_\theta^2)\right) + o(1) > 0, \tag{20}$$

*and $\Lambda$, $k_\alpha$ and $\mu_t = \mu(w_t)$ are defined respectively in equations 6, 15, and 18.*

- *The economy converges to a steady-state. The steady-state level of income $w^*$ is given by:*

$$w^* = w^{FB} \exp\left(-\frac{\lambda^{FB} - \lambda\left((1-\beta)^{1/(1-\beta)}M\right)}{1-\beta} z^2\right). \tag{21}$$

- *If $\tau/\rho$ is an increasing (decreasing) function of consumption, then $\lambda$ increases (decreases) with income. Moreover, if there exists a scalar $u$ such that $\lim_{g\to u} \tau(g)/\rho(g) = \infty$, then $\lim_{w_t\to u} \lambda(w_t) = \lambda^{FB} + \lambda^{Noise}$ where $\lambda^{Noise} \equiv (1-1/M)(\beta q/(1-q)/(1-\beta))^2\sigma_\theta^2/2$. For example under CES preferences, $\lambda$ is an increasing function of income and $\lim_{w_t\to\infty} \lambda(w_t) = \lambda^{FB} + \lambda^{Noise}$ if $\sigma > 0$, while $\lambda$ is a decreasing function and $\lim_{w_t\to 0} \lambda(w_t) = \lambda^{FB} + \lambda^{Noise}$ if $\sigma < 0$.*

To a first approximation (at the order 0 in $z$), the dynamics of income are similar to those obtained under the first best: income grows at a declining rate until it reaches a steady-state $w^*$ (assuming the wage is initially below $w^*$). Thus, the dynamics of income continue to be dominated by the neoclassical force of diminishing returns to capital – learning only generates a deviation of order $z^2$ from the neoclassical path. Though this is the case by construction in our model – learning about productivity shocks generates benefits that are small since we *assume* these shocks to be small, we conjecture that this property extends to large shocks since income admits the first best as an upper bound (starting from the same arbitrary level of income, income in the next period is lower than in the first best in which capital is more efficiently allocated) and income in the first best eventually reaches a steady-state.

Proposition 15 is illustrated in figure 6 which displays the law of motion of income along the economy's average path under CES utility (equation 19). The solid (dotted) curve corresponds to $\sigma = 0.5$ ($\sigma = -0.5$), in which case information improves (deteriorates) with income. The steady-state is located at their intersection with the 45° line (solid line). If initial income $w_0$ is below (above) $w^*$, then the wage increases (decreases) until it reaches $w^*$.

The effect of learning on income is captured by the function $\lambda$, illustrated in the bottom right panel of figure 4. We note that income may be higher in steady-state in the stock market economy compared to the first best economy because of the presence of noise (which was assumed away in the first best). Indeed, noise trading may be beneficial to income in spite of making investments less efficient because it increases the variability of the capital allocation and therefore the average income, a convex function thereof, through a Jensen inequality effect (positive noise shocks increase output more than negative shocks decrease it). This effect, reflected in the term $\mu_t^2\sigma_\theta^2$ in equation 20, vanishes as the intensity of noise trading $q$ approaches zero. If it were not for this direct influence of noise trading, steady-state income would always be lower than in the first best. The growth rate of income during the transition to the steady-state, $\Gamma(w_t) \equiv E(\widetilde{w}_{t+1})/w_t$, differs from the first best, by a factor $\exp\left[-(\lambda^{FB} - \lambda(w_t))z^2\right]$, and is lower than in the first best when the intensity of noise trading is weak.

Figure 7 depicts $\Gamma(w_t)$ for various utility functions as well as in the first best economy. When the scale effect of information dominates the substitution effect (e.g., when $\sigma > 0$ under CES utility), investors collect more information as the economy grows, which contributes to growth. As a result, the growth rate of income, while it declines, does so less quickly than in the first best:

$$\frac{d\ln\Gamma(w_t)}{d\ln w_t} = -(1-\beta) + \frac{d\lambda(w_t)}{d\ln w_t}z^2 > -(1-\beta),$$

where $-(1-\beta) = d\ln\Gamma^{FB}(w_t)/d\ln w_t$ is the change in the growth rate of income in the first best. That is, the growth rate of income is typically (i.e. when the intensity of noise trading is weak) lower than in the first best (capital is not as efficiently deployed) but it declines less quickly (the allocation of capital improves over time). Thus in this case, learning has a transitory beneficial effect on growth, that mitigates the negative neoclassical force. When the scale effect of information dominates the substitution effect (e.g., when $\sigma < 0$ under CES utility), investors collect less information as the economy grows,

which slows down growth. So, the growth rate of income falls at a faster rate than in the first best:

$$\frac{d\ln\Gamma(w_t)}{d\ln w_t} = -(1-\beta) + \frac{d\lambda(w_t)}{d\ln w_t} z^2 < -(1-\beta).$$

Here the growth rate of income is typically lower than in the first best (capital is not as efficiently deployed) and declines faster (the allocation of capital worsens over time).

We derive next various observable properties of the economy during its transition to the steady-state (for an initial wage below its steady-state level), by combining Lemmas 5 to 9 with Lemma 12. They are summarized in the following proposition.

**Proposition 16** *Suppose that the scale effect of information dominates the substitution effect (e.g. $\sigma > 0$ under CES utility). As the economy grows:*

- *The elasticity of investments to productivity shocks and TFP increase,*
- *Capital and profits are more concentrated across firms,*
- *Income inequality widens at first and then narrows,*
- *Trading on the equity market intensifies at first and then weakens,*
- *Stock market liquidity improves at first and then deteriorates,*
- *The volatility of stock prices rises, the idiosyncratic and total volatility of stock returns fall and the volatility of the market is constant.*

*The opposite patterns obtain if instead the substitution effect dominates (e.g. $\sigma < 0$ under CES utility).*

The predictions of Proposition 16 for a growing economy when the scale effect dominates can be interpreted as follows. (i) Capital is more efficiently allocated across firms, i.e. more (less) capital is channelled to more (less) productive firms. This superior efficiency leads to higher TFP, even though there is no technological progress. (ii) The economy specializes, as agents invest more selectively, leading capital and profits to become more concentrated across firms. (iii) Income inequality follows a "Kuznets curve", widening at first and then narrowing. (iv) Stock market liquidity and the share turnover increase at first and then decrease. Inequality, liquidity and turnover display similar non-monotonic behaviors because all three are driven by the extent to which investors disagree about stocks. At the early stage of development, agents follow mostly price signals since private signals are imprecise, so disagreement is low. Agents rely more on private signals as their precision improves, so disagreement rises with income.

A consensus reemerges beyond a level of income because private signals that are more precise are also more similar. (v) The volatility of stock prices rises with income as they track technology shocks more closely. As a result, stock returns, which absorb residual shocks, fluctuate less, as reflected in their idiosyncratic and total volatility. In contrast, the volatility of the market is constant. It follows that the cross-correlation of stock prices falls, while that of stock returns rises to offset, respectively, the rise in the volatility of individual stock prices and the reduction in the volatility of individual stock returns.

### 7.2 Evidence

Several aspects of the model are broadly consistent with the evidence, assuming that the scale effect of information dominates the substitution effect. First, Levine and Zervos (1998), Rousseau and Wachtel (2000) and Carlin and Mayer (2003) document that income grows faster in countries with better functioning stock markets.[32] Atje and Jovanovic (1993) estimate that this growth effect is permanent, but Harris (1997) finds that it is only transitory after controlling for possible endogeneity problems.[33] These observations support the notion developed in section 6 that the stock market, by aggregating and transmitting private information, contributes to the level of income in the long-run and to its growth rate during the transition. Moreover, Carlin and Mayer (2003) document that industries grow faster in countries with more developed stock markets, and that this relationship is stronger for industries with high R&D investments and skilled labor, such as new technologies. These findings are consistent with the notion that the stock market is useful for investing in innovative risky technologies about which opinions diverge widely.

Second, Proposition 18 predicts that allocative efficiency and TFP grow with income. Wurgler (2000) constructs cross-country estimates of the elasticity of investments to value added, our parameter $k_\alpha$. He finds that this elasticity increases with the country's degree of financial development, and in particular with the informativeness of its stock market. That is, countries with more informative stock markets increase investments more in their growing industries, and decrease investments more in their declining industries, than countries with less informative stock markets. These countries also tend to

[32] Levine and Zervos (1998) and Rousseau and Wachtel (2000) proxy for stock market development using measures of market capitalization trading volume. Carlin and Mayer (2003) use accounting standards.

[33] Aghion, Howitt and Mayer-Foulkes (2005) also document that financial development only has a transitory growth effect for sufficiently advanced economies using measures of financial intermediation such as private credit. They propose a model of agency problems and credit constraints to explain their findings.

display higher TFP. Indeed, Levine and Zervos (1998) show that stock markets promote growth in total factor productivity.[34] We stress that TFP grows in our model though there is no technological progress (the distribution of productivity shocks and the cost of information are stationary), thanks to a more efficient allocation of capital.

Third, Proposition 16 implies that the economy specializes as it grows. Empirically, Imbs and Wacziarg (2003) document that countries go through two stages of sectoral diversification. Diversification increases at first, but beyond a certain level of income, the process is reversed and economic activity starts concentrating. This pattern is consistent with our model to the extent that it applies to more advanced economies – an extension presented in the next section shows that more information is produced as incomes grows, only if income is above a threshold. In a similar vein, Kalemli-Ozcan, Sørensen and Yosha (2003) report that industrial specialization in a sample of developed countries is positively related to the share of the financial sector in GDP. This fact too is consistent with Proposition 16 to the extent that this share is positively related with information expenditures about public companies.

Fourth, Proposition 16 predicts that wealth inequality conforms to a "Kuznets curve", widening at first and then narrowing. In his seminal study, Kuznets (1955) found support for his hypothesis using both cross-country and time-series data. This pattern has been extensively examined since using new data and statistical techniques and the evidence is now mixed (e.g. Acemoglu and Robinson (2002) for a review of the evidence).

Fifth, according to Proposition 16, the trading activity and liquidity are inverted U-shape functions of income. Empirically, Levine and Zervos (1998) and Rousseau and Wachtel (2000) report that the share turnover on the stock market is positively related to output growth but do not document (nor test for) a non-monotonic pattern.

Finally, Proposition 16 implies that the volatility of stock prices rises, the idiosyncratic and total

[34] Wurgler (2000) uses a proxy for informativeness developed by Morck, Yeung and Yu (2000). They measure the extent to which stocks move together and argue that prices move in a more unsynchronized manner when they incorporate more firm-specific information. This is indeed the case in the present model (see Lemma 9). Durnev, Morck and Yeung (2004) and Durnev, Morck, Yeung and Zarowin (2003) confirm that the synchronicity measure is related to accounting estimates of stock price informativeness as well as to the efficiency of corporate investments captured by Tobin's $q$. These findings are consistent with the observations that variations in the allocation of resources account for a large fraction of the cross-country differences in TFP (Restuccia and Rogerson (2008) and Hsieh and Klenow (2009)), and that financial liberalizations are associated with increases in TFP (see for example Bekaert, Harvey and Lundblad (2011) and the references therein).

volatility of stock returns fall, the volatility of the market is constant, the cross-correlation of stock prices falls, and the cross-correlation of stock returns rises. Empirically, Morck, Yeung and Yu (2000) show that stock prices are less synchronous in richer economies. Campbell et al. (2001) document a strong increase in idiosyncratic return volatility in the U.S. from 1962 to 1997, while the volatility of the market remained stable.[35]

# 8 No-Information Trap

In the model, agents always collect private signals. This is because the cost of learning is assumed to satisfy $C'(0) = 0$, i.e. an infinitesimal amount of private information is costless. Empirically however, financial institutions only emerge once a critical level of income has been reached. In this section, we assume that $C'(0) > 0$ and show that information production only takes place for sufficiently developed economies. The following proposition describes how investors' learning decisions are altered.

**Proposition 17**
*Suppose that $C'(0) > 0$. Investors collect information if and only if $\frac{\tau(\varphi(w_t))}{\rho(\varphi(w_t))} > \frac{2M\sigma_a^2(\sigma_\alpha^2)^2}{(M-1)\beta^2}C'(0)$. In that case, the equilibrium noisiness is the unique solution to equation 18.*

If $C'(0) > 0$, then equation 18 that determines the equilibrium noisiness may admit no solution. For example, when $\rho(\varphi(w_t)$ is large relative to $\tau(\varphi(w_t))$, the marginal cost of information (the left-hand side of equation 18) may exceed its marginal benefit (the right-hand side) for all levels of noisiness. In that case, no information is collected in equilibrium as it is too costly to be profitable. The condition on $\tau/\rho$ for learning to take place leads to a condition on income. This can easily be seen in the case of CES utility, as the following lemma shows.

**Lemma 18** *Suppose that $C'(0) > 0$ and that utility is CES. Let*

$$\underline{w} \equiv \left(\frac{1-\varpi}{2\varpi}\left(\sqrt{1+\frac{8\varpi(1-\sigma)M\sigma_a^2(\sigma_\alpha^2)^2}{(1-\varpi)(M-1)\beta^2}C'(0)}-1\right)\right)^{1/\sigma}.$$

*When $\sigma > 0$, investors collect information if and only if their income exceeds the threshold $\underline{w}$. When instead $\sigma < 0$, they collect information if and only if their income is below the threshold $\underline{w}$.*

[35]The model's prediction on idiosyncratic stock return volatility appears to be at odds with the evidence reported in Campbell et al. (2001). We conjecture that increasing the number of trading rounds within each period may reverse it (without affecting the other predictions). With multiple trading rounds, when more information is produced, the current price tracks future dividends and prices more closely, thereby reducing the return volatility (as in the current model). But the volatility of future prices also increases since future prices track more closely dividends even further into the future.

The threshold $\underline{w}$ is the unique income level such that $\tau(\varphi(\underline{w}))/\rho(\varphi(\underline{w})) = C'(0)2M\sigma_a^2(\sigma_\alpha^2)^2/(M-1)/\beta^2$. When $\sigma > 0$, the scale effect of information dominates so wealthier investors collect information only if their income $w_t$ is large enough. When $\sigma < 0$, the substitution effect dominates so investors stop collecting information when their income exceeds $\underline{w}$. The properties of $\underline{w}$ mirror those of the equilibrium precision $X_t$ : the factors that increase (decrease) $X_t$ tend to decrease (increase) $\underline{w}$. Assuming that $\sigma > 0$ and that $w^* > \underline{w} > w_0$ where $w_0$ is the initial level of income, the economy goes through two stages of development. At first, it behaves as a standard neoclassical economy with no information. Once income reaches a threshold, agents start collecting privates signals and growth accelerates by a factor $\exp\left(\lambda(w_t)z^2\right)$. Thus in this case, the stock market only operates as an information processor if the economy is sufficiently developed. If instead $w_0 < w^* < \underline{w}$, then no information is ever collected.

# 9 Conclusion

This paper presents a fully integrated model of information acquisition and dissemination through prices, capital allocation and economic growth. It does so by combining, under a small risk approximation, two standard frameworks, the neoclassical overlapping generations growth model and the noisy rational expectations stock market model. The central argument in the paper is that the stock market provides a mechanism for sharing information dispersed among many investors. This mechanism is particularly useful when the information is produced privately and at a cost, because eliciting effort and truthful disclosure from investors may be difficult under these circumstances. On the stock market, investors effectively communicate through their trades. At the hart of the model lies a tension between the benefit of sharing information and the incentive to collect it in the first place. Noise trading resolves this tension by ensuring that acquiring private information remains profitable even though it gets partially revealed through prices.

The model yields several implications. First, the information sharing benefit outweighs the disincentive cost. That is, agents, though they reduce the precision of their private signals in response to a decline in the intensity of noise trading, are nevertheless better informed on the whole thanks to the increased accuracy of stock prices. The allocation of capital improves, thereby increasing TFP, and moreover, converges to the first best as the intensity of noise trading approaches zero. Second, the

learning process thus has no bearing on long run growth – it does not counter the diminishing returns to capital, but it does improve the long run level of income and therefore its transitory growth rate. Finally, the model delivers several predictions on the evolution of real and financial variables, including capital efficiency, TFP, industrial specialization, wealth inequality, stock trading intensity, liquidity and return volatility.

The paper sheds light on a force that shapes TFP, namely the ability to learn from stock prices about exogenously determined technologies. As such, it complements existing theories of endogenous growth, as well as models of the stock market that do not focus on information frictions. But it leaves aside several interesting aspects. In particular, it does not study what influence, if any, the stock market exerts on the determination of these endowed technologies. It is conceivable for example that the information contained in stock prices, or the mere expectation that future stock prices will be informative, impact the distribution of productivity shocks and the number of available stocks. In this case, the stock market may have a permanent growth effect through its information processing role. We leave these questions for future research.

# Appendix

## Proof of Lemma 1

We solve for the capital allocation $\left\{K_t^{mFB}\right\}$ chosen by a central planner who can perfectly infer the average productivity shocks $\{\widetilde{\alpha}_t^m\}$. We first note that, when $z = 0$, there are no productivity shocks so firms are identical. In that case, given the diminishing marginal product of intermediate goods, the central planner distributes capital equally across the $M$ firms: each firm is allocated $K_t^0 \equiv Lw_t/M$ units of capital, and consumption per capita equals $g_0 \equiv \beta\widetilde{G}_{t+1}/L = M\beta L^{-\beta}K_t^{0\beta}$. When $z > 0$, firm $m$'s capital stock can therefore be expressed as $K_t^{mFB} = K_t^0 \exp(\widehat{k}_t^{mFB} z)$ where $\widehat{k}_t^{mFB}$ is determined next.

The Lagrangian for the central planer's problem is:

$$E\left[U(\widetilde{g}_{l,t+1}, 1) \mid \{\widetilde{\alpha}_t^m\}\right] + \varsigma_t^{FB}(Lw_t - \sum_{m=1}^{M} K_t^{mFB}),$$

where $\varsigma_t^{FB}$ is the Lagrange multiplier on the resource constraint and $\widetilde{g}_{l,t+1} = \beta\widetilde{G}_{t+1}/L = \sum_{m=1}^{M} \beta L^{-\beta}(\widetilde{A}_t^m K_t^{mFB})^\beta$ denotes consumption per capita. The first-order condition with respect to $K_t^{mFB}$ follows:

$$\varsigma_t^{FB} = E\left[\frac{\partial U}{\partial g}(\widetilde{g}_{l,t+1}, 1).\beta^2 L^{-\beta}\widetilde{A}_t^{m\beta} K_t^{mFB(\beta-1)} \mid \{\widetilde{\alpha}_t^m\}\right].$$

The first-order condition can be expressed as:

$$\begin{aligned}
\varsigma_t^{FB} K_t^{0(1-\beta)} L^\beta/\beta^2 = & \; E\left[\tfrac{\partial U}{\partial g}(\widetilde{g}_{l,t+1},1).\exp(\beta\widetilde{a}_t^m z + (\beta-1)\widehat{k}_t^{mFB} z) \mid \{\widetilde{\alpha}_t^m\}\right] \\
= & \; E\left[\tfrac{\partial U}{\partial g}(\widetilde{g}_{l,t+1},1).\left(1+\beta\widetilde{a}_t^m z + (\beta-1)\widehat{k}_t^{mFB} z + \tfrac{1}{2}Var(\beta\widetilde{a}_t^m z \mid \{\widetilde{\alpha}_t^m\}) + o(z)\right) \mid \{\widetilde{\alpha}_t^m\}\right] \\
= & \; E\left[\tfrac{\partial U}{\partial g}(\widetilde{g}_{l,t+1},1).\left(1+\beta\widetilde{a}_t^m z + (\beta-1)\widehat{k}_t^{mFB} z + \beta^2\sigma_a^2 z/2 + o(z)\right) \mid \{\widetilde{\alpha}_t^m\}\right].
\end{aligned}$$

We expand $\frac{\partial U}{\partial g}(\widetilde{g}_{l,t+1},1)$ in a Taylor series in a neighborhood of $z=0$, i.e. for $\widetilde{g}_{l,t+1}$ around $g_0$ :

$$\frac{\partial U}{\partial g}(\widetilde{g}_{l,t+1},1) = \frac{\partial U}{\partial g}(g_0,1) + \frac{\partial^2 U}{\partial g^2}(g_0,1)(\widetilde{g}_{l,t+1}-g_0) + o(z),$$

where

$$\begin{aligned}
\widetilde{g}_{l,t+1} - g_0 = & \sum_{m=1}^{M} \beta L^{-\beta}\left[(\widetilde{A}_t^m K_t^{mFB})^\beta - K_t^{0\beta}\right] \\
= & \; \beta L^{-\beta} K_t^{0\beta} \sum_{m=1}^{M}\left[\exp(\beta\widetilde{a}_t^m z + \beta\widehat{k}_t^{mFB} z) - 1\right] \\
= & \; \beta L^{-\beta} K_t^{0\beta} \sum_{m=1}^{M}(\beta\widetilde{a}_t^m z + \beta\widehat{k}_t^{mFB} z + \tfrac{1}{2}Var(\beta\widetilde{a}_t^m z \mid \{\widetilde{\alpha}_t^m\})) + o(z) \\
= & \; \beta L^{-\beta} K_t^{0\beta} \sum_{m=1}^{M}(\beta\widetilde{a}_t^m + \beta\widehat{k}_t^{mFB} + \beta^2\sigma_a^2/2)z + o(z).
\end{aligned}$$

As a result, the first-order condition can be written as:

$$\begin{aligned}
\varsigma_t^{FB} K_t^{0(1-\beta)} L^\beta/\beta^2 = E\Bigg[\Bigg(\tfrac{\partial U}{\partial g}(g_0,1) + \tfrac{\partial^2 U}{\partial g^2}(g_0,1)\beta L^{-\beta} K_t^{0\beta} \sum_{m=1}^{M}(\beta\widetilde{a}_t^m + \beta\widehat{k}_t^{mFB} + \tfrac{1}{2}\beta^2\sigma_a^2)z\Bigg) \\
.\left(1+\beta\widetilde{a}_t^m z + (\beta-1)\widehat{k}_t^{mFB} z + \tfrac{1}{2}\beta^2\sigma_a^2 z\right) \mid \{\widetilde{\alpha}_t^m\}\Bigg] + o(z).
\end{aligned}$$

Isolating the order-$z$ terms and denoting $\varsigma_{1t}^{FB} z$ the order-$z$ component of the Lagrange multiplier yields:

$$\begin{aligned}
\varsigma_{1t}^{FB} z K_t^{0(1-\beta)} L^\beta/\beta^2 = & \; E\Bigg[\tfrac{\partial U}{\partial g}(g_0,1)\left(\beta\widetilde{a}_t^m z + (\beta-1)\widehat{k}_t^{mFB} z + \beta^2\sigma_a^2 z/2\right) \\
& + \tfrac{\partial^2 U}{\partial g^2}(g_0,1)\beta L^{-\beta} K_t^{0\beta} \sum_{m=1}^{M}(\beta\widetilde{a}_t^m + \beta\widehat{k}_t^{mFB} + \beta^2\sigma_a^2/2)z \mid \{\widetilde{\alpha}_t^m\}\Bigg] \\
= & \; \tfrac{\partial U}{\partial g}(g_0,1)\left(\beta\widetilde{\alpha}_t^m + (\beta-1)\widehat{k}_t^{mFB} + \beta^2\sigma_a^2/2\right) \\
& + \tfrac{\partial^2 U}{\partial g^2}(g_0,1)\beta L^{-\beta} K_t^{0\beta} M(\beta\overline{\widetilde{\alpha}_t} + \beta\overline{\widehat{k}_t^{FB}} + \beta^2\sigma_a^2/2).
\end{aligned}$$

Averaging this equation across stocks yields:

$$\begin{aligned}
\varsigma_{1t}^{FB} K_t^{0(1-\beta)} L^\beta/\beta^2 = & \tfrac{\partial U}{\partial g}(g_0,1)\left(\beta\overline{\widetilde{\alpha}_t} + (\beta-1)\overline{\widehat{k}_t^{FB}} + \tfrac{1}{2}\beta^2\sigma_a^2\right) \\
& + \tfrac{\partial^2 U}{\partial g^2}(g_0,1)\beta L^{-\beta} K_t^{0\beta} M(\beta\overline{\widetilde{\alpha}_t} + \beta\overline{\widehat{k}_t^{FB}} + \tfrac{1}{2}\beta^2\sigma_a^2),
\end{aligned}$$

and subtracting it form the previous one leads to:

$$0 = \frac{\partial U}{\partial g}(g_0,1)\left(\beta\widetilde{\alpha}_t^m + (\beta-1)\widehat{k}_t^{mFB} - \beta\overline{\widetilde{\alpha}_t} - (\beta-1)\overline{\widehat{k}_t^{FB}}\right).$$

A solution to this equation is $\widehat{k}_t^{mFB} = \frac{1}{1-\beta}\Delta\beta\widetilde{\alpha}_t^m + o(1)$ since $\overline{\Delta\beta\widetilde{\alpha}_t} \equiv 0$. Therefore, $K_t^{mFB} = Lw_t/M\exp(\Delta k_t^{mFB} z)$ where $\Delta k_t^{mFB} \equiv \frac{1}{1-\beta}\Delta\beta\widetilde{\alpha}_t^m + o(1)$, as stated in lemma 1.

## Proof of Lemma 2

The number of final goods produced in the first best is:

$$\widetilde{G}_{t+1} = \sum_{m=1}^{M} L^{1-\beta}(\widetilde{A}_t^m K_t^{mFB})^\beta = L w_t^\beta M^{1-\beta} \overline{\exp\left(\beta(\widetilde{a}_t z + \Delta k_t^{FB} z)\right)}.$$

Therefore, the wage and its average equal:

$$\widetilde{w}_{t+1} = (1-\beta)\widetilde{G}_{t+1}/L = (1-\beta) w_t^\beta M^{1-\beta} \overline{\exp\left(\beta z(\widetilde{a}_t + \Delta k_t^{FB})\right)},$$

$$\text{and } E(\widetilde{w}_{t+1}) = (1-\beta) w_t^\beta M^{1-\beta} E\left[\overline{\exp\left(\beta z(\widetilde{a}_t + k_t^{FB})\right)}\right],$$

where

$$\begin{aligned} E\left[\overline{\exp\left(\beta z(\widetilde{a}_t + k_t^{FB})\right)}\right] &= E\left[\exp\left(\beta z(\widetilde{a}_t^m + \tfrac{1}{1-\beta}\Delta\beta\widetilde{\alpha}_t^m)\right)\right] + o(z) \\ &= \exp\left[\tfrac{1}{2} Var\left(\beta z(\widetilde{a}_t^m + \tfrac{1}{1-\beta}\Delta\beta\widetilde{\alpha}_t^m)\right)\right] + o(z) \\ = \exp\Big\{\tfrac{1}{2} E\Big[Var\Big(\beta z(\widetilde{a}_t^m &+ \tfrac{1}{1-\beta}\Delta\beta\widetilde{\alpha}_t^m) \mid \{\widetilde{\alpha}_t^m\}\Big)\Big] + \tfrac{1}{2} Var\Big[E\Big(\beta z(\widetilde{a}_t^m + \tfrac{1}{1-\beta}\Delta\beta\widetilde{\alpha}_t^m) \mid \{\widetilde{\alpha}_t^m\}\Big)\Big]\Big\} + o(z). \end{aligned}$$

This expression reduces to:

$$\begin{aligned} E\left[\overline{\exp\left(\beta z(\widetilde{a}_t^m + k_t^{mFB})\right)}\right] &= \exp\left\{\tfrac{1}{2} E\left[Var(\beta\widetilde{a}_t^m z \mid \{\widetilde{\alpha}_t^m\})\right] + \tfrac{1}{2} Var\left[\beta z\left(\widetilde{\alpha}_t^m + \tfrac{1}{1-\beta}\Delta\beta\widetilde{\alpha}_t^m\right)\right]\right\} + o(z) \\ &= \exp\{\tfrac{1}{2} E\left[\beta^2\sigma_a^2 z\right] + \tfrac{1}{2} Var[\beta z(\widetilde{\alpha}_t^m(1 + \tfrac{\beta}{1-\beta}\tfrac{M-1}{M}) - \tfrac{\beta}{1-\beta}\tfrac{1}{M}\sum_{\substack{m'=1 \\ m'\neq m}}^{M} \widetilde{\alpha}_t^{m'})]\} + o(z) \\ &= \exp\{\tfrac{1}{2}\beta^2\sigma_a^2 z + \tfrac{1}{2} Var[\beta z(\widetilde{\alpha}_t^m(1 + \tfrac{\beta}{1-\beta}\tfrac{M-1}{M})] + \tfrac{1}{2} Var[\tfrac{\beta}{1-\beta}\tfrac{1}{M}\sum_{\substack{m'=1 \\ m'\neq m}}^{M} \widetilde{\alpha}_t^{m'})]\} + o(z) \\ &= \exp\left\{\tfrac{1}{2}\beta^2\sigma_a^2 z + \tfrac{1}{2}\beta^2\sigma_\alpha^2 z^2(1 + \tfrac{\beta}{1-\beta}\tfrac{M-1}{M})^2 + \tfrac{1}{2}\sigma_\alpha^2 z^2(\tfrac{\beta}{1-\beta})^2\tfrac{M-1}{M^2}\right\} + o(z) \\ &= \exp\left\{\tfrac{1}{2}\beta^2\sigma_a^2 z + \tfrac{1}{2}\beta^2\sigma_\alpha^2 z^2 + \tfrac{1}{2}\beta^2\sigma_\alpha^2\tfrac{M-1}{M}\tfrac{\beta(2-\beta)}{(1-\beta)^2} z^2\right\} + o(z) \\ &= \exp\left\{\tfrac{1}{2}\beta^2\sigma_a^2 z + \tfrac{1}{2}\beta^2\sigma_\alpha^2 z^2 + \lambda^{FB} z^2\right\} + o(z) \ . \end{aligned}$$

Substituting this expression into the equation for $E(\widetilde{w}_{t+1})$ leads to the law of motion for average income presented in lemma 2.

## Proof of Lemma 3

Given the conjectured capital allocation, observing the $M$ stock prices (or the $M$ capital stocks) is equivalent to observing $\Delta\xi_t^m$ for every firm $m$ where $\xi_t^m \equiv \beta a_t^m + \mu_t^m \theta_t^m$. Similarly, observing the private signals $\{s_{l,t}^m\}$ across the $M$ stocks is equivalent, for an agent $l$, to observing $\Delta s_{l,t}^m$ for every firm $m$. The first step is to relate stock returns to productivity shocks and capital.

- Stock returns

Given its capital stock $K_t^m$, firm $m$ sells $\widetilde{Y}_{t+1}^m = \widetilde{A}_t^m K_t^m$ intermediate goods for a profit $\widetilde{\Pi}_{t+1}^m = \widetilde{\rho}_{t+1}^m \widetilde{Y}_{t+1}^m = \beta L^{1-\beta} \widetilde{Y}_{t+1}^{m\beta} = \beta L^{1-\beta} (\widetilde{A}_t^m K_t^m)^\beta$. The gross return on stock $m$ is then $\widetilde{R}_{t+1}^m = \widetilde{\Pi}_{t+1}^m / K_t^m = \beta L^{1-\beta} K_t^{0\beta-1} \exp[\beta \widetilde{a}_t^m z - (1-\beta)\Delta k_t^m z]$ where $K_t^0 \equiv L w_t / M$ denotes the firm's capital stock when $z = 0$ (when $z = 0$, firms offer the same return in equilibrium since they are identical to one another, which implies that they have identical capital stocks) The log return on stock $m$ is $\ln \widetilde{R}_{t+1}^m = \ln R_t^0 + r_{t+1}^m z$ where $R_t^0 = \beta L^{1-\beta} K_t^{0\beta-1} = \beta M^{1-\beta} w_t^{\beta-1} = \varphi(w_t)/w_t$ and $r_{t+1}^m z = \beta \widetilde{a}_t^m z - (1-\beta)\Delta k_t^m z$. We show below that investors' portfolio weights depend on expected *relative* returns $E(\Delta r_{t+1}^m z \mid \mathcal{F}_{l,t})$ and on the variance of returns $Var(r_{t+1}^m z \mid \mathcal{F}_{l,t})$. These are given by:

$$E(\Delta r_{t+1}^m z \mid \mathcal{F}_{l,t}) = E(\beta \Delta \widetilde{a}_t^m z \mid \mathcal{F}_{l,t}) - (1-\beta)\Delta k_t^m z = E(\beta \Delta \widetilde{\alpha}_t^m z \mid \mathcal{F}_{l,t}) - (1-\beta)\Delta k_t^m z, \tag{22}$$

$$\text{and } Var(r_{t+1}^m z \mid \mathcal{F}_{l,t}) = Var(\beta \widetilde{a}_t^m z \mid \mathcal{F}_{l,t}) = \beta^2 \sigma_a^2 z + Var(\beta \widetilde{\alpha}_t^m z \mid \mathcal{F}_{l,t}) = \beta^2 \sigma_a^2 z + o(z). \tag{23}$$

We note that the variance of returns is constant at the order $z$ since $Var(\beta \widetilde{\alpha}_t^m z \mid \mathcal{F}_{l,t})$ is of order $z^2$. The next step is to estimate the expectation of $\Delta \widetilde{\alpha}_t^m$ using the conjectured prices (or equivalently the $\Delta \xi_t^m$'s) and private signals $s_{l,t}^m$.

- Signal extraction

For the capital allocation given in equation 14 ($\Delta k_t^m$ linear in $\Delta \widetilde{\alpha}_t^m$ and $\Delta \widetilde{\theta}_t^m$ with noisiness parameter $\mu_t^m$), the conditional mean and variance of $\Delta \widetilde{\alpha}_t^m$ are for agent $l$, whose private signal has precision $x_{l,t}^m$:

$$E(\beta \Delta \widetilde{\alpha}_t^m z \mid \mathcal{F}_{l,t}) = c_{\xi t}^m \Delta \xi_t^m z + c_{st}^m \Delta s_{l,t}^m z \tag{24}$$

$$\text{where} \quad \widehat{h}_{l,t}^m \equiv H(\mu_t^m) + x_{l,t}^m, \quad c_{\xi t}^m \widehat{h}_{l,t}^m \equiv \frac{1}{\mu_t^{m2} \sigma_\theta^2} \quad \text{and} \quad c_{st}^m \widehat{h}_{l,t}^m \equiv x_{l,t}^m.$$

$E(\beta \Delta \widetilde{\alpha}_t^m z \mid \mathcal{F}_{l,t})$ is a weighted average of priors (which equal 0), public and private signals where the weight on the private signal is increasing in $x_{l,t}^m$ and that on the public signal is decreasing in $\mu_t^m$.

- Portfolio weights

We now solve for the optimal portfolio of an investor. An agent with a wage $w_t$ and precisions $\left\{x_{l,t}^m\right\}$ maximizes $E\left[U(\widetilde{g}_{l,t+1}, j_t) \mid \mathcal{F}_{l,t}\right]$, where $\widetilde{g}_{l,t+1} = w_t \widetilde{R}_{l,t+1}$ and $j_t = 1 - \sum_{m=1}^M C(x_{l,t}^m) z$ are her consumption of final goods and leisure. Let $r_{l,t+1} z \equiv \ln \widetilde{R}_{l,t+1} - \ln R_t^0$ capture terms of order $z$ and smaller in her log portfolio return. This log portfolio return can be related to individual stock returns and portfolio weights as follows:

$$\begin{aligned}
r_{l,t+1} z = & \ln \left( \sum_{m=1}^M f_{l,t}^m \widetilde{R}_{t+1}^m / R_t^0) \right) \\
= & \ln \left( \sum_{m=1}^M f_{l,t}^m \exp(r_{t+1}^m z) \right) \\
= & \ln \left( \sum_{m=1}^M f_{l,t}^m \left(1 + r_{t+1}^m z + Var(r_{t+1}^m z \mid \mathcal{F}_{l,t})/2\right) + o(z) \right) \\
= & \sum_{m=1}^M \left( f_{l,t}^m r_{t+1}^m + \tfrac{1}{2} f_{l,t}^m (1 - f_{l,t}^m) Var(r_{t+1}^m \mid \mathcal{F}_{l,t}) \right) + o(z) \\
= & \sum_{m=1}^M \left( f_{l,t}^m r_{t+1}^m + \tfrac{1}{2} f_{l,t}^m (1 - f_{l,t}^m) \beta^2 \sigma_a^2 z \right) + o(z)
\end{aligned}$$

where we use $\sum_{m=1}^{M} f_{l,t}^m = 1$ and equation 23. Thus, the log portfolio return is approximately normal when $z$ is small (e.g. Campbell and Viceira (2002)) and its moments are given by:

$$\begin{aligned} E(r_{l,t+1}z \mid \mathcal{F}_{l,t}) &= \sum_{m=1}^{M} \left\{ f_{l,t}^m e_{l,t}^m z + \tfrac{1}{2} f_{l,t}^m (1 - f_{l,t}^m)\beta^2\sigma_a^2 z \right\} + o(z) \text{ where } e_{l,t}^m \equiv E(r_{t+1}^m \mid \mathcal{F}_{l,t}) \\ \text{and } Var(r_{l,t+1}z \mid \mathcal{F}_l) &= \sum_{m=1}^{M} f_{l,t}^{m2} Var(r_{t+1}^m z \mid \mathcal{F}_l) + o(z) = \sum_{m=1}^{M} f_{l,t}^{m2}\beta^2\sigma_a^2 z + o(z). \end{aligned} \tag{25}$$

The agent's utility can be expanded in a Taylor series in a neighborhood of $z = 0$, i.e. for $\widetilde{g}_{l,t+1}$ and $j_t$ respectively around $\varphi(w_t) = w_t R_t^0$ and 1. We denote the pair $(\varphi(w_t), 1)$ with a $*$ :

$$U(\widetilde{g}_{l,t+1}, j_t) = U(*) + \frac{\partial U}{\partial g}(*).(\widetilde{g}_{l,t+1} - \varphi(w_t)) + \frac{\partial U}{\partial j}(*).(j_t - 1) + \frac{1}{2}\frac{\partial^2 U}{\partial g^2}(*).(\widetilde{g}_{l,t+1} - \varphi(w_t))^2 + o(z).$$

Noting that $\widetilde{g}_{l,t+1} - \varphi(w_t) = \varphi(w_t)(w_t\widetilde{R}_{l,t+1}/\varphi(w_t) - 1) = \varphi(w_t)(\widetilde{R}_{l,t+1}/R_t^0 - 1) = \varphi(w_t)(\exp(r_{l,t+1}z) - 1)$ and that $j_t - 1 = -\sum_{m=1}^{M} C(x_{l,t}^m)z$ allows to write the above expression as:

$$\begin{aligned} U(\widetilde{g}_{l,t+1}, j_t) = \quad & U(*) + \tfrac{\partial U}{\partial g}(*).\varphi(w_t)(\exp(r_{l,t+1}z) - 1) - \tfrac{\partial U}{\partial j}(*).\textstyle\sum_{m=1}^{M} C(x_{l,t}^m)z \\ & + \tfrac{1}{2}\tfrac{\partial^2 U}{\partial g^2}(*).\varphi(w_t)^2(\exp(r_{l,t+1}z) - 1)^2 + o(z). \end{aligned}$$

Taking expectations and noting that $E[(\exp(r_{l,t+1}z) - 1)^2 \mid \mathcal{F}_{l,t}] = E[(r_{l,t+1}z + Var(r_{l,t+1}z \mid \mathcal{F}_l)/2 + o(z))^2 \mid \mathcal{F}_{l,t}] = Var(r_{l,t+1}z \mid \mathcal{F}_l)/2 + o(z)$ yields:

$$\begin{aligned} E(U(\widetilde{g}_{l,t+1}, j_t) \mid \mathcal{F}_{l,t}) = \quad & U(*) + \tfrac{\partial U}{\partial g}(*).\varphi(w_t)[E(r_{l,t+1}z \mid \mathcal{F}_{l,t}) + Var(r_{l,t+1}z \mid \mathcal{F}_l)/2] \\ & -\tfrac{\partial U}{\partial j}(*).\textstyle\sum_{m=1}^{M} C(x_{l,t}^m)z + \tfrac{1}{2}\tfrac{\partial^2 U}{\partial g^2}(*).\varphi(w_t)^2 Var(r_{l,t+1}z \mid \mathcal{F}_l) + o(z). \end{aligned} \tag{26}$$

Substituting in the expression for $E(r_{l,t+1}z \mid \mathcal{F}_{l,t})$ and $Var(r_{l,t+1}z \mid \mathcal{F}_l)$ given in equations 25 and maximizing this expression with respect to $f_{l,t}^m$, subject to $\sum_{m=1}^{M} f_{l,t}^m = 1$, leads to the first-order conditions:

$$\frac{\partial U}{\partial g}(*)(e_{l,t}^m + \frac{1}{2}\beta^2\sigma_a^2) + \beta^2\sigma_a^2\varphi(w_t)\frac{\partial^2 U}{\partial g^2}(*)f_{l,t}^m + o(1) = \varsigma_{l,t} \text{ for } m = 1, ...M \tag{27}$$

in which $\varsigma_{l,t}$ denotes the Lagrange multiplier on the constraint. Averaging equation 27 across stocks and noting that $\overline{f_{l,t}} \equiv \sum_{m=1}^{M} f_{l,t}^m/M = 1/M$, yields:

$$\frac{\partial U}{\partial g}(*)(\overline{e_{l,t}} + \frac{1}{2}\beta^2\sigma_a^2) + \beta^2\sigma_a^2\varphi(w_t)\frac{\partial^2 U}{\partial g^2}(*)\frac{1}{M} + o(1) = \varsigma_{l,t}. \tag{28}$$

Subtracting equation 28 from the first-order condition 27 leads to the formula for portfolio weights presented in equation 9:

$$f_{l,t}^m = \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2}\Delta e_{l,t}^m + o(1), \tag{29}$$

where $\tau(\varphi(w_t))/\varphi(w_t) = -\frac{\partial U}{\partial g}(*)/\frac{\partial^2 U}{\partial g^2}(*)/\varphi(w_t)$. Substituting in the expression for $\Delta e_{l,t}^m \equiv E(\Delta r_{t+1}^m \mid \mathcal{F}_{l,t})$ and using equations 22 and 24 leads to equation 11 for the portfolio of a rational trader. Substituting in instead $\Delta e_{l,t}^m = \Delta\theta_t^m$ yields the portfolio of a noise trader displayed in equation 13.

## Proof of Proposition 4

To prove Proposition 4, we guess that the capital allocation is given by equations 14 to 16, solve for the equilibrium and check that the guess is valid. Agents' portfolios under the conjectured capital allocation are described in lemma 3. We multiply portfolio weights by income $w_t$ and sum stock demands over all agents for each stock. The aggregate demand for stock $m$ emanating from rational traders equals:

$$\begin{aligned}\int_{l}^{Rat.} f_{l,t}^m w_t = & \int_{l}^{Rat.} w_t \left\{ \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2} \left[ \frac{x_{l,t}^m}{\widehat{h}_{l,t}^m} \Delta s_{l,t}^m + \left( \frac{1}{\widehat{h}_{l,t}^m \mu_t^{m2} \sigma_\theta^2 k_{\alpha t}^m} - (1-\beta) \right) \Delta k_t^m \right] \right\} + o(1) \\ = & \; L w_t \left\{ \frac{1-q}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2} \left[ T_t^m \beta \Delta \widetilde{\alpha}_t^m + \left( \frac{U_t^m}{\mu_t^{m2}\sigma_\theta^2 k_{\alpha t}^m} - (1-q)(1-\beta) \right) \Delta k_t^m \right] \right\} + o(1),\end{aligned} \tag{30}$$

where $T_t^m \equiv \frac{1}{L} \int_{l}^{Rat.} \frac{x_{l,t}^m}{\widehat{h}_{l,t}^m}$ and $U_t^m \equiv \frac{1}{L} \int_{l}^{Rat.} \frac{1}{\widehat{h}_{l,t}^m}$. To derive this expression, we apply the law of large numbers to the sequence $\{\frac{x_{l,t}^m}{\widehat{h}_{l,t}^m} \Delta \varepsilon_{l,t+1}^m\}$ of independent (across agents) random variables with the same mean 0 (conditional on $\Delta \widetilde{\alpha}_t^m$). It implies that $\int_l \frac{x_{l,t}^m}{\widehat{h}_{l,t}^m} \Delta \varepsilon_{l,t+1}^m = 0$ and hence that $\frac{1}{L} \int_{l}^{Rat.} \frac{x_{l,t}^m}{\widehat{h}_{l,t}^m} \Delta s_{l,t}^m = \frac{1}{L} \int_{l}^{Rat.} \frac{x_{l,t}^m}{\widehat{h}_{l,t}^m} (\beta \Delta \widetilde{\alpha}_t^m + \Delta \varepsilon_{l,t+1}^m) = \frac{1}{L} \int_{l}^{Rat.} \frac{x_{l,t}^m}{\widehat{h}_{l,t}^m} \beta \Delta \widetilde{\alpha}_t^m = T_t^m \beta \Delta \widetilde{\alpha}_t^m$.

The aggregate demand for stock $m$ emanating from noise traders equals:

$$\int_{l}^{Noise} f_{l,t}^m w_t = q L w_t \left( \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2} \Delta \theta_t^m \right) + o(1).$$

Summing up rational and noise traders' demand for stock $m$, $(\int_{l}^{Rat.} f_{l,t}^m w_t + \int_{l}^{Noise} f_{l,t}^m w_t)/P_t^m$, and equating it to the supply of shares (normalized to one) leads to:

$$L w_t \left\{ \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2} \left[ q \Delta \theta_t^m + T_t^m \beta \Delta \widetilde{\alpha}_t^m + \left( \frac{U_t^m}{\mu_t^{m2}\sigma_\theta^2 k_{\alpha t}^m} - (1-q)(1-\beta) \right) \Delta k_t^m \right] \right\} + o(1) = P_t^m.$$

Since, the left-hand side of this equation is deterministic at the order 0 in $z$ (it equals $P_t^m = K_t^m = K_t^0 + o(1) \equiv L w_t / M + o(1)$), the expression in the square bracket must be equal to zero. As a consequence,

$$\Delta k_t^m = \frac{T_t^m}{(1-q)(1-\beta) - \frac{U_t^m}{\mu_t^{m2}\sigma_\theta^2 k_{\alpha t}^m}} (\beta \Delta \widetilde{\alpha}_t^m + \frac{q}{T_t^m} \Delta \theta_t^m).$$

Comparing this expression to the conjectured capital allocation (equation 14) implies that

$$\mu_t^m T_t^m \equiv q \quad \text{and} \quad k_{\alpha t}^m = \frac{T_t^m}{(1-q)(1-\beta) - \frac{U_t^m}{\mu_t^{m2}\sigma_\theta^2 k_{\alpha t}^m}}, \tag{31}$$

which in turn implies that $k_{\alpha t}^m \equiv \frac{1}{(1-\beta)(1-q)} (T_t^m + \frac{U_t^m}{\mu_t^{m2}\sigma_\theta^2})$. Equilibrium prices are linear in $\Delta \widetilde{\alpha}_t^m$ and $\Delta \theta_t^m$, which confirms the initial guess. Moreover, if signal precisions are identical across agents for any stock $m$ ($x_{l,t}^m = X_t^m$ for all $l$), then $T_t^m$ and $U_t^m$ simplify to $T_t^m = (1-q) X_t^m / (H(\mu_t^m) + X_t^m)$ and $U_t^m = (1-q)/(H(\mu_t^m) + X_t^m)$. In this case, we obtain equations 15 and 16 displayed in Proposition 2.

## Proof of Lemma 5

The elasticity of investments to productivity shocks, $\partial(\ln K_t^m)/\partial\widetilde{\alpha}_t^m$, equals $\beta(1-1/M)k_{\alpha t}$ which decreases with $\mu_t$ since

$$\frac{\partial k_{\alpha t}}{\partial \mu_t} = -\frac{1}{1-\beta}\frac{2\frac{1-q}{q}\mu_t + \frac{\mu_t^2\sigma_\theta^2}{\beta^2\sigma_\alpha^2} - 1}{\beta^2\sigma_\alpha^2\frac{1-q}{q}H(\mu_t)^2\mu_t^4\sigma_\theta^2} < 0. \tag{32}$$

We turn to TFP. From its definition and equation 14, TFP equals $\exp\{[k_\alpha\beta^3\sigma_\alpha^2 - \beta(1-\beta)k_\alpha^2(\beta^2\sigma_\alpha^2 + \mu_t^2\sigma_\theta^2)/2]z^2(M-1)/M\}$. Therefore:

$$\frac{d\ln TFP}{d\mu_t} = \frac{\partial cov(\beta\widetilde{a}_t^m z, \beta\Delta k_t^m z)}{\partial\mu_t} - \frac{(1-\beta)}{2\beta}\frac{\partial Var(\beta\Delta k_t^m z)}{\partial\mu_t}.$$

Using using equations 34 and 35, this expression simplifies to:

$$\begin{aligned}\frac{d\ln TFP}{d\mu_t} &= -\frac{M-1}{M}\frac{\beta}{1-\beta}\left\{\frac{2\frac{1-q}{q}\mu_t + \frac{\mu_t^2\sigma_\theta^2}{\beta^2\sigma_\alpha^2} - 1}{\frac{1-q}{q}H(\mu_t)^2\mu_t^4\sigma_\theta^2} - \frac{(\frac{1-q}{q}\mu_t - 1)(\frac{1-q}{q}\mu_t + \frac{\mu_t^2\sigma_\theta^2}{\beta^2\sigma_\alpha^2})}{(\frac{1-q}{q})^2H(\mu_t)^2\mu_t^5\sigma_\theta^2}\right\}z^2 + o(z^2)\\ &= -\frac{M-1}{M}\frac{\beta}{1-\beta}\frac{(\frac{1-q}{q})^2\mu_t^2 + \frac{\mu_t^2\sigma_\theta^2}{\beta^2\sigma_\alpha^2}}{(\frac{1-q}{q})^2H(\mu_t)^2\mu_t^5\sigma_\theta^2}z^2 + o(z^2) < 0.\end{aligned}$$

Therefore, $TFP$ decreases with $\mu_t$. The Herfindahl index for capital is given by:

$$Her(K_t^m) \equiv E(K_t^{m2})/[E(K_t^m)]^2 = E[\exp(2\Delta k_t^m z)]/(E[\exp(\Delta k_t^m z)])^2 = \exp[Var(\Delta k_t^m z)],$$

which, from equation 35, decreases with noisiness $\mu_t$. A similar calculation for profits implies that its Herfindahl index, $Her(\widetilde{\Pi}_{t+1}^m)$, decreases too with noisiness.

## Proof of Lemma 6

We start by computing $E(\widetilde{w}_{t+1})$. Proceeding as in lemma 2, the wage equals $\widetilde{w}_{t+1} = (1-\beta)\widetilde{G}_{t+1}/L = (1-\beta)w_t^\beta M^{1-\beta}\overline{\exp\left(\beta z(\widetilde{a}_t^m + \Delta k_t^m)\right)}$, and its average is given by:

$$\begin{aligned}E(\widetilde{w}_{t+1}) &= (1-\beta)w_t^\beta M^{1-\beta}E\left[\overline{\exp\left(\beta z(\widetilde{a}_t^m + \Delta k_t^m)\right)}\right]\\ &= (1-\beta)w_t^\beta M^{1-\beta}E\left[\exp(\beta z(\widetilde{a}_t^m + \Delta k_t^m))\right]\\ &= (1-\beta)w_t^\beta M^{1-\beta}\exp\left[\tfrac{1}{2}Var\left(\beta z(\widetilde{a}_t^m + \Delta k_t^m)\right)\right],\end{aligned}$$

as $E(\widetilde{a}_t^m) = 0$ and $E(\Delta k_t^m) = k_\alpha E((\beta\Delta\widetilde{\alpha}_t^m + \mu_t\Delta\widetilde{\theta}_t^m)) = 0$. We evaluate next $Var\left(\beta z(\widetilde{a}_t^m + \Delta k_t^m)\right)$:

$$\begin{aligned}Var\left[\beta(\widetilde{a}_t^m z + \Delta k_t^m z)\right] &= Var\left\{E\left[\beta(\widetilde{a}_t^m z + \Delta k_t^m z) \mid \{\widetilde{\alpha}_t^m, \theta_t^m\}\right]\right\} + E\left\{Var\left[\beta(\widetilde{a}_t^m z + \Delta k_t^m z) \mid \{\widetilde{\alpha}_t^m, \theta_t^m\}\right]\right\}\\ &= Var\left\{\beta(\widetilde{\alpha}_t^m z + \Delta k_t^m z)\right\} + E\left\{Var\left[\beta(\widetilde{a}_t^m z) \mid \{\widetilde{\alpha}_t^m, \theta_t^m\}\right]\right\}\\ &= Var\left\{\beta(\widetilde{\alpha}_t^m z + \Delta k_t^m z)\right\} + \beta^2\sigma_a^2 z\\ &= Var(\beta\widetilde{\alpha}_t^m z) + Var(\beta\Delta k_t^m z) + 2cov(\beta\widetilde{\alpha}_t^m z, \beta\Delta k_t^m z) + \beta^2\sigma_a^2 z\\ &= \beta^2\sigma_\alpha^2 z^2 + Var(\beta\Delta k_t^m z) + 2cov(\beta\widetilde{\alpha}_t^m z, \beta\Delta k_t^m z) + \beta^2\sigma_a^2 z.\end{aligned} \tag{33}$$

The covariance term is given by:

$$cov(\beta\widetilde{a}_t^m z, \beta\Delta k_t^m z) = cov(\beta\widetilde{\alpha}_t^m z, \beta k_\alpha \beta\Delta\widetilde{\alpha}_t^m z) + o(z^2) = \frac{M-1}{M}\beta^3 k_\alpha \sigma_\alpha^2 z^2 + o(z^2),$$

and the variance terms by:

$$Var(\beta\widetilde{\alpha}_t^m z) = \beta^2\sigma_\alpha^2 z^2$$

$$\begin{aligned} \text{and } Var(\beta\Delta k_t^m z) &= Var[\beta k_{\alpha t}(\beta\Delta\widetilde{\alpha}_t^m + \mu_t\Delta\widetilde{\theta}_t^m)z] + o(z^2) \\ &= \beta^2 k_{\alpha t}^2[\beta^2 Var(\Delta\widetilde{\alpha}_t^m) + \mu_t^2 Var(\Delta\widetilde{\theta}_t^m)]z^2 + o(z^2) \\ &= \beta^2 k_{\alpha t}^2 \frac{M-1}{M}(\beta^2\sigma_\alpha^2 + \mu_t^2\sigma_\theta^2)z^2 + o(z^2). \end{aligned}$$

Substituting these expressions into equation 33 yields:

$$Var\left[\beta(\widetilde{a}_t^m z + \Delta k_t^m z)\right] = \beta^2\sigma_a^2 z + \beta^2\sigma_\alpha^2 z^2 + \beta^2 k_{\alpha t}^2\frac{M-1}{M}(\beta^2\sigma_\alpha^2 + \mu_t^2\sigma_\theta^2)z^2 + 2\frac{M-1}{M}\beta^3 k_{\alpha t}\sigma_\alpha^2 z^2 + o(z^2).$$

It follows that $E(\widetilde{w}_{t+1}) = \Lambda\exp(\lambda(w_t)z^2)$ where $\lambda$ and $\Lambda$ are defined in equations 20 and 6.

Next, we evaluate $\partial E(\widetilde{w}_{t+1})/\partial\mu_t$. It suffices to differentiate $\lambda$ with respect to the noisiness $\mu_t$, holding current income $w_t$ constant, since the other terms are constant:

$$2\frac{\partial\lambda}{\partial\mu_t} = \frac{\partial Var\left[\beta(\widetilde{a}_t^m z + \Delta k_t^m z)\right]}{\partial\mu_t} = \frac{\partial cov(\beta\widetilde{a}_t^m z, \beta\Delta k_t^m z)}{\partial\mu_t} + \frac{\partial Var(\beta\Delta k_t^m z)}{\partial\mu_t},$$

$$\text{where} \quad \frac{\partial cov(\beta\widetilde{a}_t^m z, \beta\Delta k_t^m z)}{\partial\mu_t} = \tfrac{M-1}{M}\beta^3\frac{\partial k_{\alpha t}}{\partial\mu_t}\sigma_\alpha^2 z^2 + o(z^2) = -\tfrac{M-1}{M}\tfrac{\beta}{1-\beta}\frac{2\frac{1-q}{q}\mu_t + \frac{\mu_t^2\sigma_\theta^2}{\beta^2\sigma_\alpha^2} - 1}{\frac{1-q}{q}H(\mu_t)^2\mu_t^4\sigma_\theta^2}z^2 + o(z^2) < 0, \tag{34}$$

$$\text{and} \quad \frac{\partial Var(\beta\Delta k_t^m z)}{\partial\mu_t} = 2\tfrac{M-1}{M}\beta^2 k_{\alpha t}\left[\frac{\partial k_{\alpha t}}{\partial\mu_t}(\beta^2\sigma_\alpha^2 + \mu_t^2\sigma_\theta^2)z^2 + k_{\alpha t}\mu_t\sigma_\theta^2\right] + o(z^2), \tag{35}$$

$$= -2\tfrac{M-1}{M}(\tfrac{\beta}{1-\beta})^2\frac{(\frac{1-q}{q}\mu_t - 1)(\frac{1-q}{q}\mu_t + \frac{\mu_t^2\sigma_\theta^2}{\beta^2\sigma_\alpha^2})}{(\frac{1-q}{q})^2 H(\mu_t)^2\mu_t^5\sigma_\theta^2}z^2 + o(z^2) < 0.$$

It follows that:

$$\begin{aligned} \frac{\partial E(\widetilde{w}_{t+1})}{\partial\mu_t} &= E(\widetilde{w}_{t+1})\frac{\partial\lambda}{\partial\mu_t}z^2 = -E(\widetilde{w}_{t+1})\frac{\frac{M-1}{M}(\frac{\beta}{1-\beta})^2}{(\frac{1-q}{q})^2 H(\mu_t)^2\mu_t^5\sigma_\theta^2}\Bigg\{(\frac{1-q}{q}\mu_t - 1)(\frac{1-q}{q}\mu_t + \frac{\mu_t^2\sigma_\theta^2}{\beta^2\sigma_\alpha^2}) \\ &\quad + \frac{1-\beta}{\beta}\frac{1-q}{q}\mu_t(2\frac{1-q}{q}\mu_t + \frac{\mu_t^2\sigma_\theta^2}{\beta^2\sigma_\alpha^2} - 1)z^2 + o(z^2)\Bigg\} < 0. \end{aligned}$$

## Proof of Lemma 7

The degree of inequality is captured by the variance of final wealth, $\widetilde{g}_{l,t+1} = w_t\widetilde{R}_{l,t+1} = w_t R_t^0\exp(r_{l,t+1}z)$. Since final wealth is approximately log-normal when $z$ is small, $Var(\widetilde{g}_{l,t+1})$ is equivalent to a Gini index, which equals $2F(\sqrt{Var(r_{l,t+1}z)/2}) - 1$ where $F$ is the cumulative distribution function for a standard normal and where $Var(r_{l,t+1}z) = Var[E(r_{l,t+1}z \mid \mathcal{F}_l)] + E[Var(r_{l,t+1}z \mid \mathcal{F}_l)] = E[\sum_{m=1}^{M} f_{l,t}^{m2}\beta^2\sigma_a^2 z] + o(z)$

given that $Var[E(r_{l,t+1}z \mid \mathcal{F}_l)]$ is of order $z^2$ and using equation 25. Substituting equation 29 into this expression leads to $Var(r_{l,t+1}z) = \frac{\beta^2\sigma_a^2}{M}z + \frac{M}{\beta^2\sigma_a^2}\frac{\tau(\varphi(w_t))^2}{\varphi(w_t)^2}E(\overline{e_{l,t}^2} - \overline{e_{l,t}}^2)z + o(z)$ where $E(\overline{e_{l,t}^2} - \overline{e_{l,t}}^2) = \overline{Var(\Delta e_{l,t}^m)}$ from equation 39 below. Moreover, $\overline{Var(\Delta e_{l,t}^m)} = Var(\Delta e_{l,t}^m) = \frac{M-1}{M}\frac{\varphi(w_t)}{\tau(\varphi(w_t))}\left(\frac{X_t}{h(X_t)^2} + \frac{q^2\sigma_\theta^2}{(1-q)^2}\right)$ because in equilibrium $\mu_t^m$ is identical across stocks (See Proposition 13). As a result:

$$Var(r_{l,t+1}z) = \frac{\beta^2\sigma_a^2}{M}z + \frac{M-1}{\beta^2\sigma_a^2}\frac{\tau(\varphi(w_t))}{\varphi(w_t)}\left(\frac{X_t}{h(X_t)^2} + \frac{q^2\sigma_\theta^2}{(1-q)^2}\right)z + o(z).$$

Differentiating this expression with respect to noisiness $\mu_t$ amounts to differentiating $X_t/h(X_t)^2 = q/[(1-q)h(X_t)\mu_t]$ (equation 16) where $h(X_t) \equiv H(\mu_t) + X(\mu_t)$:

$$\frac{\partial \ln(h(X_t)\mu_t)}{\partial \mu_t} = \frac{1-q}{q}\frac{\mu_t^2 - 2\mu_t q/(1-q) - \beta^2\sigma_\alpha^2/\sigma_\theta^2}{H(\mu_t)\mu_t^2(\mu_t(1-q)/q - 1)\beta^2\sigma_\alpha^2}.$$

The sign of this ratio is given by the sign of its numerator. It is positive for $\mu_t > \mu^+$ and negative for $\mu_t < \mu^+$ where

$$\mu^+ \equiv q/(1-q) + \sqrt{q^2/(1-q)^2 + \beta^2\sigma_\alpha^2/\sigma_\theta^2}. \tag{36}$$

Thus, $Var(r_{l,t+1}z)$ increases with noisiness $\mu_t$ over $(q/(1-q), \mu^+)$, and decreases over $(\mu^+, \infty)$.

## Proof of Lemma 8

Liquidity is given by $k_\theta(\mu_t) \equiv 1/\left(\partial(\ln K_t^m)/\partial(\widetilde{\theta}_t^m z)\right) = 1/\left((1-1/M)k_\alpha(\mu_t)\mu_t\right)$. Its derivative with respect to $\mu_t$ equals $\dfrac{\partial k_\theta}{\partial \mu_t} = k_\alpha + \mu_t\dfrac{\partial k_\alpha}{\partial \mu_t}$, where the first term reflects the usual beneficial effect of information on liquidity (improving information, i.e. lowering $\mu_t$, improves liquidity, i.e. reduces $k_\theta$), and the second term is the offsetting effect of information on the sensitivity of the capital allocation to technology shocks. Using equation 32 leads to

$$\frac{\partial k_\theta}{\partial \mu_t} = -\frac{1}{1-\beta}\frac{q}{1-q}\frac{\frac{1}{\mu_t^2\sigma_\theta^2} - \frac{1}{\beta^2\sigma_\alpha^2} + X_t}{X_t H(\mu_t)\mu_t^3\sigma_\theta^2} = -\frac{1}{1-\beta}\frac{\mu_t^2 - 2\mu_t q/(1-q) - \beta^2\sigma_\alpha^2/\sigma_\theta^2}{X_t H(\mu_t)\mu_t^4\sigma_\theta^2\beta^2\sigma_\alpha^2},$$

where the last expression uses equations 16 and 12. This expression is negative for $\mu_t > \mu^+$ and positive for $\mu_t < \mu^+$, where $\mu^+$ is defined in the proof of Lemma 7. Hence liquidity worsens ($k_\theta$ increases) when $\mu_t$ decreases over $(\mu^+, \infty)$ and improves ($k_\theta$ decreases) when $\mu_t$ decreases over $(q/(1-q), \mu^+)$.

The average value of shares traded equals $Vol = E\left[\sum_{m=1}^M \left(\int_{\substack{Rat.\\ l}} |f_{l,t}^m w_t| + \int_{\substack{Noise\\ l}} |f_{l,t}^m w_t|\right)/2\right] = Vol_{Rat} + Vol_{Noise}$ where $Vol_{Rat} = E\left[\sum_{m=1}^M \left(\int_{\substack{Rat.\\ l}} |f_{l,t}^m w_t|\right)/2\right]$ and $Vol_{noise} = E\left[\sum_{m=1}^M \left(\int_{\substack{Noise\\ l}} |f_{l,t}^m w_t|/2\right)\right]$. $f_{l,t}^m$ is approximately normally distributed so $Vol_{Rat} = (1-q)MLw_t\sqrt{\frac{1}{2\pi}Var(f_{Rat\ l,t}^m)}/2$ and $Vol_{Noise} = qMLw_t\sqrt{\frac{1}{2\pi}Var(f_{Noise,t}^m)}/2$ (e.g. He and Wang (1995)). Portfolio shares in equilibrium for a rational agent are obtained by substituting equation 14 into equation 11, setting $x_{l,t}^m = X_t$ and denoting $h(X_t) \equiv H(\mu_t) + X(\mu_t)$ :

$$
\begin{aligned}
f^m_{Rat\ l,t} &= \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2}\frac{X_t}{h(X_t)}\left(\Delta\varepsilon^m_{l,t} - \mu_t\Delta\theta^m_t\right) + o(1) \\
&= \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2}\left(\frac{X_t}{h(X_t)}\Delta\varepsilon^m_{l,t} - \frac{q}{1-q}\Delta\theta^m_t\right) + o(1) \text{ using equation 16.}
\end{aligned}
$$

Therefore, $\sqrt{Var(f^m_{Rat\ l,t})} = \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2}\sqrt{\frac{M-1}{M}}\sqrt{\frac{X_t}{h(X_t)^2} + (\frac{q}{1-q})^2\sigma_\theta^2} + o(1)$ and:

$$
Vol_{Rat} = \frac{\tau(\varphi(w_t))w_t}{\varphi(w_t)}\frac{L\sqrt{M(M-1)}}{2\sqrt{2\pi}\beta^2\sigma_a^2}\sqrt{\frac{(1-q)^2X_t}{h(X_t)^2} + q^2\sigma_\theta^2} + o(1).
$$

For noise traders, $\sqrt{Var(f^m_{Noise,t})} = \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2}\sqrt{\frac{M-1}{M}}\sqrt{\sigma_\theta^2} + o(1)$ and:

$$
Vol_{Rat} = \frac{\tau(\varphi(w_t))w_t}{\varphi(w_t)}\frac{L\sqrt{M(M-1)}}{2\sqrt{2\pi}\beta^2\sigma_a^2}\sqrt{q^2\sigma_\theta^2} + o(1).
$$

Summing both components of volume leads to:

$$
Vol = \frac{\tau(\varphi(w_t))w_t}{\varphi(w_t)}\frac{L\sqrt{M(M-1)}}{2\sqrt{2\pi}\beta^2\sigma_a^2}\left\{\sqrt{\frac{(1-q)^2X_t}{h(X_t)^2} + q^2\sigma_\theta^2} + \sqrt{q^2\sigma_\theta^2}\right\} + o(1).
$$

The share turnover is obtained by dividing by the total capitalization of the market, $\sum_{m=1}^M K^m_t = M(Lw_t/M) = Lw_t$. The trading intensity therefore equals:

$$
Turn = \frac{\tau(\varphi(w_t))}{\varphi(w_t)}\frac{\sqrt{M(M-1)}}{2\sqrt{2\pi}\beta^2\sigma_a^2}\left\{\sqrt{\frac{(1-q)^2X_t}{h(X_t)^2} + q^2\sigma_\theta^2} + \sqrt{q^2\sigma_\theta^2}\right\} + o(1).
$$

As with liquidity, the derivative of this expression with respect to $\mu_t$ is positive over $(q/(1-q), \mu^+)$, and negative over $(\mu^+, \infty)$.

## Proof of Lemma 9

Proposition 4 establishes that stock prices are given by $P^m_t = Lw_t/M\exp(\Delta k^m_t z)$. Their variance equals $Var(P^m_t) = (Lw_t/M)^2 Var(\Delta k^m_t z)$. Equation 35 implies that $\partial Var(P^m_t)/\partial\mu_t < 0$: prices are more volatile as information improves.

Stock returns are given by $\ln\widetilde{R}^m_{t+1} = \ln R^0_t + r^m_{t+1}z$ where $r^m_{t+1}z = \beta\widetilde{a}^m_t z - (1-\beta)\Delta k^m_t z$. It follows that the equally weighted return on the market equals $\overline{r}_t z \equiv \frac{1}{M}\sum_{m=1}^M r^m_t z = \beta\overline{a}^m_t z$ and its volatility is a constant $\beta^2\sigma_a^2 z/M$. Idiosyncratic return volatility is simply given by $Var(\Delta r^m_{t+1}z)$ because $\Delta r^m_{t+1}$ is uncorrelated to the market ($Cov(\Delta r^m_{t+1}z, \overline{r}_t z) = 0$).

$$
\begin{aligned}
Var(\Delta r_{t+1}^m z) &= Var(\beta \Delta \widetilde{a}_t^m z - (1-\beta)\Delta k_t^m z) \\
&= Var(\beta \Delta \widetilde{a}_t^m z - (1-\beta) k_\alpha(\mu_t^m)(\beta \Delta \widetilde{\alpha}_t^m + \mu_t^m \Delta \widetilde{\theta}_t^m) z) \\
&= Var\left\{E\left[\beta \Delta \widetilde{a}_t^m z - (1-\beta) k_\alpha(\mu_t^m)(\beta \Delta \widetilde{\alpha}_t^m + \mu_t^m \Delta \widetilde{\theta}_t^m) z \mid \{\widetilde{\alpha}_t^m, \theta_t^m\}\right]\right\} \\
&\quad + E\left\{Var\left[\beta \Delta \widetilde{a}_t^m z - (1-\beta) k_\alpha(\mu_t^m)(\beta \Delta \widetilde{\alpha}_t^m + \mu_t^m \Delta \widetilde{\theta}_t^m) z \mid \{\widetilde{\alpha}_t^m, \theta_t^m\}\right]\right\} \\
&= Var\left\{\beta \Delta \widetilde{\alpha}_t^m z - (1-\beta) k_\alpha(\mu_t^m)(\beta \Delta \widetilde{\alpha}_t^m + \mu_t^m \Delta \widetilde{\theta}_t^m) z\right\} + E\left\{Var\left[\beta(\Delta \widetilde{a}_t^m z) \mid \{\widetilde{\alpha}_t^m, \theta_t^m\}\right]\right\} \\
&= Var\left\{\beta(1-(1-\beta)k_\alpha(\mu_t^m))\Delta \widetilde{\alpha}_t^m z + (1-\beta)k_\alpha(\mu_t^m)\mu_t^m \Delta \widetilde{\theta}_t^m z)\right\} + \frac{M-1}{M}\beta^2\sigma_a^2 z \beta^2 \sigma_a^2 z \\
&= Var\left\{\frac{\beta \Delta \widetilde{\alpha}_t^m z}{\beta^2 \sigma_\alpha^2 h(X_t)}\right\} + Var\left\{\left(\frac{1}{\mu_t^m \sigma_\theta^2 h(X_t)} + \frac{1-q}{q}\right)\Delta \widetilde{\theta}_t^m z\right\} + \frac{M-1}{M}\beta^2\sigma_a^2 z \beta^2 \sigma_a^2 z,
\end{aligned}
$$

where where $h(X_t) \equiv H(\mu_t) + X(\mu_t)$. Rearranging leads to:

$$
Var(\Delta r_{t+1}^m z) = \frac{M-1}{M}\left\{\frac{h(X_t)+X_t}{h(X_t)^2} z^2 + \frac{(1-q)^2\sigma_\theta^2}{q^2} z^2 + \beta^2 \sigma_a^2 z\right\}.
$$

Differentiating this expression with respect to noisiness $\mu_t$ amounts to differentiating $(h(X_t)+X_t)/h(X_t)^2 = 1/h(X_t) + q/[(1-q)h(X_t)\mu_t]$ (equation 16):

$$
\frac{\partial}{\partial \mu_t}\left(\frac{h(X_t)+X_t}{h(X_t)^2}\right) = \frac{2}{H(\mu_t)^2 \mu_t^3}\left(\frac{1}{\sigma_\theta^2} + \frac{(1-q)^2}{q^2\beta^2\sigma_\alpha^2}\right) > 0.
$$

Therefore $\partial Var(\Delta r_{t+1}^m z)/\partial \mu_t > 0$ so idiosyncratic return volatility falls when information improves.

Total volatility is given by $Var(r_{t+1}^m z) = Var(\Delta r_{t+1}^m z + \overline{r}_t z) = Var(\Delta r_{t+1}^m z) + Var(\overline{r}_t z)$ since $Cov(\Delta r_{t+1}^m z, \overline{r}_t z) = 0$. The market volatility is constant, so total volatility behaves in the same way as idiosyncratic volatility. Finally,

$$
\begin{aligned}
Var(\overline{r}_t z) &= Var(\frac{1}{M}\sum_{m=1}^M r_t^m z) = \frac{1}{M^2}\left\{\sum_{m=1}^M Var(r_t^m z) + \sum_{m=1}^M \sum_{m'=1}^M Cov(r_{t+1}^m z, r_{t+1}^{m'} z)\right\} \\
&= \frac{1}{M}\left\{Var(r_t^m z) + (M-1)Cov(r_{t+1}^m z, r_{t+1}^{m'} z)\right\},
\end{aligned}
$$

where we use the fact that in equilibrium $\mu_t^m$ is identical across stocks (See Proposition 11). So $Cov(r_{t+1}^m z, r_{t+1}^{m'} z) = (MVar(\overline{r}_t z) - Var(r_{t+1}^m z))/(M-1)$ decreases when information improves.

## Proof of Lemma 10

We solve for an investor's optimal precision about stock $m$, $x_{l,t}^m$, given any noisiness $\mu_t^m$. We first plug the formulas for the mean and variance of portfolio returns (equations 25) into the expression for the expected utility (equation 26). We note that $\sum_{m=1}^M \Delta e_{l,t}^m = 0$ and $\sum_{m=1}^M \Delta e_{l,t}^{m2} = M(\overline{e_{l,t}^2} - \overline{e_{l,t}}^2)$ so equation 29 implies that $\sum_{m=1}^M f_{l,t}^m e_{l,t}^m = \overline{e_{l,t}^m} + \frac{\tau(\varphi(w_t))M}{\varphi(w_t)\beta^2\sigma_a^2}(\overline{e_{l,t}^2} - \overline{e_{l,t}}^2)$ and $\sum_{m=1}^M f_{l,t}^{m2} = \frac{1}{M} + \left(\frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2}\right)^2 M(\overline{e_{l,t}^2} - \overline{e_{l,t}}^2)$. After rearranging, we obtain:

$$
E(U(\widetilde{g}_{l,t+1}, j_t) \mid \mathcal{F}_{l,t}) = U(*) - \frac{\partial U}{\partial j}(*).\sum_{m=1}^M C(x_{l,t}^m) z + \frac{\partial U}{\partial g}(*).\varphi(w_t) Q_{l,t} z + o(z) \tag{37}
$$

where $Q_{l,t} \equiv E(r_{l,t+1}z \mid \mathcal{F}_l)+\frac{1}{2}Var(r_{l,t+1}z \mid \mathcal{F}_l)-\frac{\varphi(w_t)}{2\tau(\varphi(w_t))}Var(r_{l,t+1}z \mid \mathcal{F}_l) = \overline{e_{l,t}}+M\delta_t(\overline{e_{l,t}^2}-\overline{e_{l,t}}^2)+d_t$,

$$d_t \equiv \frac{\beta^2\sigma_a^2}{2}\left(1-\frac{\varphi(w_t)}{M\tau(\varphi(w_t))}\right) \quad \text{and} \quad \delta_t \equiv \frac{\tau(\varphi(w_t))}{2\varphi(w_t)\beta^2\sigma_a^2}.$$

The agent's unconditional expected utility, $E(U(\widetilde{g}_{l,t+1}, j_t))$, follows:

$$E(U(\widetilde{g}_{l,t+1}, j_t)) = U(*) - \frac{\partial U}{\partial j}(*).\sum_{m=1}^{M} C(x_{l,t}^m)z + \frac{\partial U}{\partial g}(*).\varphi(w_t)E(Q_{l,t})z + o(z) \tag{38}$$

We evaluate next $E(Q_{l,t})$. The variable $e_{l,t}^m$ is a function of $\{\Delta s_{l,t}^m\}$ and $\{k_t^m\}$, which themselves depend on $\{\Delta\widetilde{\alpha}_t^m\}$, $\{\Delta\widetilde{\theta}_t^m\}$ and $\{\Delta\widetilde{\varepsilon}_{l,t+1}^m\}$ (see equation 40 below). Like all the random variables in the model, its unconditional mean $E(e_{l,t}^m)$ equals zero. As a result, $E(\overline{e_{l,t}}) = 0$. Moreover:

$$\begin{aligned}
E(\overline{e_{l,t}^2} - \overline{e_{l,t}}^2) = \;& E(\overline{e_{l,t}^2} - 2\overline{e_{l,t}}^2 + \overline{e_{l,t}}^2) = E(\textstyle\sum_{m=1}^{M} e_{l,t}^{m2}/M - 2\overline{e_{l,t}}\sum_{m=1}^{M} e_{l,t}^m/M + \overline{e_{l,t}}^2) \\
= \;& E(\textstyle\sum_{m=1}^{M}(e_{l,t}^{m2} - 2\overline{e_{l,t}}e_{l,t}^m + \overline{e_{l,t}}^2)/M) = E(\sum_{m=1}^{M}(e_{l,t}^m - \overline{e_{l,t}})^2/M) = E(\overline{(e_{l,t}^m - \overline{e_{l,t}})^2}) \\
= \;& \overline{E((e_{l,t}^m - \overline{e_{l,t}})^2)} = \overline{Var(e_{l,t}^m - \overline{e_{l,t}}) + (E(e_{l,t}^m - \overline{e_{l,t}}))^2} = \overline{Var(e_{l,t}^m - \overline{e_{l,t}})} + \overline{(E(e_{l,t}^m - \overline{e_{l,t}}))^2}
\end{aligned}$$

$$\text{so } E(\overline{e_{l,t}^2} - \overline{e_{l,t}}^2) = \overline{Var(e_{l,t}^m - \overline{e_{l,t}})} = \overline{Var(\Delta e_{l,t}^m)} \quad \text{because } \overline{E(e_{l,t}^m)} = E(\overline{e_{l,t}}) = 0. \tag{39}$$

The next step is to compute $\overline{Var(\Delta e_{l,t}^m)}$. We first note that from equation 22 :

$$e_{l,t}^m = E(r_{t+1}^m \mid \mathcal{F}_{l,t}) = E(\beta\widetilde{\alpha}_t^m z \mid \mathcal{F}_{l,t}) - (1-\beta)k_t^m = c_{\xi t}^m\xi_t^m + c_{st}^m s_{l,t}^m - (1-\beta)k_{\alpha t}^m\Delta\xi_t^m. \tag{40}$$

It follows, since $\overline{k_t} \equiv 0$, that:

$$\Delta e_{l,t}^m = \Delta(c_{\xi t}^m\xi_t^m) + \Delta(c_{st}^m s_{l,t}^m) - (1-\beta)k_{\alpha t}^m\Delta\xi_t^m.$$

Substituting $\xi_t^m \equiv \beta a_t^m + \mu_t^m\theta_t^m$, $s_{l,t}^m = \beta\widetilde{\alpha}_t^m + \widetilde{\varepsilon}_{l,t}^m$ and replacing $c_{\xi t}^m$ and $c_{st}^m$ with their definitions (equations 22) leads to:

$$\Delta e_{l,t}^m = \mathcal{A}_{l,t}^m\beta\widetilde{\alpha}_t^m + (M-1)\frac{x_{l,t}^m}{\widehat{h}_{l,t}^m}\widetilde{\varepsilon}_{l,t}^m + \mathcal{B}_{l,t}^m\mu_t^m\theta_t^m + \sum_{\substack{n=1\\n\neq m}}^{M}\left(\mathcal{C}_{l,t}^{n,m}\beta\widetilde{\alpha}_t^n - \frac{x_{l,t}^n}{\widehat{h}_{l,t}^n}\widetilde{\varepsilon}_{l,t}^n + \mathcal{D}_{l,t}^{n,m}\mu_t^n\theta_t^n\right), \tag{41}$$

where we recall that $\widehat{h}_{l,t}^m \equiv H(\mu_t^m) + x_{l,t}^m$ and define:

$$\begin{aligned}
\mathcal{A}_{l,t}^m &\equiv (M-1)\left(1 - \frac{1}{\beta^2\sigma_\alpha^2\widehat{h}_{l,t}^m} - (1-\beta)k_{\alpha t}^m\right) \\
\mathcal{B}_{l,t}^m &\equiv (M-1)\left(\frac{1}{\mu_t^{m2}\sigma_\theta^2\widehat{h}_{l,t}^m} - (1-\beta)k_{\alpha t}^m\right) = \mathcal{A}_{l,t}^m - (M-1)\frac{x_{l,t}^m}{\widehat{h}_{l,t}^m} \\
\mathcal{C}_{l,t}^{n,m} &\equiv -1 + \frac{1}{\beta^2\sigma_\alpha^2\widehat{h}_{l,t}^n} + (1-\beta)k_{\alpha t}^m \\
\mathcal{D}_{l,t}^{n,m} &\equiv -\frac{1}{\mu_t^{n2}\sigma_\theta^2\widehat{h}_{l,t}^n} + (1-\beta)k_{\alpha t}^m = \mathcal{C}_{l,t}^{n,m} + \frac{x_{l,t}^m}{\widehat{h}_{l,t}^m}.
\end{aligned}$$

Taking the variance of equation 41 yields:

$$\begin{aligned} M^2 Var(\Delta e^m_{l,t}) &= (\beta^2\sigma_a^2 + \mu_t^{m2}\sigma_\theta^2)\mathcal{A}^{m2}_{l,t} - 2(M-1)\mu_t^{m2}\sigma_\theta^2\frac{x^m_{l,t}}{\widehat{h}^m_{l,t}}\mathcal{A}^m_{l,t} + (M-1)^2\mu_t^{m2}\sigma_\theta^2\frac{x^{m2}_{l,t}}{\widehat{h}^{m2}_{l,t}} + (M-1)^2\frac{x^m_{l,t}}{\widehat{h}^{m2}_{l,t}} \\ &+ \sum_{\substack{n=1\\ n\neq m}}^{M}\left((\beta^2\sigma_a^2 + \mu_t^{n2}\sigma_\theta^2)\mathcal{C}^{n,m2}_{l,t} + \frac{x^n_{l,t}}{\widehat{h}^{n2}_{l,t}} + 2\mu_t^{n2}\sigma_\theta^2\frac{x^m_{l,t}}{\widehat{h}^m_{l,t}}\mathcal{C}^{n,m}_{l,t} + \mu_t^{n2}\sigma_\theta^2\frac{x^{m2}_{l,t}}{\widehat{h}^{m2}_{l,t}}\right). \end{aligned}$$

Completing the sum with the terms with the $m$ superscript and rearranging yields:

$$\begin{aligned} M^2 Var(\Delta e^m_{l,t}) &= (\beta^2\sigma_a^2 + \mu_t^{m2}\sigma_\theta^2)(\mathcal{A}^{m2}_{l,t} - \mathcal{C}^{m,m2}_{l,t}) - 2\mu_t^{m2}\sigma_\theta^2\frac{x^m_{l,t}}{\widehat{h}^m_{l,t}}((M-1)\mathcal{A}^m_{l,t} + \mathcal{C}^{m,m}_{l,t}) \\ &+ M(M-2)\mu_t^{m2}\sigma_\theta^2\frac{x^{m2}_{l,t}}{\widehat{h}^{m2}_{l,t}} + M(M-2)\frac{x^m_{l,t}}{\widehat{h}^{m2}_{l,t}} \\ &+ \sum_{n=1}^{M}\left((\beta^2\sigma_a^2 + \mu_t^{n2}\sigma_\theta^2)\mathcal{C}^{n,m2}_{l,t} + \frac{x^n_{l,t}}{\widehat{h}^{n2}_{l,t}} + 2\mu_t^{n2}\sigma_\theta^2\frac{x^m_{l,t}}{\widehat{h}^m_{l,t}}\mathcal{C}^{n,m}_{l,t} + \mu_t^{n2}\sigma_\theta^2\frac{x^{m2}_{l,t}}{\widehat{h}^{m2}_{l,t}}\right). \end{aligned}$$

Noting that $\mathcal{C}^{m,m}_t = -\mathcal{A}^m_t/(M-1)$, replacing the$\mathcal{A}$ and $\mathcal{C}$ coefficients with their expressions and rearranging leads to:

$$M^2 Var(\Delta e^m_{l,t}) = -M(M-2)\left(\frac{1}{\widehat{h}^m_{l,t}} + \mathcal{K}^{m,m}_t\right) + \sum_{n=1}^{M}\left(-\frac{1}{\widehat{h}^n_{l,t}} + \mathcal{K}^{n,m}_t\right)$$

$$\text{where} \quad \mathcal{K}^{n,m}_t \equiv (\beta^2\sigma_a^2 + \mu_t^{n2}\sigma_\theta^2)\left(1-(1-\beta)k^m_{\alpha t}\right)^2 + \mu_t^{n2}\sigma_\theta^2\left(2(1-\beta)k^m_{\alpha t} - 1\right).$$

Note that $\mathcal{K}^{n,m}_t$ does not depend on the precisions chosen by agent $l$. Taking the average across all stocks yields:

$$\begin{aligned} M^2\overline{Var(\Delta e^m_{l,t})} &= -(M-2)\sum_{m=1}^{M}\left(\frac{1}{\widehat{h}^m_{l,t}} + \mathcal{K}^{m,m}_t\right) + \frac{1}{M}\sum_{m=1}^{M}\sum_{n=1}^{M}\left(-\frac{1}{\widehat{h}^n_{l,t}} + \mathcal{K}^{n,m}_t\right) \\ &= -(M-1)\sum_{m=1}^{M}\frac{1}{\widehat{h}^m_{l,t}} - (M-2)\sum_{m=1}^{M}\mathcal{K}^{m,m}_t + \frac{1}{M}\sum_{m=1}^{M}\sum_{n=1}^{M}\mathcal{K}^{n,m}_t \end{aligned}$$

since $\frac{1}{M}\sum_{m=1}^{M}\sum_{n=1}^{M}\frac{1}{\widehat{h}^n_{l,t}} = \frac{1}{M}M\sum_{n=1}^{M}\frac{1}{\widehat{h}^n_{l,t}} = \sum_{n=1}^{M}\frac{1}{\widehat{h}^n_{l,t}} = \sum_{m=1}^{M}\frac{1}{\widehat{h}^m_{l,t}}$. It follows that

$$\begin{aligned} E(Q_{l,t}) &= 0 + \frac{\delta_t}{M}\left(-(M-1)\sum_{m=1}^{M}\frac{1}{\widehat{h}^m_{l,t}} - (M-2)\sum_{m=1}^{M}\mathcal{K}^{m,m}_t + \frac{1}{M}\sum_{m=1}^{M}\sum_{n=1}^{M}\mathcal{K}^{n,m}_t\right) + d_t \\ &= -\frac{\delta_t(M-1)}{M}\sum_{m=1}^{M}\frac{1}{\widehat{h}^m_{l,t}} + \mathcal{Q}_t \end{aligned}$$

$$\text{where} \quad \mathcal{Q}_t \equiv \frac{\delta_t}{M}\left(-(M-2)\sum_{m=1}^{M}\mathcal{K}^{m,m}_t + \frac{1}{M}\sum_{m=1}^{M}\sum_{n=1}^{M}\mathcal{K}^{n,m}_t\right) + d_t.$$

Note that $\mathcal{Q}_t$ does not depend on the precisions chosen by agent $l$. We substitute this expression into equation 38 which characterizes agent $l$'s unconditional expected utility, and replace $\widehat{h}_{l,t}^m \equiv H(\mu_t^m)+x_{l,t}^m$:

$$E(U(\widetilde{g}_{l,t+1}, j_t)) = U(*) - \frac{\partial U}{\partial j}(*).\sum_{m=1}^{M} C(x_{l,t}^m)z + \frac{\partial U}{\partial g}(*).\varphi(w_t)\left(-\frac{\delta_t(M-1)}{M}\sum_{m=1}^{M}\frac{1}{H(\mu_t^m)+x_{l,t}^m} + \mathcal{Q}_t\right)z + o(z)$$

We maximize this expression with respect to $x_{l,t}^m$ taking as given the stocks' noisiness $\{\mu_t^m\}$. The first-order condition for this problem is, for every stock $m$ and agent $l$ :

$$\frac{\partial U}{\partial j}(*)C'\left(x_{l,t}^m\right) = \frac{\partial U}{\partial g}(*).\varphi(w_t)\frac{\delta_t(M-1)}{M}\frac{1}{\left(H(\mu_t^m)+x_{l,t}^m\right)^2}. \tag{42}$$

Substituting $\delta_t \equiv \frac{\tau(\varphi(w_t))}{2\varphi(w_t)\beta^2\sigma_a^2}$ and rearranging leads to equation 17 in lemma 5. Equation 42 admits a unique solution because its left hand side is monotonically increasing in $x_{l,t}^m$ starting from zero ($C'(0)=0$ by assumption), while its right hand side is monotonically decreasing towards zero. Moreover, the equation implies that signal precisions are identical across agents for any stock $m$ ($x_{l,t}^m = X_t^m$ for all $l$).

## Proof of Proposition 11

Equation 42 implies that signal precisions are identical across agents for any stock $m$, i.e. $x_{l,t}^m = X_t^m \equiv X(\mu_t^m)$ for all $l$. As a result, equations 31 which characterize stock prices for arbitrary precisions simplify to equations 15 and 16. Replacing $x_{l,t}^m$ with $X(\mu_t^m)$ on both sides of equation 42 and noting that equation 16 implies that $H(\mu_t^m)+X_t^m = H(\mu_t^m)/(1-q/(1-q)/\mu_t^m)$ leads to equation 18. This equation admits a unique solution $\mu_t^m$ for any level of income $w_t$, because its left hand side is monotonically decreasing in $\mu_t^m$ towards zero, while its right hand side is monotonically increasing from zero. Moreover, $\mu_t^m$ and therefore $X_t^m$ are identical across stocks.

## Proof of Lemma 12

Differentiating equation 18 with respect to $w_t$ yields:

$$\left\{\left(\frac{C''(X(\mu_t))}{C'(X(\mu_t))} + \frac{1}{H(\mu_t)+X(\mu_t)}\right)\frac{dX}{d\mu_t} + \frac{1}{H(\mu_t)+X(\mu_t)}\frac{dH}{d\mu_t}\right\}\frac{d\mu_t}{dw_t} = \frac{d}{dw_t}\left(\ln\frac{\tau(\varphi(w_t))}{\rho(\varphi(w_t))}\right).$$

The sign of $\frac{d\mu_t}{dw_t}$ is the opposite of that of $\frac{d}{dw_t}\left(\ln\frac{\tau(\varphi(w_t))}{\rho(\varphi(w_t))}\right)$ because $\dfrac{dX}{d\mu_t} < 0$ (equation 16) and $\dfrac{dH}{d\mu_t} < 0$ (equation 12). Moreover, $\varphi$ is increasing (equation 10). So $\frac{d\mu_t}{dw_t} < 0$ ($> 0$) if $\tau/\rho$ is increasing (decreasing).

## Proof of Lemma 13

We start by computing $dX_t/dq$, which captures the *ex ante* disincentive effect on the precision of private information. We write equation 18 as $\rho(\varphi(w_t))C'(X_t)((1-q)/q)^2\mu_t^2{X_t}^2 = \tau(\varphi(w_t))(M-1)/(2M\beta^2\sigma_a^2)+o(1)$ using equation 16. We take logs, differentiate this equation with respect to $q$, holding $w_t$ constant, and obtain:

$$\frac{C''(X_t)}{2C'(X_t)}\frac{dX_t}{dq} + \frac{1}{q(1-q)} + \frac{1}{\mu_t}\frac{d\mu_t}{dq} + \frac{1}{X_t}\frac{dX_t}{dq} = 0. \tag{43}$$

We decompose $d\mu_t/dq$ into *ex post* and *ex ante* components, $\frac{d\mu_t}{dq} = \frac{\partial \mu_t}{\partial q}_{X_t \text{ fixed}} + \frac{\partial \mu_t}{\partial X_t}_{q \text{ fixed}} * \frac{dX_t}{dq}$. We differentiate equation 16 to evaluate $\frac{\partial \mu_t}{\partial q}_{X_t \text{ fixed}}$ and $\frac{\partial \mu_t}{\partial X_t}_{q \text{ fixed}}$ and substitute the results into the previous expression for $d\mu_t/dq$:

$$\frac{d\mu_t}{dq} = \frac{\mu_t}{H_t + X_t + \frac{2}{\mu_t^2 \sigma_\theta^2}} \left\{ -\frac{H_t + X_t}{q(1-q)} + * \frac{dX_t}{dq} \frac{H_t}{X_t} \right\}.$$

Substituting back into equation 43 and rearranging leads to

$$\frac{dX_t}{dq} = \frac{2X_t}{q(1-q)\mu_t^2 \sigma_\theta^2 \mathcal{N}} > 0$$

where $\mathcal{N} \equiv X_t + \frac{2}{\mu_t^2 \sigma_\theta^2} + \frac{\epsilon_{C'}(X_t)}{2} \left( H_t + X_t + \frac{2}{\mu_t^2 \sigma_\theta^2} \right) > 0$ and $\epsilon_{C'}(X) \equiv \frac{XC''(X)}{C'(X)} > 0$.

The *ex ante* disincentive effect on the total precision is given by:

$$\frac{d(H_t + X_t)}{dq} = -\frac{2}{\mu_t^3 \sigma_\theta^2} \frac{d\mu_t}{dq} + \frac{dX_t}{dq} = -\frac{\epsilon_{C'}(X_t))(H_t + X_t)}{q(1-q)\mu_t^2 \sigma_\theta^2 \mathcal{N}} < 0.$$

Hence, less information is produced ($X_t$ falls) but the total precision, $H_t + X_t$, rises nevertheless (because more information is shared through stock prices) when the fraction of noise traders $q$ decreases.

## Proof of Lemma 14

We evaluate equation 18 when the fraction of noise traders is close to zero. When $q \approx 0$, equation 16 can be approximated as $X(\mu_t) \approx \frac{H(\mu_t)}{\mu_t} \frac{q}{1-q} \approx \frac{H(\mu_t)}{\mu_t} q$ so $C'(X_t) \approx C''(0) \frac{H(\mu_t)}{\mu_t} q$ and $1/(H(\mu_t) + X(\mu_t))^2 \approx \frac{1}{H(\mu_t)^2}(1 - \frac{1}{\mu_t} \frac{2q}{1-q}) \approx \frac{1}{H(\mu_t)^2}(1 - \frac{2q}{\mu_t})$. Substituting these expressions into equation 18 yields:

$$\frac{\mu_t}{H(\mu_t)} \approx 2 \left( C''(0) \frac{M\beta^2 \sigma_a^2}{M-1} \frac{\rho(\varphi(w_t))}{\tau(\varphi(w_t))} + 1 \right) q + o(1). \tag{44}$$

We guess that $\mu_t$ is close to zero so $H(\mu_t) \approx \frac{1}{\mu_t^2 \sigma_\theta^2}$. Substituting back into equation 44 and rearranging leads to:

$$\mu_t \approx \left\{ 2\sigma_\theta^2 \left( C''(0) \frac{M\beta^2 \sigma_a^2}{M-1} \frac{\rho(\varphi(w_t))}{\tau(\varphi(w_t))} + 1 \right) \right\}^{1/3} q^{1/3} + o(1),$$

which confirms that $\mu_t$ is close to zero. This formula implies that $H(\mu_t)$ grows to infinity when $q$ approaches zero: agents' information becomes perfect thanks to its revelation through stock prices even though the precision of their private signals goes to zero. As a result, the capital allocation and the income process converge to those of the first best.

## Proof of Proposition 15

The first part of the proposition (equations 19 and 20) was established in the proof of lemma 6. These equations imply that the steady-state level of income along the average path solves $w^* = \Lambda w^{*\beta} \exp(\lambda(w^*) z^2)$. To determine $w^*$ at the order $z^2$, we replace $w^*$ in $\lambda(w^*)$ with its order-zero component, which is identical to the order-zero component of $w^{FB}$, i.e. $(1-\beta)^{1/(1-\beta)} M$ (see equations 6 and 8). This leads to equation 21.

The last part of the proposition obtains by combining lemma 6 with lemma 12. Lemma 12 states that $\frac{d\mu_t}{dw_t} < 0$ (noisiness falls with income) if $\tau/\rho$ is an increasing function, while lemma 6 shows that $\dfrac{\partial\lambda}{\partial\mu_t} < 0$ (income grows on average when noisiness is lower). Together, they imply that $\dfrac{d\lambda}{dw_t} > 0$. If instead If $\tau/\rho$ is a decreasing function, then $\dfrac{d\lambda}{dw_t} < 0$.

If $\lim_{g\to u}\tau(g)/\rho(g) = \infty$, then equation 18 implies that $\lim_{w_t\to u}\mu_t = q/(1-q)$, which corresponds to the perfect-information case. In that case, $\lim_{w_t\to u}\lambda(w_t) = \lambda^{FB} + \lambda^{Noise}$. Alternatively, if $\lim_{g\to u}\tau(g)/\rho(g) = 0$, then $\lim_{w_t\to u}\mu_t = \infty$ (no-information case) and $\lim_{w_t\to u}\lambda(w_t) = \lambda^{Noise}$.

### Proof of Proposition 16

Proposition 16 follows directly from combining Lemmas 5 to 10 with Lemma 13.

### Proof of Proposition 17

The proof of the proposition follows directly from the discussion following Proposition 15.

### Proof of Lemma 18

Under CES preferences, $\tau(g)/\rho(g) = \omega g^\sigma(\varpi g^\sigma + 1 - \varpi)/(1-\sigma)/(1-\varpi)^2$. Substituting this expression into the condition in Proposition 15 leads to $w_t^\sigma > \dfrac{1-\varpi}{2\varpi}\left(\sqrt{1 + \dfrac{8\varpi(1-\sigma)M\sigma_a^2(\sigma_\alpha^2)^2}{(1-\varpi)(M-1)\beta^2}C'(0)} - 1\right) \equiv \underline{w}^\sigma$ and to lemma 18.

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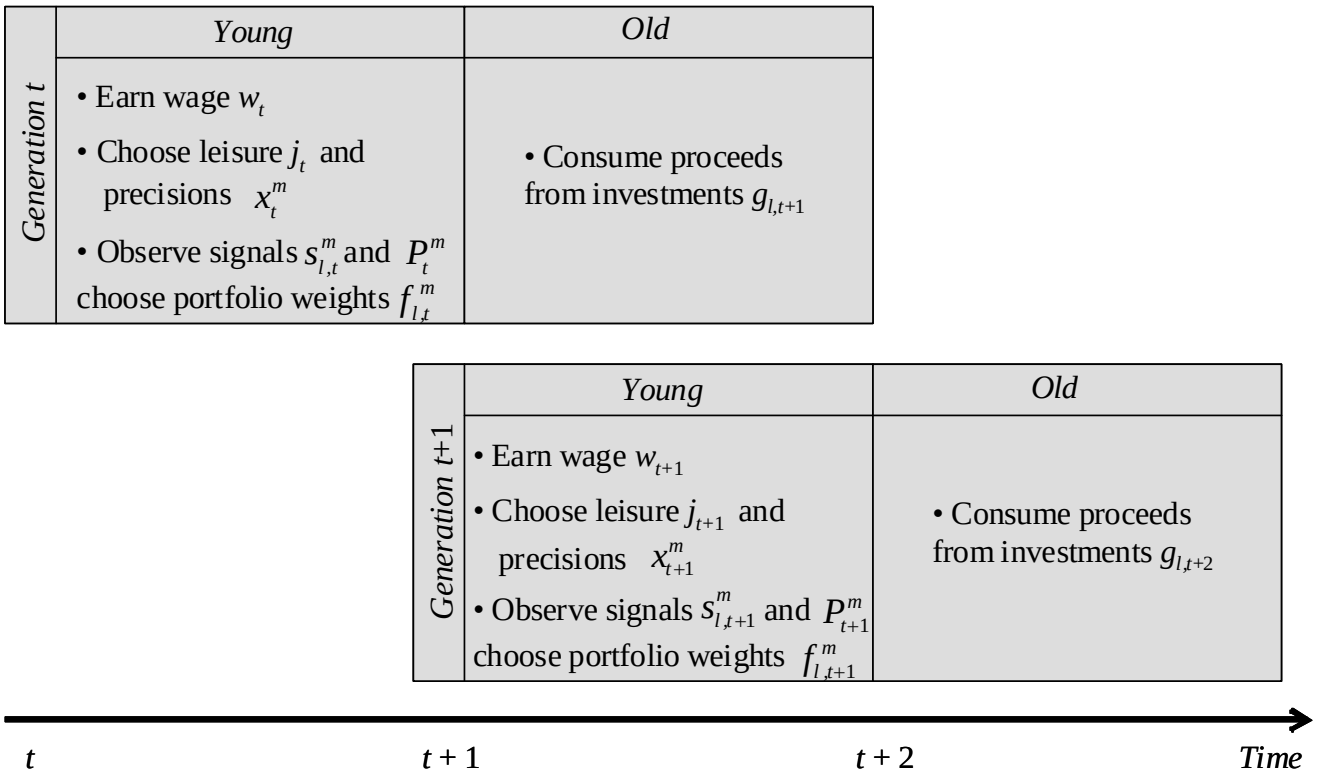


Figure 1: Timing.

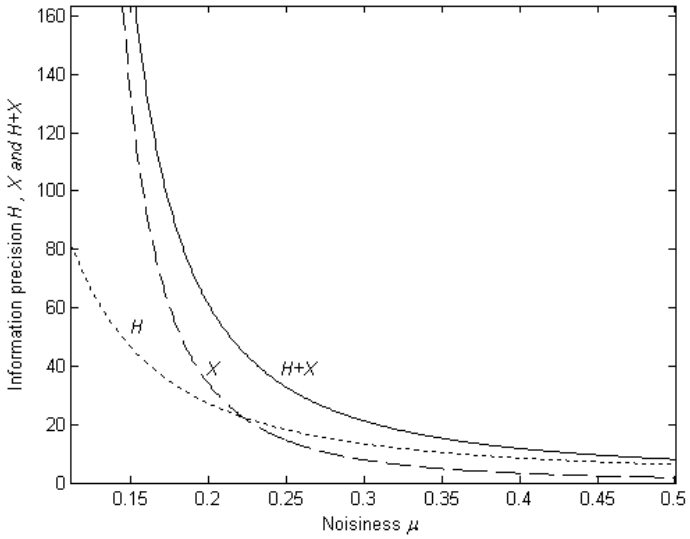


Figure 2: Signal precisions and the noisiness of the price system. The picture depicts the precision of the stock price $H$ (dotted curve), the precision of an investor's private signal $X$ (dashed curve) and her total precision $H + X$ (solid curve) as a function of the stock price noisiness $\mu$. Utility is CES $(U(g, j) \equiv (\varpi g^{\sigma} + (1 - \varpi) j^{\sigma})^{1/\sigma})$ with $\sigma = 0.5$. The other parameters are $\beta = 2/3$, $C(x) = x^2$, $q = 0.1$, $\sigma_a^2 = 0.01$, $\sigma_\theta^2 = \sigma_\alpha^2 = 1$, $\omega = 0.5$, $M = 50$ and $z = 0.5$.

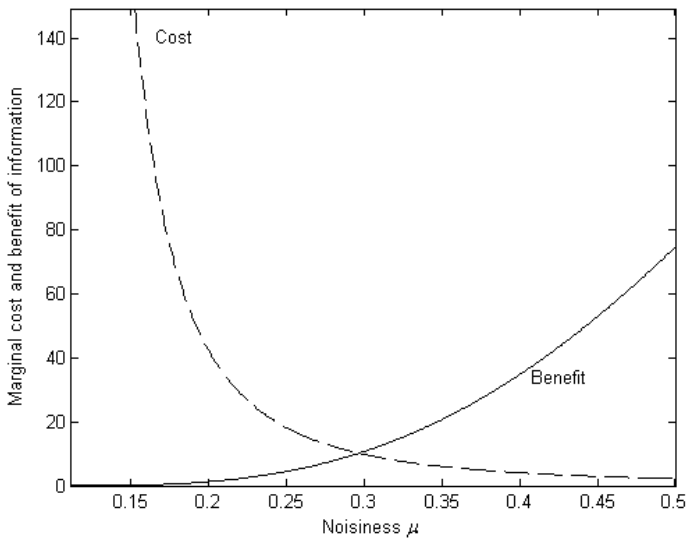


Figure 3: The benefit and cost of information in equilibrium. The picture depicts the marginal benefit of private information (solid curve) and its marginal cost (dashed curve) in equation 18 as a function of the equilibrium noisiness $\mu$. See Figure 2 for the parameters of the picture.

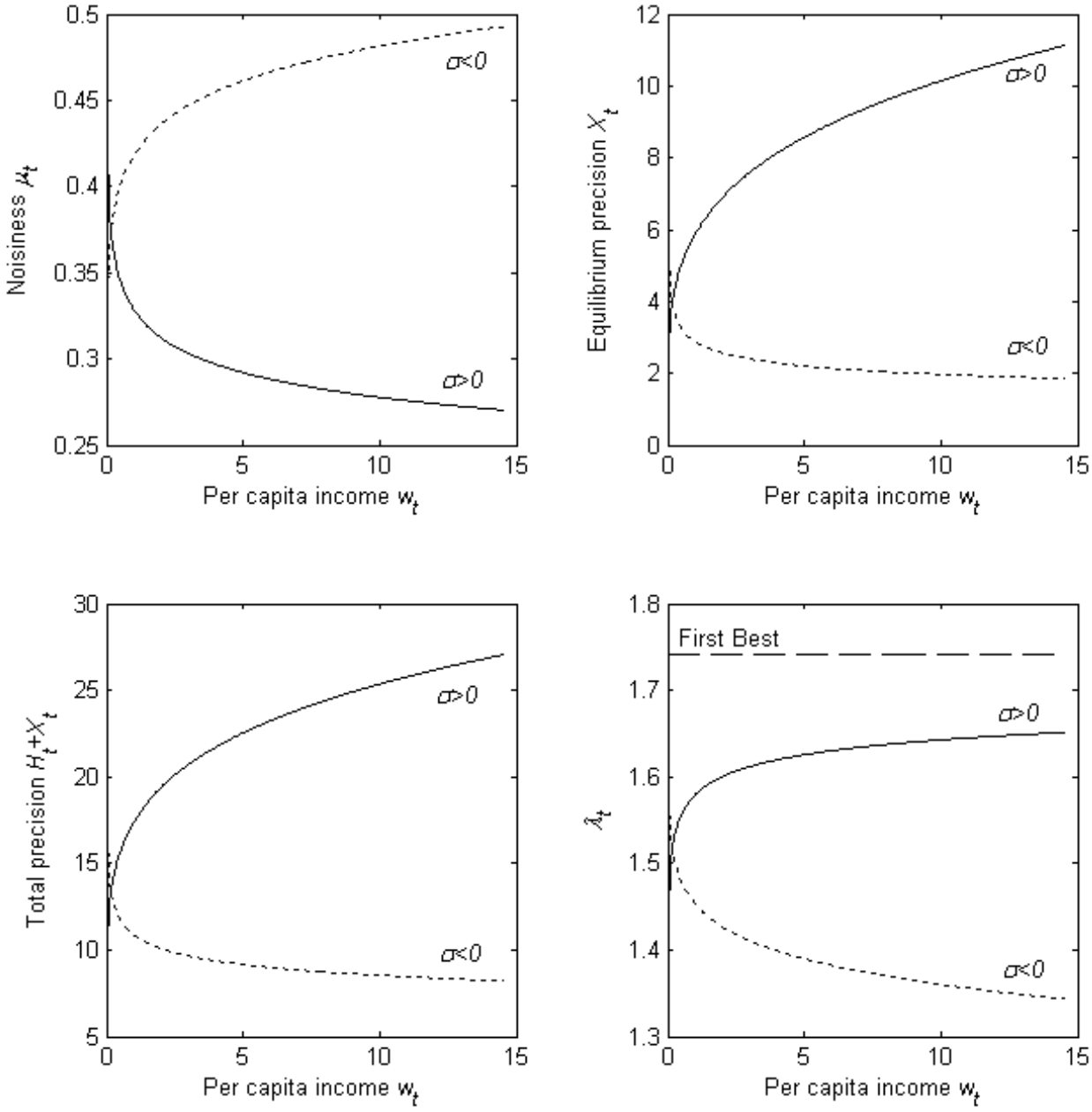


Figure 4: The impact of income on the equilibrium. The picture depicts the equilibrium noisiness $\mu_t$ (top left panel), the precision of private information $X_t$ (top right panel), the total precision $H_t+X_t$ (bottom left panel) and $\lambda_t$ which captures the effect of learning on income (bottom right panel) as a function of current income $w_t$. See Figure 2 for the parameters of the picture. The solid curves correspond to $\sigma = 0.5$ and the dotted curves to $\sigma = -0.5$.

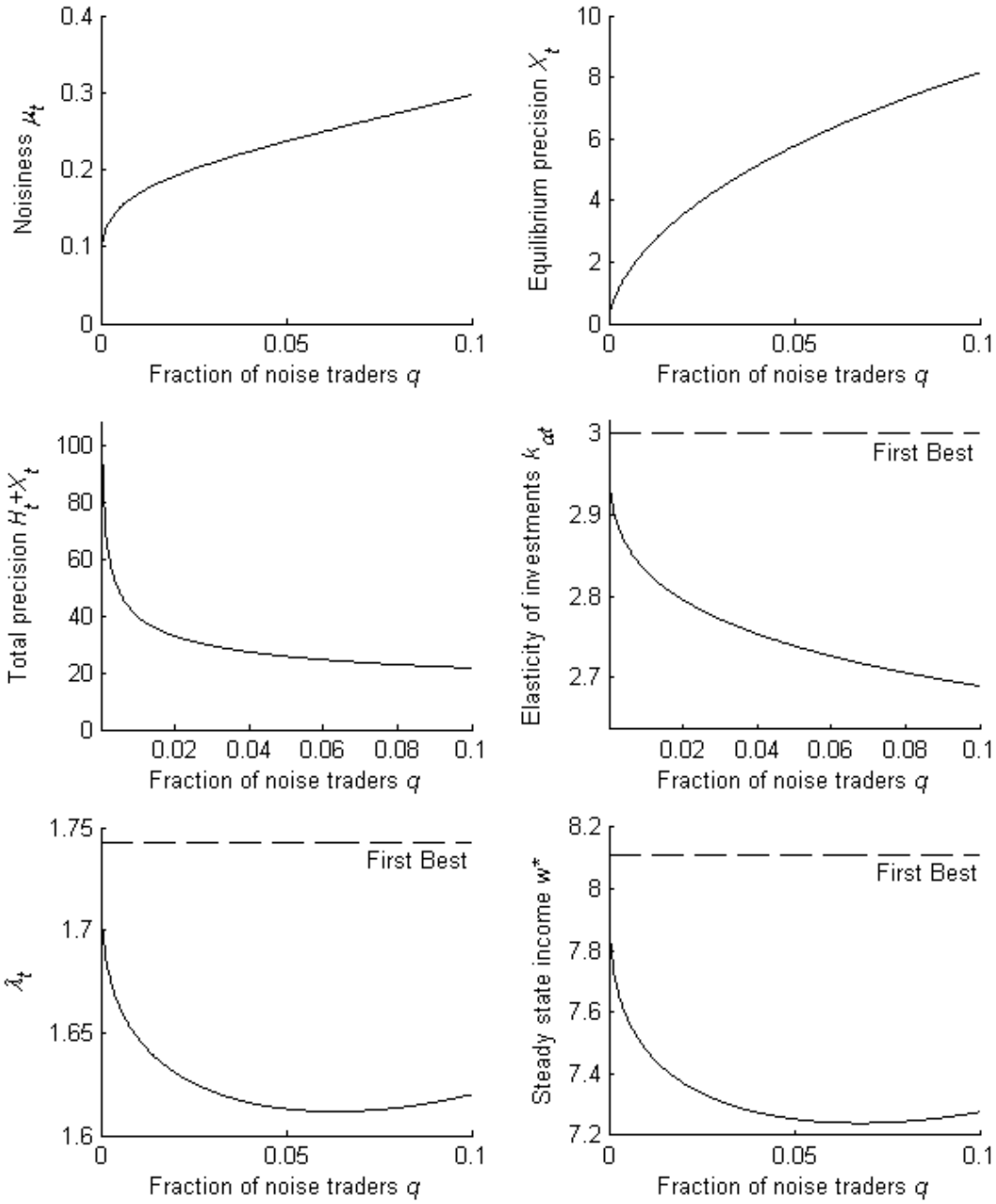


Figure 5: The impact of the fraction of noise traders on the equilibrium. The picture depicts the equilibrium noisiness $\mu_t$ (top left panel), the precision of private information $X_t$ (top right panel), the total precision $H_t + X_t$ (middle left panel), the elasticity of investments to productivity shocks $k_{\alpha t}$ (middle right panel), $\lambda_t$ which captures the effect of learning on income (bottom left panel) and the steady state level of income $w^*$ (bottom right panel) as a function of the fraction of noise traders $q$. See Figure 2 for the parameters of the picture.

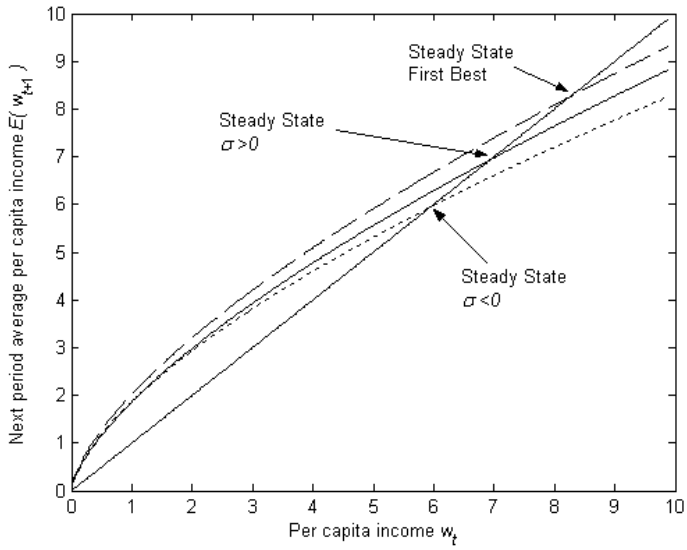


Figure 6: The dynamics of income in an economy along its average path. The curves represent the average income in period $t+1$, $E(w_{t+1})$, as a function of income in period $t$, $w_t$. See Figure 2 for the parameters of the picture. The solid curve corresponds to $\sigma = 0.5$ and the dotted curve to $\sigma = -0.5$. The dashed curve corresponds to the first-best economy. The economies' steady-states are located at the intersections of these curves with the 45° line, represented as a solid line.

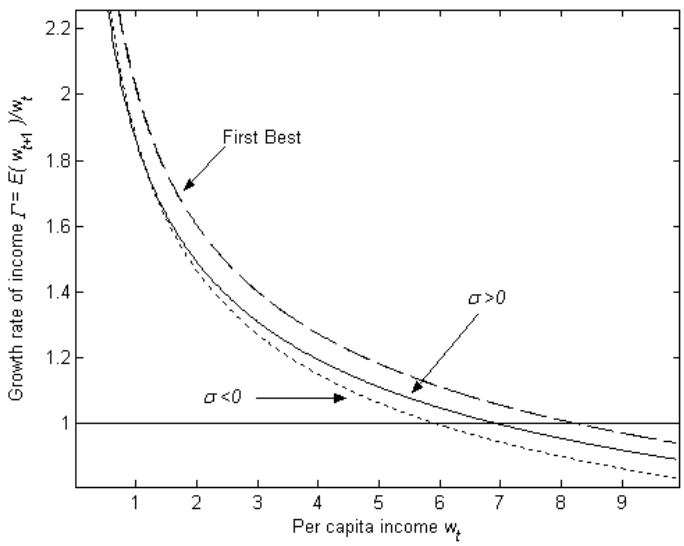


Figure 7: The growth rate of income. The picture depicts the growth rate of income, $\Gamma(w_t) \equiv E(\widetilde{w}_{t+1})/w_t$, during the transition to the steady-state. See Figure 2 for the parameters of the picture. The solid curve corresponds to $\sigma = 0.5$ and the dotted curve to $\sigma = -0.5$. The dashed curve corresponds to the first-best economy.

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