Competitive Bundling in a Bertrand Duopoly

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In competitive industries, some firms bundle their products, whereas others unbundle them; still other firms occupy a niche position and offer only a subset of products. No general theory has been advanced to explain this variety of bundling strategies. We characterize the strategies of two symmetric firms competing (in a Bertrand fashion) with regard to two homogeneous components. We model their decisions as a two-stage non-cooperative game. Firms in the first stage select their product offerings, which may include any single-component product and/or the bundle; in the second stage, firms simultaneously set their products' prices. We show that three types of equilibria always emerge in equilibrium: (i) Differentiated duopoly, in which one firm offers the bundle and the other firm offers only a single-component product; (ii) Monopoly, in which one firm offers the full set of products and the other firm stays out of the business; (iii) Perfect competition, in which both firms offer both single-component products (possibly along with the bundle) and compete on price, driving their profits down to zero. Hence, bundling can be anti-competitive (if it is used to defend a monopolistic position) or hyper-competitive (if it leads to head-to-head competition). In case of a differentiated duopoly, the bundler has a profit advantage which remains moderate when customer valuations are negatively correlated and highly heterogeneous but might grow out of bound otherwise. Therefore, bundling does not necessarily hurt customer welfare or competition, but under the wrong set of circumstances, it could be a terrible competitive weapon.

Key words: Industrial Organization; Bundling; Bertrand Competition; Non-cooperative Game Theory

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1. Introduction

Bundling is the practice of selling several products (or services) as a combination for a single price (Adams and Yellen 1976). It is prevalent in many industry sectors, including physical goods (e.g., gift baskets, car options, fast-food menu combos), services (insurance, fast food, telecommunications, retail banking), and digital platforms (Google, Amazon).

Bundling offers several benefits (Eppen et al. 1991). First, it can lead to economies of scope by reducing the costs of production, transaction, and administration. Second, bundling can expand demand by improving product performance or combining complementary products. Bundling also enables firms to extract more customer surplus by reducing heterogeneity in customer valuations—a form of price discrimination.

Although bundling offers multiple benefits in principle, competing firms do not appear to employ a uniform bundling strategy. For instance, telecom companies compete through
bundling, unbundling, and re-bundling their current services. Traditionally, broadband
Internet, TV, and telephone services have been offered as “double-play” or “triple-play”
bundles. Facing intense competition from multichannel video programming distributors
(MVPDs) and changing consumption habits due to the rise of over-the-top (OTT) ser-
VICES, such as Netflix and Amazon Video, telecom companies have started offering smaller,
targeted “skinny” bundles (e.g., Sling TV, DirectNow) in which fewer TV channels are bun-
dled for a lower price. Yet, in a complete reversal of strategy, Verizon recently announced
it would break its traditional triple-play bundle to offer greater mix and match (Krouse
2020). In retail banking, incumbent banks typically offer bundles of services—such as
checking and saving accounts, credit cards, and mortgages—whereas new entrants (e.g.,
N26, Chime, TransferWise), which are mostly digital, often focus on a particular product
or customers segment (Bakos et al. 2005). Traditional newspapers, such as the New York
Times, are bundles of news of broad interest, whereas new entrants take a more focused
approach, such as The Athletic, which covers only sport-related news. In entertainment
streaming services, Disney has responded to Netflix’s growing importance in the segment
of “stories and entertainment” by offering Hulu both as a standalone product and as part
of a bundle with Disney+ (brand and family) and ESPN+ (sports).¹

As these examples suggest, different bundling strategies are adopted under competition,
and they evolve over time. A common perception of bundling is that it is anti-competitive
because it either lowers consumer surplus (Schmalensee 1984), provides a monopolist lever-
age to foreclose sales in a secondary market (Whinston 1990), or pre-empts entry (Nalebuff
2004). A classical case study was the charge against Microsoft’s bundling of its browser
(Internet Explorer) with its operating system (Windows). In a curious repeat of history,
Slack, a team communication software, recently filed a lawsuit against Microsoft for its
bundling of Teams with Office 365.

To explain the numerous strategies observed in practice and determine whether or not
they are anti-competitive, we develop a theory of competitive bundling that exploits a
Bertrand competition model. We consider two symmetric firms that first choose their prod-
uct offering and then compete on price in a Bertrand-Nash pricing game. Unlike the extant
literature, we do not restrict firms’ actions, i.e., firms can choose any possible offering.

Although firms are ex-ante identical, they may differentiate in equilibrium. Within the framework of that stylized model, we investigate several questions: What bundling strategies emerge in equilibrium? Would firms always seek to avoid head-to-head competition by differentiating their offering? Would they even co-exist in the market? Is bundling always anti-competitive? How are equilibrium outcomes affected by the structure of customer valuations (correlation, heterogeneity)?

We address these questions by considering a symmetric Bertrand duopoly in which firms can offer two undifferentiated components (e.g., washer and dryer; burgers and fries) either separately or as a bundle. Given two components, A and B, a product may consist of a single component (i.e., either A or B) or the bundle (AB). Firms can choose any offering of products they wish, i.e., any subset—including the null set—of \{A, B, AB\}. To make our model as streamlined as possible, we make similar assumptions to Adams and Yellen (1976). On the supply side we assume zero costs of including products in an offering and zero costs of transaction and delivery—as is common to many information goods. On the demand side, we assume that customers purchase the products that yield them the greatest surplus, that they purchase at most one unit of each component, and their valuations of the bundle are equal to the sum of their valuations of the respective components. We first assume uniform valuations for the components that are either independent (IV), perfectly negatively correlated (NV), or perfectly positively correlated (PV). Motivated by highly segmented business-to-business markets, we then study a market with two equal-size atomic customer segments (2CS) whose valuations are perfectly negatively correlated. We finally consider a general distribution of valuations under the restriction that firms can offer at most one product.

We obtain the following results. First, when firms can choose any offering, three types of equilibria always emerge in equilibrium:

- **Differentiated duopoly**: One firm offers the bundle, and the other firm only offers a single-component product, which is not offered independently by the bundler. The bundler earns more profit, which suggests first-mover advantages to bundling.
- **Monopoly**: One firm offers the full set of products as a monopolist and the other firm stays out of the business.
- **Perfect competition**: Both firms offer both single-component products (possibly along with the bundle) and compete on price, driving their profits down to zero.
A fourth type of equilibrium emerges in the case of two atomic customer segments with relatively homogenous valuations, namely non-competing mono-product monopolies, in which firms offer distinct single-component products and act as monopolists in their respective markets. When firms may offer at most one product, there is always a differentiated duopoly equilibrium; other equilibria are possible, but they are weakly dominated. Although the literature identified the differentiated duopoly as the outcome of a game with restricted action sets (Carbajo et al. 1990, Nalebuff 2004), it is reassuring that this outcome still emerges when the action set is fully unconstrained. Yet, it is also worth pointing out that it is only one out of three (or four) possible types of outcomes, so focusing on that outcome tells an incomplete story.

Second, we find that bundling can be at the same time anti-competitive and hyper-competitive. The profit differential between the bundler and the other firm is known to be significant in the differentiated duopoly equilibrium outcome (Nalebuff 2004), but the situation can be much worse in a monopoly or irrelevant under perfect competition. Moreover, forcing a monopolist to unbundle may not be the solution. Indeed, it is precisely because the monopolist offers the single-component products along with the bundle that the other firm opts not to enter the market.

Third, we find that the degree of correlation and heterogeneity in customer valuations matter for competitive dynamics. For a monopolist, negative correlation in valuations is desirable because it enables the firm to extract more surplus from consumers through bundling; however in a duopoly, it also opens a wider door to market entry, as it makes a rival’s position more viable in the presence of fixed entry costs. In the differentiated duopoly outcome, the bundler’s relative profit advantage remains moderate when customer valuations are negatively correlated and highly heterogenous, but it could grow out of bound when their valuations are positively correlated or relatively homogeneous. Hence, when determining whether or not a firm’s bundling strategy is anticompetitive, the regulator should assess the degree of correlation and heterogeneity in customer valuations.

The rest of this paper proceeds as follows. Section 2 reviews the literature on bundling and outlines our contribution. In Section 3, we model bundling and pricing as a two-stage non-cooperative game. We characterize the equilibrium bundling and pricing strategies under uniform valuations in Section 4, in a market with two customer segments with negatively correlated valuations in Section 5, and under general distribution of valuations,
but under the restriction of single-product offerings in Section 6. Section 7 summarizes our results. All proofs and supporting results are given in the appendix.

2. Literature Review

Since the seminal work by Stigler (1963), the economics and management literature has explored the numerous benefits and pitfalls of bundling—first from a monopolist’s perspective and, more recently, in oligopolistic settings. For surveys of the literature, see Stremersch and Tellis (2002), Kobayashi (2005), and Venkatesh and Mahajan (2009).

For a monopolist, bundling offers several benefits in terms of product performance, as well as economies of scope in production, distribution, and promotional activities (Eppen et al. 1991, Evans and Salinger 2005). Bundling may also expand demand in response to complementarities among the bundle components (Venkatesh and Kamakura 2003) or to its greater perceived value (Sharpe and Staelin 2010). More subtly, bundling is a form of price discrimination because it renders customer valuations less heterogeneous (Stigler 1963, Adams and Yellen 1976, Schmalensee 1984, McAfee et al. 1989, Salinger 1995, Bakos and Brynjolfsson 2000, Raghunathan and Sarkar 2016). In particular: a monopolist can extract more surplus from its customers—especially when their valuations of the different components are negatively correlated—by offering a bundle of components, for which there is little heterogeneity in valuations, than by offering the components separately. This insight is especially relevant for goods that have zero marginal costs, such as information goods (Bakos and Brynjolfsson 1999). However, bundling is less attractive in the presence of “double marginalization” in the supply chains for physical goods (Bhargava 2012, Girju et al. 2013), of congestion for physical services (Wu and Yang 2019), of ample demand relative to the inventory (Abdallah et al. 2019), of digital piracy for information goods (Wu et al. 2019), and of asymmetric network externalities (Prasad et al. 2010).

Bundling also gives the monopolist leverage in other markets. Thus, a firm that is a monopolist on one component, but competes with other firms on another component, can leverage its monopolist position by bundling the two components together and foreclosing rivals’ sales, thereby increasing its market power, in a competitive market (Whinston 1990). In fact, bundling can even pre-empt the entry of potential competitors or force the exit of current ones (Carlton and Waldman 2002, Nalebuff 2004, Peitz 2008). In addition, bundling enables the monopolist to shift “slack” resources from one market to the other, giving it
control over the rate, direction, and timing of innovation across markets (Choi 1996). Bakos and Brynjolfsson (2000) find that bundling allows large bundlers of information goods to outbid smaller ones in securing upstream content, discourages competitors’ entry in the bundler’s market while favoring entry of the bundler in adjacent markets, and discourages innovation by niche players. However, leverage has its limits: if the market for the second component is perfectly competitive (instead of a differentiated oligopoly), then there is no benefit to bundling (Schmalensee 1982).

An emerging stream of research has explored how bundling affects the intensity of competition in oligopolistic markets, which is the focus of our study. Yet, so far the literature has studied specific settings, as summarized in Table 1. Considering a firm that is a monopolist in one market and competes with another firm in another market, Carbajo et al. (1990) observe that bundling softens price competition; even under PV, bundling is profitable because it leads rivals to price less aggressively. Therefore, bundling benefits competitors in this case, which is at odds with the leverage effect. Yet, Nalebuff (2004) warns that the bundler’s profit advantage can be so large that it may pre-empt the market entry of the competitor in case of fixed entry costs. We demonstrate the robustness of that equilibrium outcome by showing that it remains an equilibrium under a broader action set, but also noting that it is only one out of three types of equilibrium outcomes. We also highlight that the bundler’s profit advantage, characterized by Nalebuff (2004) in the IV case, may be much smaller if customers have very heterogenous preferences, but may grow out of

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<td>{} A, B, AB}</td>
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</tbody>
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*Note: AB refers to the bundle, A&B refers to offering products A and B separately, A&AB refers to offering both product A and the bundle AB.*
bound if they are relatively homogenous. Our broader scope of analysis helps contextualize the statement that bundling may be anti-competitive since both more severe and softer assessments can be made. Building on the setup described by Schmalensee (1982), Chen (1997) considers two symmetric firms that offer a component in a duopoly (rather than in a monopoly) and another component in a perfectly competitive market. As in Carbajo et al. (1990) and Nalebuff (2004), bundling emerges in equilibrium even though it is not optimal for a monopolist in the focal setting (Schmalensee 1982). In contrast to Chen (1997), who considers the price in the second market as exogenous, we endogenize its price formation by considering a broader set of feasible offering decisions. Consequently, the bundling firm always earns more profit than the firm that offers a single-component product in our setting (i.e., contra Chen 1997). Similar to our study, Zhou et al. (2020) consider a setting in which both components are sold in duopolies; likewise, these authors find that bundling can soften competition. Yet, they assume that firms are asymmetric, allowing one firm to make its product offering decision before the other firm, and restrict the firms to offer both products, either separately or as a bundle. Hence, much of the emerging literature on the effects of bundling on competition presupposes some form of asymmetry between firms (with respect to market power or market presence) and restricts their decisions to a subset of the possible offerings. In contrast, we consider two symmetric firms that operate in two duopolies and are allowed to choose any offering.


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2 In fact, Bhargava (2013, p. 2182) argues that even in monopoly bundling, “many researchers have disallowed mixed bundling and focused on pure bundling (even though mixed bundling is known to be better, usually).”
that “introducing product differentiation is necessary for studying competitive bundling if firms have similar cost conditions, [for] otherwise, prices would settle at marginal costs and there would be no meaningful scope for bundling” (footnote 9), we establish that this implication may not hold when firms are free to choose their product offerings. In fact, product differentiation arises as an equilibrium outcome of the game.

In sum, the literature on competitive bundling has assumed either some form of differentiation among firms and/or products or some form of asymmetry in the markets in which they operate—or it has imposed restrictions on the types of offerings. Yet in its purest form, Bertrand competition presupposes complete symmetry (Bertrand 1883). Given the foundational nature of Bertrand competition, a natural first step in any study of competitive bundling should be to investigate its practice under complete symmetry of firms, products, and markets while considering the exhaustive set of offering strategies. Taking that step is precisely the objective of our study.

3. Model

We consider a Bertrand duopoly with two symmetric firms, indexed by $i \in \{1, 2\}$, that can offer two components (e.g., washer and dryer; burgers and fries) separately or as a bundle. Thus, each firm can offer three products, indexed by $k \in \{1, 2, 3\}$: product 1, consisting of component 1; product 2, consisting of component 2; and/or product 3, consisting of the bundle of components 1 and 2. Both firms operate under perfect information and are fully rational. We shall use $-i = 3 - i$ to denote the firm other than firm $i$ and $-k = 3 - k$ to denote the single-component product other than product $k$ ($k \in \{1, 2\}$). We first introduce the supply side, then the demand side, and finally the resulting equilibria. Our assumptions mirror those in the literature (Adams and Yellen 1976) extended to a duopoly setting.

Supply. Bundling decisions tend to be irreversible: once a firm decides on a product offering, it must usually remain committed to that decision for a substantial amount of time. We accordingly model this scenario as a two-stage game.

In the first stage, which we call the bundling game, firms choose their product offering (or bundling) strategy simultaneously and non-cooperatively. We assume that products can be included in a firm’s offering at no cost. For any $i \in \{1, 2\}$, let $z_i = (z_{ik})_{k=1,2,3}$ be firm $i$’s offering decision; here $z_{ik} = 1$ if firm $i$ offers product $k$ in its offering and $z_{ik} = 0$ otherwise. Let $Z$ denote the set of feasible offerings. In contrast to the extant literature, we
assume that firms can offer any subset of products, i.e., $Z = \{0, 1\}^3 = \{0, 1\} \times \{0, 1\} \times \{0, 1\}$.

In particular, firms are allowed to practice “partial mixed bundling,” i.e., offering one component independently and the other component only through the bundle, which is sometimes referred to as “tie-in sales” (Bhargava 2013). In Section 6, we consider a situation where firms can offer at most one product, i.e., $Z = \{(0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1)\}$, in case there is a large fixed cost to market a product.

In the second stage, the *pricing game*, firms choose their pricing decisions (again simultaneously and non-cooperatively) and compete in a Bertrand fashion. Let $p_i = (p_{ik})_{k=1,2,3}$ denote firm $i$’s vector of pricing decisions, which is assumed to be non-negative. If a firm does not offer a product, we assume it sets its price to infinity, i.e., $p_{ik} < \infty \iff z_{ik} = 1$, $\forall i \in \{1, 2\}$, $k \in \{1, 2, 3\}$. We assume that firms have the same marginal costs, which we normalize to zero—as is typical of many information goods (Bakos and Brynjolfsson 1999).

**Demand.** We consider a unit-size market of customers with heterogeneous valuations. We assume no complementarity or substitutability in customer valuations; accordingly, customer valuations for the bundle are equal to the sum of their valuations for each component of the bundle. Without loss of generality, we assume that valuations for each component lie between 0 and 1. We assume that customers buy at most one unit of each component, i.e., that there is full satiation in consumption. If they purchase a product, customers purchase it at its lowest available price; accordingly, the market price of product $k \in \{1, 2, 3\}$, denoted by $p_k$, is equal to $\min_i p_{ik}$. Customers optimize their purchasing decision to maximize their net surplus with ties being broken in favor of the bundle. For any $k \in \{1, 2, 3\}$, let $\zeta_k(v, p) \in \{0, 1\}$ indicate whether a customer with component valuations $v = (v_\kappa)_{\kappa=1,2}$ buys product $k$ given the market prices $p = (p_\kappa)_{\kappa=1,2,3}$. Therefore,

$$
\zeta_3(v, p) = 1 \iff p_3 \leq \min\{v_1, p_1\} + \min\{v_2, p_2\} \quad (1)
$$

for $k \in \{1, 2\}: \zeta_k(v, p) = 1 \iff v_k \geq p_k$ and $p_3 > \min\{v_1, p_1\} + \min\{v_2, p_2\}$. \quad (2)

Let $F(v)$ denote the joint cumulative distribution function of customers. Accordingly, the demand for product $k$ at price $p$ equals $D_k(p) = \int \zeta_k(v, p) dF(v)$ and customer total welfare equals

$$
W(p) = \int ((v_1 - p_1)\zeta_1(v, p) + (v_2 - p_2)\zeta_2(v, p) + (v_1 + v_2 - p_3)\zeta_3(v, p)) dF(v). \quad (3)
$$
Demand $D(p)$ turns out to be left-continuous in $p$ except at $p_1 + p_2 = p_3$ when $P[v_1 \geq p_1, v_2 \geq p_2] > 0$; hence discontinuities appear only if all three products are available.

In Section 4, we consider uniformly distributed valuations with the following structures of correlation:

- IV: independent uniform valuations; i.e., $F(v) = v_1 v_2$;
- NV: perfect negatively correlated uniform valuations, summing up to 1; i.e., $F(v) = \max\{v_1 + v_2 - 1, 0\}$;
- PV: perfect positively correlated uniform valuations; i.e., $F(v) = \min\{v_1, v_2\}$.

In Section 5, we consider two customer segments (2CS) of equal size with perfectly negatively correlated valuations and the same total valuation for the bundle, normalized to 1; i.e., $F(v) = (\mathbb{1}_{v_1 \geq v} + \mathbb{1}_{v_1 \geq 1-v}) (\mathbb{1}_{v_2 \geq v} + \mathbb{1}_{v_2 \geq 1-v}) / 4$, with $0 < v < 1/2$. Finally, we consider a general distribution of valuations in Section 6.

**Price Equilibria.** For any $i \in \{1, 2\}$ and $k \in \{1, 2, 3\}$, let $x_{ik}(p_{ik}; p_{-i,k})$ denote firm $i$’s market share on product $k$ when firm $i$ sets its price at $p_{ik}$ and its competitor sets it at $p_{-i,k}$. Under the assumption that firms equally share the market in case of a tie, firm $i$’s market share on product $k$ is equal to

$$x_{ik}(p_{ik}; p_{-i,k}) = \mathbb{1}_{p_{ik} < p_{-i,k}} + \frac{1}{2} \mathbb{1}_{p_{ik} = p_{-i,k} \land p_{ik} < \infty};$$

(4)

here, $\mathbb{1}_{y > 0} = 1$ if $y > 0$ and 0 if $y \leq 0$. Accordingly, firm $i$’s profit given its prices $p_i$ and offering $z_i$ and its competitor’s prices $p_{-i}$ and offering $z_{-i}$ equals:

$$\pi_i(p_i; p_{-i}, z_i, z_{-i}) = \sum_{k=1}^{3} p_{ik} x_{ik}(p_{ik}; p_{-i,k}) D_k(\min\{p_i, p_{-i}\}).$$

Given firms’ offerings ($z_i, z_{-i}$), firm $i \in \{1, 2\}$ sets its price to maximize its profit accounting for $-i$’s prices $p_{-i}$. The Bertrand-Nash pricing equilibrium is thus determined by solving:

$$p_i^* = \arg \max_{p_i} \pi_i(p_i; p_{-i}, z_i, z_{-i}) \quad i \in \{1, 2\}$$

(5)

s.t. $p_{ik} < \infty \iff z_{ik} = 1$ \quad $\forall k \in \{1, 2, 3\}$.

For any $i \in \{1, 2\}$, let $\pi_i(z_i; z_{-i}) = \pi_i(p_i^*; p_{-i}^*, z_i, z_{-i})$ denote the equilibrium profit in the pricing game. If both firms offer the same product, i.e., if $z_{1k} = z_{2k} = 1$ for some $k \in \{1, 2, 3\}$, then, due to Bertrand competition, no firm makes a positive profit from the sales of that product in equilibrium (see Lemma A-2 in appendix).
Table 2  Firms’ Payoffs under IV

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<td>0, 0</td>
<td>0, 0</td>
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</tr>
</tbody>
</table>

Note. The shaded cells correspond to bundling equilibria.

**Bundling Equilibria.** Anticipating the equilibrium in the pricing game, firms choose their bundling strategy to maximize their profit:

\[
    z_i^* = \arg \max_{z_i \in Z} \pi_i(z_i; z_{-i}^*) \quad i \in \{1, 2\}.
\]  

(6)

The bundling game turns out to always have a pure-strategy Nash equilibrium in the cases we consider (and so we ignore any mixed-strategy equilibria). Yet it might have multiple pure-strategy equilibria, in which case we employ various refinements, such as Pareto dominance, trembling-hand perfection (Selten 1975), and sequential entry in the presence of infinitesimal costs of offering products.

4. Uniform Valuations

We solve the game (5)–(6) by backward induction. For each of the \(2^3 \times 2^3 = 64\) possible combination of offerings (since \(Z = \{0,1\}^3\), we first characterize the Nash equilibria of the pricing game. We then identify the equilibrium strategies in the bundling game while incorporating the equilibrium outcomes of the pricing subgames.

Based on the pricing equilibria characterized in Lemmas A-4-A-8 in appendix, Tables 2-4 present the firms’ equilibrium payoffs, for each possible combination of offerings, in the cases of IV, NV, and PV. The next proposition characterizes the bundling equilibria, which are highlighted in Tables 2-4.
Table 3  Firms’ Payoffs under NV

<table>
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Note. The shaded cells correspond to bundling equilibria.

Table 4  Firms’ Payoffs under PV

<table>
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Note. The shaded cells correspond to bundling equilibria.

Proposition 1. Under IV, NV, and PV, the bundling equilibria are:
• Differentiated duopoly generating customer welfare $W^{IV} \approx 0.471571$, $W^{NV} = 7/18$, and $W^{PV} = 26/49$:
  — One firm offers the bundle only, i.e., $z_{i} = (0,0,1)$, and the other firm offers a single component, i.e., either $z_{-i} = (1,0,0)$ or $z_{-i} = (0,1,0)$;
One firm offers the bundle and a single-component product, i.e., either $z_i = (1, 0, 1)$ or $z_i = (0, 1, 1)$, and the other offers a distinct single-component product, i.e., either $z_{-i} = (0, 1, 0)$ or $z_{-i} = (1, 0, 0)$, respectively;

- Monopoly generating customer welfare $W^{IV} = \frac{2(6 + \sqrt{2})}{27}$, $W^{NV} = 0$, and $W^{PV} = \frac{1}{2}$:
  
  - One firm offers all three products, i.e., $z_i = (1, 1, 1)$, and the other firm offers nothing, i.e., $z_{-i} = (0, 0, 0)$;
  
  - Under PV only, one firm could offer the two standalone components without bundle, i.e., $z_i = (1, 1, 0)$ and the other firm offers nothing, i.e., $z_{-i} = (0, 0, 0)$;

- Undifferentiated duopoly generating customer welfare $W^{IV} = W^{NV} = W^{PV} = 1$: Each firm offers both components separately, with or without bundle, i.e., either $z_i = (1, 1, 0)$ or $z_i = (1, 1, 1)$ for $i = 1, 2$.

According to Proposition 1, there are three types of equilibria, which we discuss in reverse order. Under one type of equilibrium, firms offer both components separately, with or without bundle, and compete head to head, driving market prices for both components to zero. In this case, the bundling decision is irrelevant because the single components are offered at marginal cost (assumed to be zero). Hence, even if firms have numerous opportunities to differentiate their offerings, they may not necessarily take advantage of them and instead opt to compete directly against each other. This equilibrium is reminiscent of the head-to-head competition in the fast-food industry, where fast-food chains offer similar sandwiches and drinks separately or as a bundle (combo).

Customers benefit from this lack of differentiation since their welfare is maximized. The caveat is that this equilibrium is Pareto-dominated; thus if firms could coordinate, they may opt to operate under a different type of equilibrium. In this case, the regulator should not prevent bundling (which is irrelevant), but rather prevent coordination.

Under the second type of equilibrium, one firm operates as a natural monopoly: This firm offers all products (a strategy that is often referred to as “mixed bundling”) to prevent the other firm from offering anything—effectively preventing market entry. (In the particular case of PV, the fact that the monopolistic firm offers the bundle is irrelevant since all customers are indifferent between purchasing the bundle or both components separately.) The potential rival indeed knows that it would not make a profit after entry irrespective of its offering, as it anticipates head-to-head competition against the incumbent. In fact,
this is the only equilibrium that survives in a model of *sequential entry* with infinitesimal costs of including products in an offering. To illustrate this situation in practice, Uber announced that it would offer access to bikes and scooters, public transportation, and self-driving cars in addition to human drivers. As a result, Lyft faces two choices: either exit the market, as predicted by this type of equilibrium which is what almost happened, or build the same platform, which is what Lyft seems to follow, potentially leading the firms toward head-to-head competition as in the first type of equilibrium.\(^3\)

Customer welfare is very low under this monopolistic equilibrium, in contrast to the head-to-head competition equilibrium. This seems challenging for the regulator to handle. Entry cannot happen because all products are offered; thus, one cannot force the monopolist to unbundle its offering because it is already doing it.

In the third type of equilibrium, firms have differentiated offerings: One firm offers a single-component product, and the other firm offers the bundle, with or without the other single-component product. Once a firm offers the bundle, the other firm prefers to avoid engaging in a price war and instead offers only a single component. Conversely, if one firm offers a single-component product, then the other firm finds it attractive to offer the bundle—that is, to expand horizontally. Thus the asymmetry in bundling strategies results from the firms’ attempt to soften price competition by differentiating their product offerings. This outcome, characterized by Carbajo et al. (1990), Chen (1997), Nalebuff (2004), and Zhou et al. (2020) in restricted settings (see Table 1), remains an equilibrium when firms are allowed to choose any product offering. Moreover, this equilibrium turns out to be *trembling-hand perfect*.

It is instructive to compare these two outcomes: (i) when one firm offers the bundle and a single-component product and the other firm offers only the other component (e.g., \(z_1 = (1, 0, 1)\) and \(z_2 = (0, 1, 0)\)); and (ii) when both firms bundle in addition to offering their respective single-component products (\(z_1 = (1, 0, 1)\) and \(z_2 = (0, 1, 1)\)). One might suppose that it is preferable to have more options, which is indeed the case in a monopolistic setting (Adams and Yellen 1976, McAfee et al. 1989). Yet in a duopoly, both firms earn more profit if one of them decides to omit the bundle from its offering. We remark that, because the pricing decision is noncommittal, this decision is not equivalent to setting the price

of the bundle so high that no customer purchases it; given that the firm actively selling the bundle makes more profit, any hard commitment to a pricing scheme would not be sustainable. It would indeed be too tempting for the firm that is not selling the bundle to lower its price and capture its competitor’s profit. In other words, including the bundle in a product offering without any intention of selling it will inevitably lead firms to engage in a price war and, hence, earn zero profit. In line with the saying that “strategy is not only about which projects to choose but also about which projects not to choose,” a firm that faces a bundling competitor might be better off deliberately excluding the bundle from its offering to preclude any later temptation to fight for value capture.

Similarly, if one firm offers the bundle and the other firm offers only a single-component product, then both firms are worse off when the bundler offers the same single-component product in its offering. In principle, that inclusion may cost the bundler nothing because both components are already included in its bundle; yet doing so would lead to a price war on that product, reducing the bundler’s pricing advantage. Thus, once the bundling firm includes the single-component product in its offering, it can no longer commit not to compete on price with the other firm.

There exists another similar equilibrium (but not trembling-hand perfect) in which the bundler offers the other single-component product. In a celebrated case study on bundling (Berenson 2006), Pfizer offered torcetrapib initially only in combination with Lipitor, a statin that competes with Merck’s Zocor. After being criticized for that bundling strategy, Pfizer decided to offer Lipitor separately. Within the framing of our model, Pfizer’s offering switched from \((0,0,1)\) to \((1,0,1)\), while Merck’s offering was \((0,1,0)\). Obviously, the real friction in practice that prevented Pfizer from initially offering torcetrapib as a standalone product might have been its development cost, assumed to be zero in our model.

The firm that bundles earns much greater profit, which suggests first-mover advantage to bundling.\(^4\) Comparing Tables 2-4 indicates that the firm that bundles earns eight times more profit under PV, about five times more profit under IV, and four times more profit under NV. In the presence of fixed entry costs, this equilibrium will be harder to sustain under PV than under NV. To illustrate this type of equilibrium with a recent example, consider the competitive dynamics between Microsoft’s Teams software, which is integrated

\(^4\) This was already noted by Carbajo et al. (1990) and Nalebuff (2004) respectively in the PV and IV cases.
into the firm’s Office 365 suite, and Slack. Slack initially didn’t feel threatened, claiming differentiation, even stating that “Teams is not a competitor to Slack.” However, they recently reverted back and filed an antitrust complaint against Microsoft (Lohr 2020). Obviously, our stylized model does not capture all relevant dimensions in practice, such as established user base, sales channels, or complementary effects, but it illustrates the precarious position of the firm that focuses on a single-component product.

Customer welfare under this differentiated duopoly equilibrium lies somewhere between the welfare under the perfect competition equilibrium and the welfare under the monopoly equilibrium. For customer welfare to improve, the bundling firm should be forced to provide the single component offered by the other firm separately from the bundle, as if Microsoft were forced to price Teams separately. Even so, customer welfare would only grow from 0.472 to 0.625 under IV if the other firm does not change its offering. (Interestingly, the rival’s profit would then drop to zero. So it is not clear that Slack has much to gain from Microsoft’s unbundling of Teams from Office if that is ensured by a price war.) Absent development costs, the rival may then respond by changing its offering and either offer the other component (e.g., as if Slack were to develop its own office suite) and then compete head to head with the bundler (which would benefit customers) or exit the market (which would hurt customers). In this case, forcing unbundling may be a double-edged sword.

In summary, Proposition 1 reveals that the set of equilibria is much richer than characterized in the literature once firms are given full freedom on their choice of offerings. Although firms are ex-ante symmetric, they may choose to differentiate in equilibrium. The literature has focused on the differentiated duopoly outcome and highlighted its anti-competitive nature, differentiation of offerings could lead to a monopoly, which is even worse from a customer welfare standpoint. From a pure welfare perspective, a desirable outcome is head-to-head competition, which remains possible even if it has been overlooked by the literature.

5. Two Customer Segments with Perfectly Negatively Correlated Valuations

Motivated by business-to-business markets (e.g., large enterprise IT projects), we next consider two customer segments (2CS) of equal size with perfectly negatively correlated

5 https://www.youtube.com/watch?v=W3CpVK83az4, accessed August 14, 2020
valuations and the same total valuation for the bundle, normalized to 1. This model of valuation is similar to that in Adams and Yellen (1976). Customer group 1 values component 1 at \( v \) and component 2 at \( 1 - v \), and customer group 2 has the opposite valuations, with \( 0 < v < 1/2 \). The next proposition characterizes the bundling equilibria in this case.

**Proposition 2.** Under 2CS, the bundling equilibria are:

- **Differentiated duopoly:**
  - One firm offers the bundle only, i.e., \( z_i = (0, 0, 1) \), and the other firm offers a single component, i.e., either \( z_{-i} = (1, 0, 0) \) or \( z_{-i} = (0, 1, 0) \);
  - One firm offers the bundle and a single-component product, i.e., either \( z_i = (1, 0, 1) \) or \( z_i = (0, 1, 1) \), and the other offers a distinct single-component product, i.e., either \( z_{-i} = (0, 1, 0) \) or \( z_{-i} = (1, 0, 0) \), respectively;
  - When \( v \geq 1/3 \), if one firm offers only a single component, the other firm may offer both components in any form, i.e., any \( z_i \) such that either \( z_{i3} = 1 \) or \( z_{i1} = z_{i2} = 1 \).

- **Monopoly:**
  - One firm offers all three products, i.e., \( z_i = (1, 1, 1) \), and the other firm offers nothing, i.e., \( z_{-i} = (0, 0, 0) \);
  - When \( v \geq 1/3 \), the monopolist bundler may choose not to carry other products, i.e., any \( z_i \) such that \( z_{i3} = 1 \) and \( z_{-i} = (0, 0, 0) \) is an equilibrium.

- **Undifferentiated duopoly:**
  - Both firms offer both components, either separately or as a bundle, i.e, either \( z_i = (1, 1, 0) \) or \( z_i = (1, 1, 1) \);
  - When \( v \geq 1/3 \), a bundling firm may choose to drop the standalone products, i.e, either \( z_i = (1, 1, 0) \) or any \( z_i \) such that \( z_{i3} = 1 \).

  - When \( v \geq 1/3 \), non-competing mono-product monopolies, i.e., \( z_i = (1, 0, 0) \) and \( z_{-i} = (0, 1, 0) \).

According to Proposition 2, the number of equilibria is significantly larger when \( v \geq 1/3 \) than when \( v < 1/3 \). When \( v < 1/3 \), which happens when the customer segments are quite heterogenous, we obtain the same equilibrium outcomes as in Proposition 1, namely: a differentiated duopoly (and this equilibrium is trembling-hand perfect when the bundler offers only the bundle), a monopoly (which is the only equilibrium that survives in a model of sequential entry with infinitesimal costs of offering a product), and perfect competition.
(which is Pareto-dominated). Hence, the results obtained in the case of uniform valuations are robust to greater polarization in valuations.

Among the three types of equilibria, the differentiated duopoly is unique in the sense that its pricing subgame does not have a pure-strategy Nash equilibrium; it has only a mixed-strategy Nash equilibrium. When \( z_i = (0, 0, 1) \) and \( z_{-i} = (1, 0, 0) \), the customer who has the lowest valuation for product 1 (since \( v < 1/2 \) is captive to firm \( i \), so the two firms will be competing for the other customer. Firm \( i \) will “mix” between two strategies, sometimes attempting to attract the non-captive customer (when the market price for product 1 is high), sometimes attempting to extract as much surplus from the captive customer without attempting to capture the other one (when the market price for product 1 is low). Hence, this equilibrium is associated with volatile prices and customer switches.

When customer valuations are more homogeneous, i.e., when \( 1/3 \leq v < 1/2 \), the same three types of equilibrium outcomes emerge, but under a broader set of offerings. In particular, in the differentiated duopoly, the firm that offers both components does not need to bundle them. In the monopoly, the active firm does not have to offer all three products; just offering the bundle is enough to deter entry. Similarly, in the situation of perfect competition, any firm that offers the bundle may opt to drop offering the components separately. In addition, a fourth type of equilibrium emerges when \( 1/3 \leq v < 1/2 \), namely one in which firms choose to focus on a distinct single-component product market, thereby avoiding competition by operating as local monopolies in their respective markets. Consequently, when \( 1/3 \leq v < 1/2 \), 55 out of 64 combinations of offerings result in equilibria. Despite this large number of equilibria, the game retains its predictive power given that many equilibria are payoff-equivalent—what matters is that four types of equilibrium outcomes emerge.

Moreover, these equilibrium outcomes are almost the same as if valuations were perfectly positively correlated.\(^6\) Hence, when the two customer segments have relatively homogenous valuations, in the sense that a segment’s valuation of a component is no larger than twice the other segment’s valuation for the same component (i.e., \( v/(1-v) \leq 2 \)), it is as if they had the same valuations. Therefore, in a duopoly with two customer segments, moderate heterogeneity in customer valuations is not enough to induce different market structures.

\(^6\) With perfectly positively correlated valuations, the only additional outcome is a monopoly, in which the monopolist offers both components separately without bundle, as in Table 4. Details are omitted.
Next, we study the profit split in the differentiated duopoly outcome. As in Proposition 1, the firm that bundles earns more than the one that offers a single-component product. A plot of the profit differential (Figure 1) based on Lemma A-12 in the appendix reveals that the bundler’s profit advantage is convexly increasing in \( v \). In particular, when \( v \to 0 \), the bundler earns twice the profit of its rival. When \( v = 1/4 \), it earns four times more (as in NV). When \( v \geq 1/3 \), the rival earns zero, so the relative profit advantage is infinite. Hence, the more homogenous the valuations of the two segments, the greater the bundler’s relative profit advantage over its rival, complementing our earlier result with regard to the correlation structure under uniform valuations. From a regulatory perspective, the convex nature of this profit differential makes antitrust decisions sensitive to the degree of heterogeneity in valuations: While it is certainly acceptable that a firm that offers two components earns twice as much as a firm that offers only one component (when \( v \approx 0 \)), profit differentials that go out of bound deserve scrutiny. Therefore, bundling may not necessarily be detrimental in the presence of heterogeneous segments of customers, but it could be severely anti-competitive when the segments are more homogenous.

6. Single-Product Offerings under a General Distribution of Valuations

In cases where it is expensive to package and sell multiple products, firms may be constrained to offer, at most, one product. In that case, \( Z = \{(0,0,0), (1,0,0), (0,1,0), (0,0,1)\} \), which we refer to as the single-product offering game. In Tables 2-4, eliminating the rows and columns where \( \sum_k z_{ik} > 1 \) for any \( i \in \{1,2\} \) shows that the only equilibria left are settings in which one firm offers the bundle and the other firm offers a single-component product. Similarly under 2CS, only such equilibria remain when \( v < 1/3 \); when \( v \geq 1/3 \),
there are other equilibria such as a monopoly (i.e., $z_i = (0, 0, 1)$ and $z_{-i} = (0, 0, 0)$), head-to-head competition on the bundle (i.e., $z_i = z_{-i} = (0, 0, 1)$), and non-competing mono-product monopolies (i.e., $z_i = (1, 0, 0)$ and $z_{-i} = (0, 1, 0)$).

The next proposition assesses the robustness of these findings by considering a general distribution of valuations $F(v)$.

**Proposition 3.** For any distribution $F(v)$, the single-product offering game has at least two equilibria, in which one firm offers the bundle and the other offers a single-component product. In such equilibria, the bundling firm earns more profit than the non-bundling firm.

Hence, the emergence of the differentiated duopoly as an equilibrium outcome and the profit advantage of the bundler in that equilibrium are general results, which do not depend on any specific distribution of valuations. This result complements and generalizes Chen (1997), who considers a limited set of offerings, namely $Z = \{(1, 0, 0), (0, 0, 1)\}$, under the condition that $p^*_2 = 0$ (i.e., the market for product 2 is perfectly competitive). We generalize this result by considering an expanded set of actions and complement it by endogenizing the price formation on product 2’s market. Although we identify a similar equilibrium, we find here that the bundler earns more profit, in contrast to Chen (1997), who shows that the bundler earns less profit when $p^*_2 = 0$.

7. Conclusions

In this paper, we take a first step to understand the competitive bundling dynamics by studying a stylized model of competitive bundling in a Bertrand duopoly. We consider two symmetric firms that simultaneously and non-cooperatively make offering (bundling) decisions followed by pricing decisions. In contrast to the extant literature, we assume that firms can offer any combination of products. Within this setting, we characterize the equilibrium bundling strategies under uniform valuations and various correlation structures, with two customer segments with perfectly negatively correlated valuations, and under a general distribution of valuations when firms can offer, at most, one product.

We find that three types of equilibria always emerge in equilibrium: a **differentiated duopoly**, in which one firm offers the bundle and the other firm offers only a single-component product; a **monopoly**, in which one firm offers the full set of products as a monopolist and the other firm stays out of the business; and **perfect competition**, in which both firms offer both single-component products (possibly along with the bundle) and
compete on price, driving their profits down to zero. When customers have relatively homogenous valuations, a fourth type of equilibrium emerges—namely *non-competing mono-product monopolies*—in which firms offer distinct single-component products and act as monopolists in their respective markets. One type of equilibrium leading to the differentiated duopoly outcome is when one firm offers only the bundle and the other firm offers only a single-component product. This equilibrium is not only trembling-hand perfect, it also arises for any distribution of valuations when firms are restricted to offer, at most, one product. Although the extant literature identified that particular equilibrium in restricted settings, we find that different equilibria may lead to the same outcome (e.g., the bundler could also offer a distinct single-component product) and that this outcome is only one out of three (or four) possible outcomes.

Now that we have provided the full picture on the set of possible equilibria under competitive bundling, we clarify on the debate about the anti-competitive nature of bundling, which so far had been centered on the differentiated duopoly outcome. With our comprehensive characterization of equilibrium outcomes, this debate may indeed appear irrelevant in some cases (if the outcome ends up being perfect competition) and much more severe than what was previously conceived in other cases (if the outcome ends up being a monopoly). Even within the outcome of a differentiated duopoly, the bundler’s profit advantage over the other firm was considered to be high when it was in the range of 4–7 under uniform valuations (Nalebuff 2004). However, it could be both milder or much more severe with only two customer segments: If these segments have highly heterogenous, negatively correlated valuations, then the bundler’s profit advantage is marginal. But if these segments have relatively homogenous valuations, in the sense that a segment does not value a component more than twice what the other segment does, then the bundler’s profit advantage grows out of bound. Hence, when determining whether or not a firm’s bundling strategy is anticompetitive, the regulator should assess the degree of correlation and heterogeneity in customer valuations.

Through our comprehensive analysis, our objective was to provide a more complete picture of competitive bundling in a duopoly. Our goal was to highlight that, on the one hand, differentiation does not have to be assumed ex-ante as it may arise in equilibrium but also that, on the other hand, a differentiated duopoly is not the only outcome of the game. To pursue this effort, it would be worth relaxing some of the simplifying assumptions, such
as considering different marginal costs as in Carbajo et al. (1990)—especially if the focus
is on physical goods, complementarity effects, asymmetric valuations (Bhargava 2013),
alternate (e.g., Cournot) forms of competition, more than two products, and more than two
firms. With digitalization of information, media companies tend to expand horizontally,
i.e., to operate across multiple markets, so the study of bundling is more relevant than
ever.

References
Abdallah T, Asadpour A, Reed J (2019) Revenue management with bundles, INFORMS Annual Meeting
presentation.
Ahn I, Yoon K (2012) Competitive mixed bundling of vertically differentiated products. The BE Journal of
Economic Analysis & Policy 12(1).
com/2006/07/26/business/26drug.html.
1021.
Carlton DW, Waldman M (2002) The strategic use of tying to preserve and create market power in evolving


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Appendix: Proofs and Supplementary Results

EC.1. Preliminaries

**Lemma A-1.** \(D(p)\) is left-continuous except at \(p_1 + p_2 = p_3\) when \(P[v_1 \geq p_1, v_2 \geq p_2] > 0\).

**Proof.** By (1)-(2),

\[
D_k(p) = \int_{p_k}^{p_2} \int_{\max(0, p_3 - v_1)}^{p_2} dF(v) + \int_{p_1}^{1} \int_{\max(0, p_3 - p_k)}^{1} dF(v) \mathbb{1}_{[v_3 > p_1 + p_2]}
\]

For \(k = 1, 2\):

\[
D_1(p) = \int_{p_1}^{1} \int_{0}^{\min(p-k, p_3 - p_k)} dF(v) + \int_{p_1}^{1} \int_{p_2}^{1} dF(v) \mathbb{1}_{[v_3 > p_1 + p_2]}
\]

Because \(F(v)\) is right-continuous, the only point where \(D(p)\) is not left-continuous is at \(p_1 + p_2 = p_3\) when \(P[v_1 \geq p_1, v_2 \geq p_2] > 0\). Indeed, when \(p_1 + p_2\) drops slightly below \(p_3\), \(D_k(p)\), for \(k = 1, 2\), discontinuously increase by \(P[v_1 \geq p_1, v_2 \geq p_2]\) and \(D_3(p)\) drops by the same amount. \(\square\)

**Lemma A-2.** For any \(k = 1, 2, 3\), if \(z_{1k} = z_{2k} = 1\) and \(D_k(p^*) > 0\), then \(p_k^* = 0\).

**Proof.** To obtain a contradiction, suppose that there exists an equilibrium \(p^*\) such that \(D_k(p^*) > 0\) and \(p_k^* > 0\). Suppose first that, for any \(i \in \{1, 2\}\), \(p_{i,k}^* > p_{i,k}^*\). In that case, firm \(i\) could lower its price \(p_{i,k}\) to be equal to \(p_{i,k}^*\) without changing anything to \(D(p^*)\) and increase its profit by \(p_{i,k}^*D_k(p^*)/2 > 0\) by (4), a contradiction.

Next, suppose that \(p_{1k}^* = p_{2k}^* = p_k^*\) and that \(D(p)\) is left-continuous in \(p_k\) at \(p^*\). In that case, firm \(i\) could strictly increase its sales of product \(k\) by \(D_k(p^*)/2\) by setting \(p_{i,k}\) infinitesimally below \(p_{i,k}^*\), which is feasible given that \(p_k^* > 0\), without reducing its sales of the other products by more than an infinitesimal amount by (4) and continuity of \(D(p)\). As a result, firm \(i\)'s profit would increase by about \(p_{1,k}^*D_k(p^*)/2 > 0\), a contradiction.

Finally, suppose that \(p_{1k}^* = p_{2k}^* = p_k^*\) and that \(D(p)\) is not left-continuous in \(p_k\) at \(p^*\), which, by Lemma A-1, holds when \(p_1^* + p_2^* = p_3^*\) and \(P[v_1 \geq p_1^*, v_2 \geq p_2^*] > 0\). Without loss of generality, suppose also that \(p_{3,k}^* > p_{3,k}^*\). In that case, because \(p_3^* > 0\) (since \(p_k^* > 0\)), firm \(i\) could drop its price \(p_{3,k}\) infinitesimally below \(p_3^*\) and increase its sales of product \(3\) by at least \(P[v_1 \geq p_1^*, v_2 \geq p_2^*]/2 > 0\), a contradiction. Next, assume that \(z_{13} = 0\) for some firm \(i\), i.e., \(x_{-i,3}(p_{-i,3}^*; p_{3,k}^*) = 1\) by (4). In that case, firm \(i\) could drop its price \(p_{i,k}\) infinitesimally below \(p_{i,k}^*\), increase its sales of product \(k\) by \(P[v_1 \geq p_1^*, v_2 \geq p_2^*] > 0\), and strictly increase its profit, a contradiction. \(\square\)

**Lemma A-3.** If \(z_{i3} = 1\) for any \(i \in \{1, 2\}\), then firm \(i\) is always (weakly) better-off selling the bundle; that is \(x_i(p_{i,3}^*, p_{-i,3}^*)D_3(p^*) > 0\).

**Proof.** To obtain a contradiction, suppose that selling the bundle makes firm \(i\) strictly worse off. Then \(D_3(p^*) = 0\), i.e., \(\zeta_{3}(v, p) = 0\) for all \(v\). Without loss of generality, assume that \(p_3^* = \infty\). Then,

\[
\pi_i(p_i^*; p_{-i}) = \sum_{k=1}^{2} p_{i,k}^* x_k(p_{i,k}, p_{-i,k})D_k(p^*)
\]
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\[ D^I_3(p) =  \begin{cases} 
1 - \frac{p^2}{2} & \text{if } 0 \leq p_3 \leq 1 \\
\frac{(2-p_3)^2}{2} & \text{if } 1 \leq p_3 \leq 2,
\end{cases} \]

**EC.2. Uniform Valuations**

**Lemma A-4.** When \( \max\{z_1, z_2\} = (1, 0, 0) \),

- if \( z_i = (1, 0, 0) \) and \( z_{-i} = (0, 0, 0) \), then for \( v \in \{IV, NV, PV\} \), \( \pi^v_i(z_i; z_{-i}) = 1/4 \), \( \pi^v_{-i}(z_{-i}; z_i) = 0 \), and \( W^v(z) = 1/8 \);

- if \( z_i = (1, 0, 0) \) for \( i = 1, 2 \), then for \( v \in \{IV, NV, PV\} \), \( \pi^v_i(z_i; z_{-i}) = 0 \) for \( i = 1, 2 \) and \( W^v(z) = 1/2 \).

Symmetric results hold when \( \max\{z_1, z_2\} = (0, 1, 0) \).

**Proof.** When \( \max\{z_1, z_2\} = (1, 0, 0) \), for any \( v \in \{IV, NV, PV\} \), \( D^I_1(p) = 1 - p_1 \) for \( 0 \leq p_1 \leq 1 \) by (2). Hence:

- If \( z_i = (1, 0, 0) \) and \( z_{-i} = (0, 0, 0) \), then \( p^*_i = \arg\max_{0 \leq p_i \leq 1} p_1 D^I_1(p_1) = 1/2 \).

- If \( z_i = (1, 0, 0) \) for \( i = 1, 2 \), the result follows by Lemma A-2. \( \square \)

**Lemma A-5.** When \( \max\{z_1, z_2\} = (0, 0, 1) \),

- if \( z_i = (0, 0, 1) \) and \( z_{-i} = (0, 0, 0) \), \( \pi^V_i(z_i; z_{-i}) = (2/3)\sqrt{2} \), \( \pi^{IV}_{-i}(z_{-i}; z_i) = 0 \), and \( W^{IV}(z) = 1 - 8\sqrt{6}/27 \);

- \( \pi_i^{NV}(z_i; z_{-i}) = 1 \), \( \pi^{NV}_{-i}(z_{-i}; z_i) = 0 \), and \( W^{NV}(z) = 0 \); \( \pi_i^{PV}(z_i; z_{-i}) = 1/2 \), \( \pi^{PV}_{-i}(z_{-i}; z_i) = 0 \), and \( W^{PV}(z) = 1/4 \);

- if \( z_i = (0, 0, 1) \) for \( i = 1, 2 \), then for \( v \in \{IV, NV, PV\} \), \( \pi^v_i = 0 \) for \( i = 1, 2 \) and \( W^v(z) = 1 \).

**Proof.** When \( \max\{z_1, z_2\} = (0, 0, 1) \), by (1),
The results follow from Lemmas A-2 and A-4. When 

\[ D_3^{NV}(p) = 1 \quad \text{if } 0 \leq p_3 \leq 1, \]
\[ D_3^{PV}(p) = 1 - \frac{p_3}{2} \quad \text{if } 0 \leq p_3 \leq 2. \]

Hence:

- If \( z_i = (0,0,1) \) and \( z_{-i} = (0,0,0) \), then \( p_3^{IV} = \arg \max_{0 \leq p_3 \leq 2} p_3D_3^{IV}(p_3) = \sqrt{2/3} \)
- \( p_3^{NV} = \arg \max_{0 \leq p_3 \leq 1} p_3D_3^{NV}(p_3) = 1 \)
- \( p_3^{PV} = \arg \max_{0 \leq p_3 \leq 2} p_3D_3^{PV}(p_3) = 1. \)

- If \( z_i = (0,0,1) \) for \( i = 1,2 \), the result follows by Lemma A-2. □

**Lemma A-6.** When \( \max \{z_1, z_2\} = (1,1,0) \),

- if \( z_i = (1,1,0) \) and \( z_{-i} = (0,0,0) \), then for \( v \in \{IV,NV,PV\} \), \( \pi_i^v(z_i; z_{-i}) = 1/2, \pi_{-i}^v(z_{-i}; z_i) = 0, \) and \( W^v(z) = 1/4; \)
- if \( z_i = (1,0,0) \) and \( z_{-i} = (0,1,0) \), then for \( v \in \{IV,NV,PV\} \), \( \pi_i^v(z_i; z_{-i}) = 1/4, \pi_{-i}^v(z_{-i}; z_i) = 0, \) and \( W^v(z) = 1/4; \)
- if \( z_i = (1,1,0) \) and \( z_{-i} = (1,0,0) \) or \( z_{-i} = (0,1,0) \), then for \( v \in \{IV,NV,PV\} \), \( \pi_i^v(z_i; z_{-i}) = 1/4, \pi_{-i}^v(z_{-i}; z_i) = 0, \) and \( W^v(z) = 5/8; \)
- if \( z_i = (1,1,0) \) for \( i = 1,2 \), then for \( v \in \{IV,NV,PV\} \), \( \pi_i^v(z_i; z_{-i}) = 0 \) for \( i = 1,2 \) and \( W^v(z) = 1. \)

**Proof.** The results follow from Lemmas A-2 and A-4. □

**Lemma A-7.** When \( \max \{z_1, z_2\} = (1,0,1) \),

- if \( z_i = (1,0,1) \) and \( z_{-i} = (0,0,0) \), then \( \pi_i^{IV}(z_i; z_{-i}) = 59/108, \pi_{-i}^{IV}(z_{-i}; z_i) = 0, \) and \( W^{IV}(z) = 173/648; \)
- \( \pi_i^{NV}(z_i; z_{-i}) = 1, \pi_{-i}^{NV}(z_{-i}; z_i) = 0, \) and \( W^{NV}(z) = 1/2, \pi_i^{PV}(z_i; z_{-i}) = 0, \) and \( W^{PV}(z) = 1/4; \)
- if \( z_i = (0,0,1) \) and \( z_{-i} = (1,0,0) \), then \( \pi_i^{IV}(z_i; z_{-i}) \approx 0.369018, \pi_{-i}^{IV}(z_{-i}; z_i) \approx 0.667047, \) and \( W^{IV}(z) \approx 0.471571; \pi_i^{NV}(z_i; z_{-i}) = 4/9, \pi_{-i}^{NV}(z_{-i}; z_i) = 1/9, \) and \( W^{NV}(z) = 7/18, \pi_i^{PV}(z_i; z_{-i}) = 16/49, \pi_{-i}^{PV}(z_{-i}; z_i) = 2/49, \) and \( W^{PV}(z) = 26/49; \)
- if \( z_i = (1,0,1) \) and \( z_{-i} = (1,0,0) \), then for \( v \in \{IV,NV,PV\} \), \( \pi_i^v(z_i; z_{-i}) = 1/4, \pi_{-i}^v(z_{-i}; z_i) = 0, \) and \( W^v(z) = 5/8; \)
- if \( z_i = (1,0,1) \) and \( z_{-i} = (0,0,1) \), then for \( v \in \{IV,NV,PV\} \), \( \pi_i^v(z_i; z_{-i}) = \pi_{-i}^v(z_{-i}; z_i) = 0 \) and \( W^v(z) = 1; \)
- if \( z_i = (1,0,1) \) for \( i = 1,2 \), then for \( v \in \{IV,NV,PV\} \), \( \pi_i^v(z_i; z_{-i}) = 0 \) for \( i = 1,2 \) and \( W^v(z) = 1. \)

Symmetric results hold when \( \max \{z_1, z_2\} = (0,1,1) \).
**Proof.** When \( \max\{z_1, z_2\} = (1, 0, 1) \), using (1)-(2), for any \( p_1 \in [0, 1] \) and \( p_3 \in [0, 2] \) (under IV and PV) and \( p_2 \in [0, 1] \) (under NV),

\[
D^I_1(p) = \begin{cases} 1 - p_1 & \text{if } p_1 \leq p_3 - 1 \\ (1 - p_1)(p_3 - p_1) & \text{if } p_3 - 1 \leq p_1 \leq p_3 \\ 0 & \text{if } p_3 \leq p_1, \end{cases}
\]

\[
D^N_1(p) = \begin{cases} p_3 - p_1 & \text{if } p_1 \leq p_3 \\ 0 & \text{if } p_3 \leq p_1, \end{cases}
\]

\[
D^P_1(p) = \begin{cases} 1 - p_1 & \text{if } p_1 \leq p_3 - 1 \\ p_3 - 2p_1 & \text{if } p_3 - 1 \leq p_1 \leq \frac{p_3}{2} \\ 0 & \text{if } \frac{p_3}{2} \leq p_1. \end{cases}
\]

and

\[
D^I_3(p) = \begin{cases} 1 - \frac{p_2}{2} & \text{if } p_3 \leq p_1 \\ 1 - p_3 + p_1 - \frac{p_2}{2} & \text{if } p_1 \leq p_3 \leq 1 \\ (1 - p_1)(1 - p_3 + p_1) + \frac{1}{2}(p_1 - p_3 + 1)^2 & \text{if } 1 \leq p_3 \leq 1 + p_1 \\ 0 & \text{if } 1 + p_1 \leq p_3, \end{cases}
\]

\[
D^N_3(p) = \begin{cases} 1 & \text{if } p_3 \leq p_1 \\ 1 - p_3 + p_1 & \text{if } p_1 \leq p_3, \end{cases}
\]

\[
D^P_3(p) = \begin{cases} 1 - \frac{p_2}{2} & \text{if } p_3 \leq 2p_1 \\ 1 - p_3 + p_1 & \text{if } 2p_1 \leq p_3 \leq 1 + p_1 \\ 0 & \text{if } 1 + p_1 \leq p_3. \end{cases}
\]

Therefore:

- If \( z_i = (1, 0, 1) \) and \( z_{-i} = (0, 0, 0) \),
  \[
  p^I = \arg\max_{0 \leq p_1 \leq 1, 0 \leq p_3 \leq 2} p_1 D^I_1(p) + p_3 D^I_3(p) = (2/3, 5/6)
  \]

- If \( z_i = (0, 0, 1) \) and \( z_{-i} = (1, 0, 0) \): Consider first the case IV. Naturally, \( p^I_{-i,1}(p_3) \leq p_3 \) and \( p^I_{i,3}(p_1) \leq 1 + p_1 \) for otherwise firms would make zero sales. Furthermore, the function \( p_3((1 - p_1)(1 - p_3 + p_1) + \frac{1}{2}(p_1 - p_3 + 1)^2) \) is concave and has a negative derivative at \( p_3 = 1 \); hence, \( p^I_{i,3}(p_1) \leq 1 \). Therefore,

\[
\begin{align*}
p^I_{-i,1}(p_3) &= \arg\max_{0 \leq p_1 \leq p_3} p_1(1 - p_1)(p_3 - p_1) \\
p^I_{i,3}(p_1) &= \arg\max_{p_1 \leq p_3 \leq 1} p_3 \left(1 - p_3 + p_1 - \frac{p_2}{2}\right).
\end{align*}
\]
Next, we show that for both players, the first-order optimality conditions are necessary and sufficient for optimality. On the one hand, the function $p_3(1 - p_3 + p_1 - \frac{p_1^2}{2})$ is concave in $p_3$. On the other hand, the function $p_1(1 - p_1)(p_3 - p_1)$ is concave in $p_1$, if and only if $6p_1 \leq 2(1 + p_3)$. Because the function $p_1(1 - p_1)(p_3 - p_1)$ is non-negative when $0 \leq p_1 \leq p_3$ and equal to zero at the boundaries of that interval, it has at most one stationary point, and this point corresponds to a global maximum over that interval. Solving the first-order optimality conditions yields $(p_1^*, p_3^*) \approx (0.244934, 0.607469)$.\footnote{Nalebuff (2004)[p.180] argues that the solution should be $(p_1^{IV}, p_3^{IV}) \approx (0.24, 0.59)$ yielding profits $\pi^{IV} \approx 0.366$ and $\pi_1^{IV} \approx 0.064$, but that conclusion is based on an erroneous statement of the first-order conditions, which should read instead $x^* = (1 + p_0 - p_3^2/2)/2$.}

Finally, we consider the case PV. Carbajo et al. (1990) show that $(p_1^{PV}, p_3^{PV}) = (1/7, 4/7)$.

- If $z_i = (1, 0, 1)$ and $z_{-i} = (0, 0, 0)$, then for any $v \in \{IV, NV, PV\}$, $p_i^v = 0$ by Lemma A-2 and, therefore, $D_i^v(p_1, 0) = 0$ for any $p_1$.

- If $z_i = (1, 0, 1)$ for $i = 1, 2$, then for any $v \in \{IV, NV, PV\}$, $p_i^v = p_3^v = 0$ by Lemma A-2. \(\square\)

**Lemma A-8.** When $\max\{z_1, z_2\} = (1, 1, 1)$,

- if $z_i = (1, 1, 1)$ and $z_{-i} = (0, 0, 0)$, then $\pi_i^{IV}(z_i; z_{-i}) = 2(6 + \sqrt{2})/27$, $\pi_i^{IV}(z_{-i}; z_i) = 0$, and $W^{IV}(z) = (15 + 4\sqrt{2})/81$; $\pi_i^{IV}(z_i; z_{-i}) = 1$, $\pi_{-i}^{IV}(z_{-i}; z_i) = 0$, and $W^{IV}(z) = 0$; $\pi_i^{PV}(z_i; z_{-i}) = 1/2$, $\pi_{-i}^{PV}(z_{-i}; z_i) = 0$, and $W^{PV}(z) = 1/4$;

- if $z_i = (0, 0, 1)$ and $z_{-i} = (1, 1, 0)$, then for $v \in \{IV, NV, PV\}$, $\pi_i^v(z_i; z_{-i}) = \pi_{-i}^v(z_{-i}; z_i) = 0$ and $W^v(z) = 1$;

- if $z_i = (0, 1, 1)$ and $z_{-i} = (1, 0, 0)$ or if $z_i = (1, 0, 1)$ and $z_{-i} = (0, 1, 0)$, then $\pi_i^{IV}(z_i; z_{-i}) \approx 0.369018$, $\pi_i^{IV}(z_{-i}; z_i) \approx 0.0670476$, and $W^{IV}(z) \approx 0.471571$; $\pi_i^{IV}(z_i; z_{-i}) = 4/9$, $\pi_{-i}^{IV}(z_{-i}; z_i) = 1/9$, and $W^{IV}(z) = 7/18$; $\pi_i^{PV}(z_i; z_{-i}) = 16/49$, $\pi_{-i}^{PV}(z_{-i}; z_i) = 2/49$, and $W^{PV}(z) = 26/49$;

- if $z_i = (1, 0, 1)$ or $z_i = (0, 1, 1)$ and $z_{-i} = (1, 1, 0)$, then for $v \in \{IV, NV, PV\}$, $\pi_i^v(z_i; z_{-i}) = \pi_{-i}^v(z_{-i}; z_i) = 0$ and $W^v(z) = 1$;

- if $z_i = (1, 1, 1)$ and $z_{-i} = (1, 0, 0)$ or $z_{-i} = (0, 1, 0)$, then for $v \in \{IV, NV, PV\}$, $\pi_i^v(z_i; z_{-i}) = 1/4$, $\pi_{-i}^v(z_{-i}; z_i) = 0$, and $W^v(z) = 5/8$;
\[ D^I_k(p) = \begin{cases} (1 - p_k) & \text{if } p_k < p_3 - p_{-k} \\ (1 - p_k)(p_3 - p_k) & \text{if } p_3 - p_{-k} \leq p_k \leq p_3 \\ 0 & \text{if } p_3 \leq p_k, \end{cases} \]

\[ D^N_k(p) = \begin{cases} 1 - p_k & \text{if } p_k < p_3 - p_{-k} \\ p_3 - p_k & \text{if } p_3 - p_{-k} \leq p_k \leq p_3 \\ 0 & \text{if } p_3 \leq p_k, \end{cases} \]

\[ D^V_k(p) = \begin{cases} 1 - p_k & \text{if } p_k < p_3 - p_{-k} \\ p_3 - 2p_k & \text{if } p_3 - p_{-k} \leq p_k \leq \frac{p_3}{2} \\ 0 & \text{if } \max \{p_3 - p_{-k}, \frac{p_3}{2}\} \leq p_k, \end{cases} \]

and

\[ D^I_3(p) = \begin{cases} 1 - \frac{p_3^2}{2} & \text{if } p_3 \leq \min \{p_1, p_2\} \\ 1 - p_3 + \min \{p_1, p_2\} - \frac{(\min \{p_1, p_2\})^2}{2} & \text{if } \min \{p_1, p_2\} \leq p_3 \leq \max \{p_1, p_2\} \\ (1 - p_3 + p_1)(1 - p_3 + p_2) - \frac{1}{2}(p_1 + p_2 - p_3)^2 & \text{if } \max \{p_1, p_2\} \leq p_3 \leq p_1 + p_2 \\ 0 & \text{if } p_1 + p_2 < p_3, \end{cases} \]

\[ D^N_3(p) = \begin{cases} 1 & \text{if } p_3 \leq \min \{p_1, p_2\} \\ 1 - p_3 + \min \{p_1, p_2\} & \text{if } \min \{p_1, p_2\} \leq p_3 \leq \max \{p_1, p_2\} \\ 1 - 2p_3 + p_1 + p_2 & \text{if } \max \{p_1, p_2\} \leq p_3 \leq p_1 + p_2 \\ 0 & \text{if } p_1 + p_2 < p_3, \end{cases} \]

\[ D^V_3(p) = \begin{cases} 1 - \frac{p_3^2}{2} & \text{if } p_3 \leq 2 \min \{p_1, p_2\} \\ 1 - p_3 + \min \{p_1, p_2\} & \text{if } 2 \min \{p_1, p_2\} \leq p_3 \leq p_1 + p_2 \\ 0 & \text{if } p_1 + p_2 < p_3. \end{cases} \]

Therefore:

- If \( z_i = (1, 1, 1) \) and \( z_{-i} = (0, 0, 0) \),

\[ p^I = \arg \max_{0 \leq p_1 \leq 1, 0 \leq p_2 \leq 1, 0 \leq p_3 \leq 2} p_1 D^I_1(p) + p_2 D^I_2(p) + p_3 D^I_3(p) = \left( \frac{2}{3}, \frac{2}{3}, \frac{4 - \sqrt{2}}{3} \right) \]
\[ p^{NV} = \arg \max_{0 \leq p_1 \leq 1, 0 \leq p_2 \leq 1, 0 \leq p_3 \leq 1} p_1 D_1^{NV}(p) + p_2 D_2^{NV}(p) + p_3 D_3^{NV}(p) = (1, 1, 1) \]
\[ p^{PV} = \arg \max_{0 \leq p_1 \leq 1, 0 \leq p_2 \leq 1, 0 \leq p_3 \leq 1} p_1 D_1^{PV}(p) + p_2 D_2^{PV}(p) + p_3 D_3^{PV}(p) = \left( \frac{1}{2}, \frac{1}{2}, \frac{3}{2} \right). \]

- If \( z_1 = (0, 0, 1) \) and \( z_{-1} = (1, 1, 0) \): Consider first the case IV. By symmetry of \( D_1^{IV}(p) \) and \( D_2^{IV}(p) \), \( p^{IV}_{i,1}(p_3) = p^{IV}_{i,2}(p_3) = p_i \). Hence, firm \( -i \)'s problem can be simplified to a one-variable optimization problem. To obtain a contradiction, assume that there exists an equilibrium such that both firms earn positive profit. Accordingly, \( p_k > 0 \) for \( k \in \{1, 2\} \) and \( p_3 > 0 \). Moreover, \( p_k < p_3 \) for \( k \in \{1, 2\} \) and \( p_3 \leq p_1 + p_2 \) for otherwise firms would make zero sales. Accordingly, if there were such an equilibrium, \( p^{IV}_i(p_3) \) would maximize \( 2p(1 - p)(p_3 - p) \) subject to \( p_3/2 \leq p < \min\{1, p_3\} \).

The function \( 2p(1 - p)(p_3 - p) \) has two stationary points, namely \( (1 + p_3)/3 \pm \sqrt{1 - p_3 + p_3^2}/3 \), and is concave if \( p \leq (1 + p_3)/3 \) and convex otherwise. Because \( (1 + p_3)/3 - \sqrt{1 - p_3 + p_3^2}/3 \leq p_3/2 \) and \( (1 + p_3)/3 + \sqrt{1 - p_3 + p_3^2}/3 \geq \min\{1, p_3\} \), the function \( 2p(1 - p)(p_3 - p) \) is decreasing on the interval \( p \in [p_3/2, \min\{1, p_3\}] \), and it is maximized on this range at \( p = p_3/2 \). Hence, if there were such an equilibrium, \( p^{IV}_i(p_3) \) would be equal to \( p_3/2 \). However, since \( p_3 > 0 \), firm \( -i \) could lower its price below \( p_3/2 \) and substantially increase its market share, a contradiction. Hence, in equilibrium, at least one of the firms earns zero profit. In fact, both firms earn zero profit, otherwise, the firm that earns zero profit could undercut the price of the firm that earns positive profit and capture some of that profit.

Next, consider the case NV. As in the IV case, firm \( -i \)'s problem can be simplified to a one-variable optimization problem. To obtain a contradiction, assume that there exists an equilibrium such that both firms earn positive profit, i.e., \( p_1 > 0 \), \( p_2 > 0 \), and \( p_3 > 0 \); moreover, \( p_k < p_3 \) for \( k \in \{1, 2\} \) and \( p_3 \leq p_1 + p_2 \). Accordingly, if there were such an equilibrium, \( p^{IV}_i(p_3) \) would maximize \( 2p(p_3 - p) \) subject to \( p_3/2 \leq p < p_3 \), i.e., \( p^{IV}_i(p_3) \) would be equal to \( p_3/2 \). However, since \( p_3 > 0 \), firm \( -i \) could lower its price below \( p_3/2 \) and substantially increase its market share, a contradiction. As a result, in equilibrium, at least one of the firms earns zero profit. In fact, both firms earn zero profit, otherwise, the firm that earns zero profit could undercut the price of the firm that earns positive profit and capture some of that profit.

Finally, we consider the case PV. As in the IV and NV cases, firm \( -i \)'s problem can be simplified to a one-variable optimization problem. To obtain a contradiction, assume that there exists an equilibrium such that both firms earn positive profit, i.e., \( p_1 > 0 \), \( p_2 > 0 \), and \( p_3 > 0 \), \( p_k < p_3 \) for \( k \in \{1, 2\} \), and \( p_3 \leq p_1 + p_2 \). Hence, we need to have \( p < p_3/2 \), a contradiction. As a result, in equilibrium, at least one of the firms earns zero profit. In fact, both firms earn zero profit, otherwise, the firm that earns zero profit could undercut the price of the firm that earns positive profit and capture some of that profit.

- If \( z_1 = (0, 1, 1) \) and \( z_{-1} = (1, 0, 0) \): By Lemma A-3, firm \( i \) will always opt to generate sales from the bundle. Thus, \( p^{i_1}_{3}(p_1) - p^{i_2}_{2}(p_1) \leq p_1 \) for all \( v \in \{IV, NV, PV\} \). Moreover, \( p^{i+1}_{i,1}(p_2, p_3) \leq p_3 \) for \( v \in \{IV, NV\} \) and \( p^{i+1}_{i,1}(p_2, p_3) \leq p_3/2 \), otherwise, firm \( -i \) would make zero sales.

In addition, firm \( i \) would price product 2 less aggressively than firm \( -i \)'s pricing of product 1, otherwise, firm \( i \) would end up competing against itself, similar to the case where \( z_1 = (0, 0, 1) \) and \( z_{-1} = (1, 1, 0) \). Hence, \( p^i_2 \geq p^i_1 \) for all \( v \in \{IV, NV, PV\} \).
Furthermore, the introduction of competition will lead to a price decrease relative to the case where 
\( z = (0, 1, 1) \) and \( z_{-i} = (0, 0, 0) \) characterized in Lemma A-7. In particular, \( p_{i}^{IV} \leq 0.607469, p_{N}^{IV} \leq 2/3, \) and \( p_{3}^{PV} \leq 4/7. \)

Consider first the case IV. Under the domain restrictions,

\[
\begin{align*}
    p_{i}^{IV} \left( p_2, p_3 \right) &= \arg \max_{p_3 - p_2 \leq p_1 \leq p_3} p_1 (1 - p_1)(p_3 - p_1) \\
    \left( p_{12}^{IV} (p_1), p_{i3}^{IV} (p_1) \right) &= \arg \max_{p_1 \leq p_2 \leq p_3, p_1 \leq p_3 - p_2} p_3 \left( 1 - p_3 + p_1 - \frac{p_2^2}{2} \right) \mathbb{I}_{[p_3 = p_2]} + p_2 (1 - p_2)(p_3 - p_2) \\
    &\quad + p_3 \left( (1 - p_3 + p_1)(1 - p_3 + p_2) - \frac{1}{2} (p_1 + p_2 - p_3)^2 \right) \mathbb{I}_{[p_3 > p_2]}.
\end{align*}
\]

The derivative of the function \( p_2(1 - p_2)(p_3 - p_2) + p_3 \left( (1 - p_3 + p_1)(1 - p_3 + p_2) - \frac{1}{2} (p_1 + p_2 - p_3)^2 \right) \) with respect to \( p_2 \) is equal to zero when \( p_2 = p_3 \). Given that its second derivative is non-positive if and only if \( 6p_2 - 3p_3 - 2 \leq 0 \) and that \( p_{3}^{IV} \leq 0.607469 \), we conclude that setting \( p_2 = p_3 \) maximizes firm \( i \)'s profit. Hence, the equilibrium is identical to the case where \( z = (0, 0, 1) \) and \( z_{-i} = (0, 1, 0) \), as characterized in Lemma A-7.

Next, consider the case NV. Under the domain restrictions,

\[
\begin{align*}
p_{i}^{NV} \left( p_2, p_3 \right) &= \arg \max_{p_3 - p_2 \leq p_1 \leq p_3} p_1 (1 - p_1) \\
\left( p_{12}^{IV} (p_1), p_{i3}^{IV} (p_1) \right) &= \arg \max_{p_1 \leq p_2 \leq p_3, p_1 \leq p_3 - p_2} p_3 \left( 1 - p_3 + p_1 \right) \mathbb{I}_{[p_3 = p_2]} + p_2 (1 - p_2)(p_3 - p_2) \\
    &\quad + p_3 \left( (1 - p_3 + p_1)(1 - p_3 + p_2) + p_3 \right) \mathbb{I}_{[p_3 > p_2]}.
\end{align*}
\]

The derivative of the function \( p_2(p_3 - p_2) + p_3 \left( 1 - 2p_3 + p_1 + p_2 \right) \) with respect to \( p_2 \) is equal to zero when \( p_2 = p_3 \). Given that its second derivative is negative, we conclude that setting \( p_2 = p_3 \) maximizes firm \( i \)'s profit. Hence, the equilibrium is identical to the case where \( z = (0, 0, 1) \) and \( z_{-i} = (0, 1, 0) \), as characterized in Lemma A-7.

Finally, consider the case PV. Under the domain restrictions, \( p_2 \geq p_3 - p_1 \geq p_3/2 \). Hence, firm \( i \) generates no sales from product 2, and the equilibrium is identical to the case where \( z = (0, 0, 1) \) and \( z_{-i} = (0, 1, 0) \), as characterized in Lemma A-7.

The case where \( z = (1, 0, 1) \) and \( z_{-i} = (0, 1, 0) \) can be treated in a similar fashion.

- if \( z = (1, 0, 1) \) and \( z_{-i} = (1, 0, 0) \): For any \( v \in \{ IV, NV, PV \} \), \( p_i \) is 0 by Lemma A-2. To obtain a contradiction, assume that there exists an equilibrium in which both firms make positive profit. Accordingly, \( p_i^v(0, p_3) < p_3 \), otherwise, firm \( -i \) would generate no profitable sales, and \( p_i^v(0, p_2) \geq p_1 \), otherwise, firm \( i \) would generate no profitable sales, a contradiction. As a result, in equilibrium, at least one of the firms earns zero profit. In fact, both firms earn zero profit, otherwise, the firm that earns zero profit could undercut the price of the firm that earns positive profit and capture some of that profit.

The case where \( z = (0, 1, 1) \) and \( z_{-i} = (1, 0, 0) \) can be treated in a similar fashion.

- If \( z = (1, 1, 1) \) and \( z_{-i} = (0, 0, 0) \): For any \( v \in \{ IV, NV, PV \} \), \( p_i \) is 0 by Lemma A-2. Accordingly,

\[
(p_i^v(0), p_3^v) = \arg \max_{0 \leq p_2 \leq 1, 0 \leq p_3} p_2 \left( 1 - p_2 \right) \mathbb{I}_{[p_2 < p_3]} + p_3 \left( 1 - p_3 \right) \mathbb{I}_{[p_3 \geq p_3]}
\]
and the equilibrium is identical to the case where \( z_i = (1, 1, 0) \) and \( z_{-i} = (1, 0, 0) \) characterized in Lemma A-6 or \( z_i = (1, 0, 1) \) and \( z_{-i} = (1, 0, 0) \) characterized in Lemma A-7.

The case where \( z_i = (1, 1, 1) \) and \( z_{-i} = (0, 1, 0) \) can be treated in a similar fashion.

- If \( z_i = (1, 1, 1) \) and \( z_{-i} = (1, 1, 0) \): For any \( v \in \{ IV, NV, PV \} \), \( p_i^v = p_2^v = 0 \) by Lemma A-2. Hence, either \( p_i^v = 0 \) or \( D_i^v(0, 0, p_3) = 0 \).
- If \( z_{13} = z_{23} = 1 \): For any \( v \in \{ IV, NV, PV \} \), \( p_3^v = 0 \) by Lemma A-2. Hence, either \( p_k^v = 0 \) or \( D_k^v(p_1, p_2, 0) = 0 \) for any \( k \in \{1, 2\} \).

**Proof of Proposition 1.** Combining Lemmas A-4-A-8 leads to Tables 2-4, from which we deduce the equilibria. The trembling-hand perfect equilibria are identified by maximizing each player's payoff in case of random deviations by the other player (Selten 1975). \( \square \)

### EC.3. Two Customer Segments with Perfectly Negatively Correlated Valuations (2CS)

**Lemma A-9.** Under 2CS, when \( \max\{z_1, z_2\} = (1, 0, 0) \),

- if \( z_i = (1, 0, 0) \) and \( z_{-i} = (0, 0, 0) \), then \( \pi_i^*(z_i; z_{-i}) = \max\{v, (1-v)/2\} \) and \( \pi_{-i}^*(z_{-i}; z_i) = 0 \);
- if \( z_i = (1, 0, 0) \) for \( i = 1, 2 \), then \( \pi_i^*(z_i; z_{-i}) = 0 \) for \( i = 1, 2 \).

Symmetric results hold when \( \max\{z_1, z_2\} = (0, 1, 0) \).

**Proof.** When \( \max\{z_1, z_2\} = (1, 0, 0) \), by (2),

- if \( z_i = (1, 0, 0) \) and \( z_{-i} = (0, 0, 0) \), then \( p_i^v = v \) if \( v \geq (1-v)/2 \) and \( p_i^v = 1 - v \) otherwise.
- if \( z_i = (1, 0, 0) \) for \( i = 1, 2 \), the result follows by Lemma A-2. \( \square \)

**Lemma A-10.** Under 2CS, when \( \max\{z_1, z_2\} = (0, 0, 1) \),

- if \( z_i = (0, 0, 1) \) and \( z_{-i} = (0, 0, 0) \), then \( \pi_i^*(z_i; z_{-i}) = 1 \) and \( \pi_{-i}^*(z_{-i}; z_i) = 0 \);
- if \( z_i = (0, 0, 1) \) for \( i = 1, 2 \), then \( \pi_i^*(z_i; z_{-i}) = 0 \) for \( i = 1, 2 \).

**Proof.** When \( \max\{z_1, z_2\} = (0, 0, 1) \), by (1),

- if \( z_i = (0, 0, 1) \) and \( z_{-i} = (0, 0, 0) \), then \( p_i^v = 1 \).
- if \( z_i = (0, 0, 1) \) for \( i = 1, 2 \), the result follows by Lemma A-2. \( \square \)

**Lemma A-11.** Under 2CS, when \( \max\{z_1, z_2\} = (1, 1, 0) \),

- if \( z_i = (1, 1, 0) \) and \( z_{-i} = (0, 0, 0) \), then \( \pi_i^*(z_i; z_{-i}) = \max\{2v, (1-v)\} \) and \( \pi_{-i}^*(z_{-i}; z_i) = 0 \);
- if \( z_i = (1, 0, 0) \) and \( z_{-i} = (0, 1, 0) \), then \( \pi_i^*(z_i; z_{-i}) = \max\{v, (1-v)/2\} \) and \( \pi_{-i}^*(z_{-i}; z_i) = \max\{v, (1-v)/2, z_i \} \);
- if \( z_i = (1, 1, 0) \) and \( z_{-i} = (1, 0, 0) \) or \( z_{-i} = (0, 1, 0) \), then \( \pi_i^*(z_i; z_{-i}) = \max\{v, (1-v)/2\} \) and \( \pi_{-i}^*(z_{-i}; z_i) = 0 \);
- if \( z_i = (1, 1, 0) \) for \( i = 1, 2 \), then \( \pi_i^*(z_i; z_{-i}) = 0 \) for \( i = 1, 2 \).

**Proof.** The results follow from Lemmas A-2 and A-9. \( \square \)
LEMMA A-12. Under 2CS, when \( z_i = (0,0,1) \) and \( z_{-i} = (1,0,0) \) then there exists a pure-strategy Nash equilibrium in the pricing game provided that \( v \geq 1/3 \). Otherwise, if \( v < 1/3 \), then there exists only a mixed-strategy Nash equilibrium in the pricing game. With these equilibria, the payoffs are as follows.

- For \( v \geq 1/3 \): \( \pi^*_i(z_i; z_{-i}) = v \) and \( \pi^*_i(z_{-i}; z_i) = 0 \).
- For \( 1/4 < v < 1/3 \): \( \pi^*_i(z_i; z_{-i}) = 1 - 2v \) and \( \pi^*_i(z_{-i}; z_i) = (1 - 3v)/2 \).
- For \( v \leq 1/4 \): \( \pi^*_i(z_i; z_{-i}) = 1/2 \) and \( \pi^*_i(z_{-i}; z_i) = 1/4 - v/2 \).

Proof. Since \( z_i \) and \( z_{-i} \) are kept fixed, hereafter, we shall omit these arguments from the profit functions and best-response correspondences; we also use the simplified notation \( p_i = p_{i3}, p_{-i} = p_{-i_k}, k \in \{1, 2\} \). First, we analyze the best-response correspondence of firm \( i \) then that of firm \( -i \). Throughout this characterization of best responses, we assume that prices must be set in infinitesimal increments of \( \delta > 0 \).

Firm \( i \). We begin by analyzing the best-response correspondence for different ranges of firm \( i \)'s prices—namely, when \( p_{-i} > 1 - v, v \leq p_{-i} \leq 1 - v, \) and \( 0 \leq p_{-i} < v \).

- If \( p_{-i} > 1 - v \), then, by (1), \( \zeta_i(v, 1 - v, p) = 1 \) if and only if \( p_i \leq 1 \) and \( \zeta_i(1 - v, v, p) = 1 \) if and only if \( p_i \leq 1 \). As a result,
  \[
  \pi^*_i(p_i; p_{-i}) = 1 \quad \text{and} \quad p^*_i(p_{-i}) = 1.
  \]

- If \( v \leq p_{-i} \leq 1 - v \), then, by (1), \( \zeta_i(v, 1 - v, p) = 1 \) if and only if \( p_i \leq 1 \) and \( \zeta_i(1 - v, v, p) = 1 \) if and only if \( p_i \leq p_{-i} + v \). As a result,
  \[
  \pi^*_i(p_i; p_{-i}) = \max \left\{ p_{-i} + v, \frac{1}{2} \right\} \quad \text{and} \quad p^*_i(p_{-i}) = \begin{cases} p_{-i} + v & \text{if } p_{-i} \geq \frac{1}{2} - v, \\ 1 & \text{if } p_{-i} \leq \frac{1}{2} - v. \end{cases}
  \]

- If \( 0 \leq p_{-i} < v \), then by (1), \( \zeta_i(v, 1 - v, p) = 1 \) if and only if \( p_i \leq p_{-i} + 1 - v \) and \( \zeta_i(1 - v, v, p) = 1 \) if and only if \( p_i \leq p_{-i} + v \). As a result,
  \[
  \pi^*_i(p_i; p_{-i}) = \max \left\{ p_{-i} + v, \frac{1}{2} (p_{-i} + 1 - v) \right\} \quad \text{and} \quad p^*_i(p_{-i}) = \begin{cases} p_{-i} + v & \text{if } p_{-i} \geq 1 - 3v; \\ p_{-i} + 1 - v & \text{if } p_{-i} \leq 1 - 3v. \end{cases}
  \]

Combining all cases, we have the following best responses for firm \( i \):

\[
\text{if } v \geq \frac{1}{3} : \quad p^*_i(p_{-i}) = \begin{cases} 1 & \text{if } p_{-i} > 1 - v, \\ p_{-i} + v & \text{if } 1 - v \geq p_{-i} \geq 0; \end{cases} \quad (A-1)
\]
\[
\text{if } \frac{1}{3} \geq v > \frac{1}{4} : \quad p^*_i(p_{-i}) = \begin{cases} p_{-i} + v & \text{if } 1 - v \geq p_{-i} \geq 1 - 3v, \\ p_{-i} + 1 - v & \text{if } 1 - 3v \geq p_{-i} \geq 0; \end{cases} \quad (A-2)
\]
\[
\text{if } \frac{1}{4} \geq v : \quad p^*_i(p_{-i}) = \begin{cases} p_{-i} + v & \text{if } 1 - v \geq p_{-i} \geq \frac{1}{2} - v, \\ 1 & \text{if } \frac{1}{2} - v \geq p_{-i} \geq 0. \end{cases} \quad (A-3)
\]

Firm \( -i \). Next, we identify firm \( -i \)'s best-response correspondence. Toward that end, we consider the following four distinct sets of values for \( p_i \).

- If \( p_i > 1 \), then by (2), \( \zeta_i(v, 1 - v, p) = 1 \) if and only if \( p_{-i} \leq v \) and \( \zeta_i(1 - v, v, p) = 1 \) if and only if \( p_{-i} \leq 1 - v \). As a result,
  \[
  \pi^*_i(p_{-i}; p_i) = \max \left\{ v, \frac{1}{2} (1 - v) \right\} \quad \text{and} \quad p^*_i(p_i) = \begin{cases} v & \text{if } v \geq \frac{1}{3}; \\ 1 - v & \text{if } v \leq \frac{1}{3}. \end{cases}
  \]
• If \( 1 - v < p_i \leq 1 \), then by (2), \( \zeta_1(v, 1 - v, p) = 1 \) if and only if \( p_{-i} \leq v \) and \( p_{-i} + 1 - v < p_i \) and \( \zeta_3(1 - v, v, p) = 1 \) if and only if \( p_{-i} \leq 1 - v \) and \( p_{-i} + v < p_i \). As a result, for some infinitesimal \( \delta > 0 \) we have

\[
\pi^*_i(p_{-i}; p_i) = \max \left\{ p_i - (1 - v) \cdot \frac{1}{2}(p_i - v) \right\} \quad \text{and} \quad p^*_i(p_i) = \begin{cases} p_i - (1 - v) - \delta & \text{if } p_i > 2 - 3v, \\ p_i - v - \delta & \text{if } p_i \leq 2 - 3v. \end{cases}
\]

• If \( v < p_i \leq 1 - v \), then by (2), \( \zeta_1(v, 1 - v, p) = 0 \) for all \( p_{-i} \) and \( \zeta_3(1 - v, v, p) = 1 \) if and only if \( p_{-i} \leq 1 - v \) and \( p_{-i} + v < p_i \). As a result, for some infinitesimal \( \delta > 0 \) we have

\[
\pi^*_i(p_{-i}; p_i) = \frac{1}{2}(p_i - v) \quad \text{and} \quad p^*_i(p_i) = p_i - v - \delta.
\]

• If \( 0 \leq p_i \leq v \), then by (2), \( \zeta_1(v, 1 - v, p) = 0 \) and \( \zeta_3(1 - v, v, p) = 0 \) for all \( p_{-i} \). Hence, firm \(-i\) does not capture any customer at any non-negative price \( p_{-i} \geq 0 \), so we may write

\[
\pi^*_i(p_{-i}; p_i) = 0 \quad \text{and} \quad p^*_i(p_i) = \{p_{-i} | p_{-i} \geq 0\}.
\]

Combining all cases, we have the following best responses for firm \(-i\):

\[
\text{if } v \geq \frac{1}{3}: \quad p^*_i(p_i) = \begin{cases} v & \text{if } p_i > 1, \\ p_i - (1 - v) - \delta & \text{if } 1 \geq p_i > 1 - 3v, \\ p_i - v - \delta & \text{if } 1 - 3v \geq p_i > v, \\ (0, \infty) & \text{if } v \geq p_i \geq 0; \end{cases} \tag{A-4}
\]

\[
\text{if } v \leq \frac{1}{3}: \quad p^*_i(p_i) = \begin{cases} 1 - v & \text{if } p_i > 1, \\ p_i - v - \delta & \text{if } 1 \geq p_i > v, \\ (0, \infty) & \text{if } v \geq p_i \geq 0. \end{cases} \tag{A-5}
\]

**Existence of Pure-Strategy Nash Equilibria.** Combining the best responses shows that the existence of a Nash equilibrium depends on the value of \( v \).

1. If \( v \geq \frac{1}{3} \) then, for any \( \delta > 0 \), there exists a pure-strategy Nash equilibrium \( (p^*_i, p^*_{-i}) = (v, 0) \), which yields \( \pi^*_i = v \) and \( \pi^*_{-i} = 0 \). This equilibrium is not unique, but it is payoff-equivalent to any other equilibria for this game.

2. If \( v < \frac{1}{3} \), there exists no pure-strategy Nash equilibrium because the best-response correspondences do not cross.

**Support of Non-Dominated Strategies.** In what follows, we assume that \( v < 1/3 \); thus, we focus on cases where no pure-strategy Nash equilibrium exists. We first identify the support of non-dominated strategies based on the best-response correspondences by undertaking a process of elimination of strictly dominated strategies. Just as in our characterization of the best responses, we assume that prices must be set in increments of \( \delta > 0 \).

1. \( 1/4 < v < 1/3 \): In the first iteration, firm \(-i\) starts off with the full set of pricing strategies (i.e., \( p_{-i}^{(1)} \in [0, \infty) \)). According to (A-2), \( p_{-i}^{(1)}(p_{-i}) \in [1 - 2v, 1] \) for any \( p_i \in [0, \infty) \). In the next iteration by (A-5), \( p_{-i}^{(2)}(p_i) \in [1 - 3v - \delta, 1 - v - \delta] \) for any \( p_i \in [1 - v, 1] \). In turn, firm \( i \) sets \( p_i^{(3)}(p_{-i}) \in [1 - 2v, 1 - \delta] \). Firm \(-i\) then responds with \( p_{-i}^{(3)}(p_i) \in [1 - 3v - \delta, 1 - v - 2\delta] \). It is easy to show by induction that, after \( t \leq [(4v - 1)/\delta] \) iterations, \( p_i^{(t)} \in [1 - 2v, 1 - (t - 1)\delta] \) and \( p_{-i}^{(t)} \in [1 - 3v - \delta, 1 - v - (t - 1)\delta] \). At the next iteration, the support of non-dominated strategies will stop shrinking and be equal to

\[
p_i^{(t)}(p_{-i}) \in [1 - 2v - 4v] \quad \text{and} \quad p_{-i}^{(t)}(p_i) \in [1 - 3v - \delta, 2 - 5v - \delta]. \tag{A-6}
\]
2. \( v \leq 1/4 \): In the first iteration, firm \(-i\) starts off with the full set of pricing strategies (i.e., \( p_{-i}^{(1)} \in [0, \infty) \)). By (A-3), \( p_{-i}^{(1)}(p_{-i}) \in [1/2, 1] \). In the next iteration, firm \(-i\) sets \( p_{-i}^{(2)}(p_{i}) \in [1/2 - v - \delta, 1 - v - \delta] \) to which firm \( i \) responds with \( p_{i}^{(2)} \in [1/2, 1] \). Hence, the process converges after one iteration. At that point, the support of non-dominated strategies is equal to

\[
\begin{align*}
p_{i}^{*}(p_{-i}) & \in \left[ \frac{1}{2}, 1 \right] \quad \text{and} \quad p_{-i}^{*}(p_{i}) \in \left[ \frac{1}{2} - v - \delta, 1 - v - \delta \right].
\end{align*}
\]

(A-7)

**Mixed-Strategy Equilibrium Characterization.** Although there are no pure-strategy Nash equilibria when \( v < 1/3 \) for any \( \delta > 0 \), the following cumulative distribution functions constitute a mixed-strategy Nash equilibrium when \( \delta \to 0 \):

\[
G_{i}(p_{i}) = \begin{cases}
p_{i} - a & \text{for } a \leq p_{i} < b, \\
1 & \text{for } p_{i} = b;
\end{cases}
\]

(A-8)

\[
G_{-i}(p_{-i}) = 2 - \frac{b}{p_{-i} + b - a} \quad \text{for } c \leq p_{-i} \leq c + b - a.
\]

(A-9)

Here

\[
\begin{align*}
a & = 1 - 2v, \quad b = 2 - 4v - \delta, \quad \text{and} \quad c = 1 - 3v - \delta \quad \text{when } \frac{1}{4} < v \leq \frac{1}{3}, \\
a & = \frac{1}{2}, \quad b = 1, \quad \text{and} \quad c = \frac{1}{2} - v - \delta \quad \text{when } v \leq \frac{1}{4}.
\end{align*}
\]

(A-10) (A-11)

For any \( p_{i} \in [a, b] \), let \( \pi_{i}(p_{i}; G_{-i}) \) denote firm \( i \)'s profit when firm \(-i\) employs the randomizing profile \( G_{-i} \), over \( c \leq p_{-i} \leq c + b - a \). Similarly for any \( p_{-i} \in [c, c + b - a] \), we use \( \pi_{-i}(p_{-i}; G_{-i}) \) to denote firm \(-i\)'s profit when firm \( i \) employs the randomizing profile \( G_{i}(p_{i}) \) over \( a \leq p_{i} \leq b \). The distributions (A-8) and (A-9) define a Nash equilibrium if \( \pi_{i}(p_{i}; G_{-i}) = \pi_{i}(a; G_{-i}) \) for all \( p_{i} \in [a, b] \) and \( \pi_{i}(p_{i}; G_{-i}) \leq \pi_{i}(a; G_{-i}) \) for all \( p_{i} \notin [a, b] \) and if \( \pi_{-i}(p_{-i}; G_{-i}) = \pi_{-i}(c; G_{-i}) \) for all \( p_{i} \in [c, c + b - a] \) and \( \pi_{-i}(p_{-i}; G_{-i}) \leq \pi_{-i}(c; G_{-i}) \) for all \( p_{i} \notin [c, c + b - a] \).

We establish this result by first considering the case where \( 1/4 < v < 1/3 \), then the case where \( v \leq 1/4 \).

1. If \( 1/4 < v < 1/3 \), then \( \zeta_{i}(v, 1 - v, p) = 1 \) for all \( p_{i} \in [a, b] \) and \( p_{-i} \in [c, c + b - a] \) because \( \min\{v, p_{-i}\} + 1 - v \geq p_{i} \) when \( p_{i} \leq 2 - 4v - \delta \) and \( p_{-i} \geq 1 - 3v - \delta \). In contrast, \( \zeta_{i}(1 - v, v, p) = 1 \) if and only if \( p_{i} \leq p_{-i} + v \).

It follows that, for any \( p_{i} \in [a, b] \):

\[
\begin{align*}
\pi_{i}(p_{i}; G_{-i}) & = \lim_{\delta \to 0} \int_{c}^{c+b-a} \pi_{i}(p_{i}; p_{-i}) dG_{-i}(p_{-i}) = \lim_{\delta \to 0} \left( \frac{1}{2} p_{i} \left( 1 + \int_{p_{i}-v}^{c+b-a} dG_{-i}(p_{-i}) \right) \right) \\
& = \lim_{\delta \to 0} \left( \frac{1}{2} p_{i} (1 + G_{-i}(c + b - a) - G_{-i}(p_{i} - v)) \right) \\
& = \lim_{\delta \to 0} \left( \frac{1}{2} p_{i} \left( 2 - 4v - \delta \right) \right) = 1 - 2v.
\end{align*}
\]

(A-12)

Following our previously outlined iterative process of elimination of dominated strategies, we find that \( \pi_{i}(p_{i}; G_{-i}) < \pi_{i}(a; G_{-i}) \) for all \( p_{i} < a \) or \( p_{i} > b + \delta \). Finally, for any \( p_{i} \in (b, b + \delta] \), \( \zeta_{i}(v, 1 - v, p) = 1 \) if and only if \( p_{i} \leq p_{-i} + 1 - v \) and \( \zeta_{i}(1 - v, v, p) = 1 \) if and only if \( p_{i} \leq p_{-i} + v \). So for any \( p_{i} \in (b, b + \delta] \), we have

\[
\begin{align*}
\pi_{i}(p_{i}; G_{-i}) & = \lim_{\delta \to 0} \int_{c}^{c+b-a} \pi_{i}(p_{i}; p_{-i}) dG_{-i}(p_{-i}) = \lim_{\delta \to 0} \left( \frac{1}{2} p_{i} \left( \int_{p_{i}-(1-v)}^{c+b-a} dG_{-i}(p_{-i}) + \int_{p_{i}-v}^{c+b-a} dG_{-i}(p_{-i}) \right) \right) \\
& = \lim_{\delta \to 0} \left( \frac{1}{2} p_{i} \left( 2G_{-i}(c + b - a) - G_{-i}(p_{i} - (1-v)) - G_{-i}(p_{i} - v) \right) \right) \\
& = \lim_{\delta \to 0} \left( \frac{1}{2} p_{i} \left( \frac{2 - 4v - \delta + 4 - 8v - 2p_{i} - 3\delta}{p_{i} + \delta} - \frac{1}{p_{i} - (1-2v) + \delta} \right) \right) = 1 - 2v = \pi_{i}(a; G_{-i}).
\end{align*}
\]
As a consequence, when $\delta \to 0$ we see that $\pi_i(p_i; G_{-i}) = \pi_i(a; G_{-i})$ for all $p_i \in [a, b]$ and $\pi_i(p_i; G_{-i}) \leq \pi_i(a; G_{-i})$ for all $p_i \notin [a, b]$.

Because $\zeta_1(v, 1 - v, p) = 0$ for all $p_i \in [a, b]$ and $p_{-i} \in [c, c + b - a + \delta]$ and $\zeta_1(1 - v, v, p) = 1$ if and only if $p_i > p_{-i} + v$ (since $p_{-i} \leq 1 - v$ for all $p_{-i} \leq c$), we have, for any $p_i \in [c, c + b - a + \delta]$,

$$
\pi_i(p_{-i}; G_i) = \lim_{\delta \to 0} \left( \frac{1}{2} \int_{p_{-i} + v + \delta}^{b} dG_i(p_i) \right) = \lim_{\delta \to 0} \left( \frac{1}{2} p_{-i} (G_i(b) - G_i(p_{-i} + v + \delta)) \right) = \lim_{\delta \to 0} \left( \frac{1}{2} p_{-i} \left( 1 - \frac{p_{-i} + \delta - (1 - 3v)}{p_{-i}} \right) \right) = \frac{1 - 3v}{2}.
$$

(A-13)

Again following the iterative process of elimination of dominated strategies, we obtain $\pi_i(p_i; G_i) < \pi_i(c; G_i)$ for all $p_i < c$ or $p_{-i} > c + b - a + \delta$. So as $\delta \to 0$, we have $\pi_i(p_{-i}; G_{-i}) = \pi_i(c; G_{-i})$ for all $p_{-i} \in [c, c + b - a]$ and $\pi_i(p_{-i}; G_{-i}) \leq \pi_i(c; G_{-i})$ for all $p_i \notin [c, c + b - a]$.

2. If $v \leq 1/4$, then $\zeta_3(v, 1 - v, p) = 1$ if and only if $p_i - p_{-i} \leq 1 - v$ whenever $p_{-i} \leq v$. We assume in the following that $\delta \leq (1 - 4v)/2$. Hence, for any $p_i \in [a, b]$ and $p_{-i} \in [c, c + b - a]$ we have $p_i - p_{-i} \leq b - c = 1/2 + v + \delta \leq 1 - v$. It follows that $\zeta_3(v, 1 - v, p) = 1$ for all $p_i \in [a, b]$ and $p_{-i} \in [c, c + b - a]$. Similarly, $\zeta_3(1 - v, v, p) = 1$ if and only if $p_{-i} \geq p_i - v$. So for any $p_i \in [a, b]$,

$$
\pi_i(p_i; G_{-i}) = \lim_{\delta \to 0} \int_{a}^{c + b - a} \pi_i(p_i; p_{-i}) dG_{-i}(p_{-i}) = \lim_{\delta \to 0} \left( \frac{1}{2} p_i \left( 1 + \int_{p_{-i} - v}^{c + b - a} dG_{-i}(p_{-i}) \right) \right) = \lim_{\delta \to 0} \frac{1}{2} p_i \left( 1 + G_{-i}(c + b - a) - G_{-i}(p_i - v) \right) = \lim_{\delta \to 0} \left( \frac{1}{2} p_i \left( \frac{1}{p_i + \delta} \right) \right) = \frac{1}{2}.
$$

(A-14)

Following the same iterative process of elimination of dominated strategies as before gives that $\pi_i(p_i; G_{-i}) < \pi_i(a; G_{-i})$ for all $p_i < a$ or $p_i > b$. Therefore, as $\delta$ approaches zero, $\pi_i(p_i; G_{-i}) = \pi_i(a; G_{-i})$ for all $p_i \in [a, b]$ and $\pi_i(p_i; G_{-i}) \leq \pi_i(a; G_{-i})$ for all $p_i \notin [a, b]$.

Because $\zeta_1(1 - v, v, p) = 1$ if and only if $p_{-i} < p_i - v$. For any $p_i \in [c, c + b - a]$,

$$
\pi_i(p_{-i}; G_i) = \lim_{\delta \to 0} \left( \frac{1}{2} \int_{a}^{b} \pi_i(p_{-i}; p_i) dG_i(p_i) \right) = \lim_{\delta \to 0} \left( \frac{1}{2} p_{-i} (G_i(b) - G_i(p_{-i} + v + \delta)) \right) = \lim_{\delta \to 0} \left( \frac{1}{2} p_{-i} \left( 1 - \frac{p_{-i} + \delta + v - 1/2}{p_{-i}} \right) \right) = \frac{1 - v}{2}.
$$

(A-15)

Following our iterative process of elimination of dominated strategies, we obtain that $\pi_i(p_i; G_i) < \pi_i(c; G_i)$ for all $p_i < c$ or $p_{-i} > c + b - a$. So as $\delta \to 0$, we have $\pi_i(p_{-i}; G_{-i}) = \pi_i(c; G_{-i})$ for all $p_{-i} \in [c, c + b - a]$ and $\pi_i(p_{-i}; G_{-i}) \leq \pi_i(c; G_{-i})$ for all $p_i \notin [c, c + b - a]$.

\[\square\]

**Lemma A-13.** Under 2CS, when $\max\{z_1, z_2\} = (1, 0, 1)$,

- if $z_1 = (1, 0, 1)$ and $z_{-i} = (0, 0, 0)$, then $\pi_i^*(z_i; z_{-i}) = 1$ and $\pi_{-i}^*(z_{-i}; z_i) = 0$;

- if $z_1 = (0, 0, 1)$ and $z_{-i} = (1, 0, 0)$, then
Symmetric results hold when max ec15:

Proof.

If if \( z_i = (1, 0, 1) \) and \( z_{-i} = (0, 0, 0) \): By setting \( p_3 = 1 \) and \( p_1 \geq p_3 \), both customers purchase the bundle. Since for any \( p_i, \pi_i(p_i; p_{-i}) \leq 1 \), this is the best outcome achievable.

If if \( z_i = (0, 0, 1) \) and \( z_{-i} = (1, 0, 0) \): See Lemma A-12.

If if \( z_i = (1, 0, 1) \) and \( z_{-i} = (0, 0, 0) \): According to Lemma A-2, either \( D_i(p^*) = 0 \) or \( p_i^* = 0 \); thus, \( \pi_i^*(p_{-i}^*; p_i^*; p_{-i}) = 0 \). First assume that \( p_i^* > 0 \), which implies that \( \zeta_i(v, p) = 0 \) for all \( v \), i.e., by (2), either \( p_i^* > v \) or \( p_i^* + 1 - v \geq p_i^* \); and either \( p_i^* > 1 - v \) or \( p_i^* + v \geq p_i^* \). For this to be an equilibrium, firm \(-i\) should have no opportunity to drop its price \( p_{-i,1} \) and capture market share; that is, \( p_i \leq v \). When \( p_i \leq v \), by (1), \( \zeta(v, 1 - v, p) = 1 \) and \( \zeta(v, 1 - v, p) = 1 \). Thus, \( \pi_i(p_{11}, p_{03}; p_{-1,1}) = p_3 \). Maximizing firm \( i \)'s profit over all \( p_i \leq v \) yields \( p_i^* = v \) and \( \pi_i(p_{11}^*, p_{03}^*; p_{-1,1}^*) = v \). Next, assume that \( p_i^* = 0 \). In that case, by (1), \( \zeta_i(v, 1 - v, p) = 1 \) if and only if \( 1 - v \geq p_3 \) and \( \zeta_i(1 - v, v, p) = 1 \) if and only if \( v \geq p_3 \). Maximizing firm \( i \)'s profit with respect to \( p_3 \) yields \( \pi_i(p_{11}^*, p_{03}^*; p_{-1,1}^*) = \max\{v, (1 - v)/2\} \). Combining both cases and accounting for the fact that firm \( i \) can choose to set \( p_i^* \) to zero, we obtain that \( \pi_i(p_{11}^*, p_{03}^*; p_{-1,1}^*) = \max\{v, (1 - v)/2\} \).

If if \( z_i = (1, 0, 1) \) and \( z_{-i} = (0, 0, 1) \): According to Lemmas A-2 and A-3, \( p_3 = 0 \); thus \( \pi_i^*(p_{-i,2}; p^*_{03}) = 0 \). By (2), \( \zeta_i(v, p^*) = 0 \) for all \( p_i^* \) since \( \min\{v_1, p_i^*\} + v_2 \geq 0 \). Hence, \( D_i(p^*) = 0 \). Therefore, \( \pi_i(p_i^*; p_{-i}^*) = 0 \).

If if \( z_i = (1, 0, 1) \) for \( i = 1, 2 \): The result follows by Lemma A-2. \( \square \)

**Lemma A-14.** Under 2CS, when \( \max\{z_1, z_2\} = (1, 1, 1) \),

- if \( z_i = (1, 1, 1) \) and \( z_{-i} = (0, 0, 0) \), then \( \pi_i^*(z_i; z_{-i}) = 1 \) and \( \pi_{-i}(z_{-i}; z_i) = 0 \);
- if \( z_i = (0, 0, 1) \) and \( z_{-i} = (1, 1, 0) \), then \( \pi_i^*(z_i; z_{-i}) = \pi_{-i}(z_{-i}; z_i) = 0 \);
- if \( z_i = (0, 1, 1) \) and \( z_{-i} = (1, 0, 0) \) or if \( z_i = (1, 0, 1) \) and \( z_{-i} = (0, 1, 0) \), then equilibrium profits are the same as when \( z_i = (0, 0, 1) \) and \( z_{-i} = (1, 0, 0) \);
- if \( z_i = (1, 0, 1) \) or \( z_i = (0, 1, 1) \) and \( z_{-i} = (1, 1, 0) \), then \( \pi_i^*(z_i; z_{-i}) = \pi_{-i}(z_{-i}; z_i) = 0 \);
- if \( z_i = (1, 1, 1) \) and \( z_{-i} = (1, 0, 0) \) or \( z_{-i} = (0, 1, 0) \), then equilibrium profits are the same as when \( z_i = (1, 0, 1) \) and \( z_{-i} = (1, 0, 0) \);
- if \( z_i = (1, 1, 1) \) and \( z_{-i} = (1, 1, 0) \), then \( \pi_i^*(z_i; z_{-i}) = \pi_{-i}(z_{-i}; z_i) = 0 \);
- if \( z_{13} = z_{23} = 1 \), then \( \pi_i^*(z_i; z_{-i}) = \pi_{-i}(z_{-i}; z_i) = 0 \).

**Proof.** If \( z_i = (1, 1, 1) \) and \( z_{-i} = (0, 0, 0) \): By setting \( p_3 = 1 \) and \( p_k \geq p_3 \) for \( k = 1, 2 \), both customers purchase the bundle. Since for any \( p_i, \pi_i(p_i; p_{-i}) \leq 1 \), this is the best outcome achievable.
• If $z_i = (0, 0, 1)$ and $z_{-i} = (1, 1, 0)$: By symmetry $p_1^* = p_2^*$. To obtain a contradiction, suppose that $p_1 + p_2 > 0$. If $p_1 + p_2 > p_3$, both customers purchase only the bundle, $\pi_i(p_3; p_1, p_2) > 0$ and $\pi_{-i}(p_1, p_2; p_3) = 0$. Whereas if $p_1 + p_2 \leq p_3$, no customer purchases the bundle, $\pi_i(p_3; p_1, p_2) = 0$, and $\pi_{-i}(p_1, p_2; p_3) > 0$. Therefore, firms get into a price war and no equilibrium is possible unless $p_1 = p_2 = p_3 = 0$.

• If $z_i = (0, 1, 1)$ and $z_{-i} = (1, 0, 0)$: By Lemma A-3, $D_3(p^*) > 0$. Because valuations are perfectly negatively correlated, both customers receive the same surplus from the bundle and are potential buyers of the bundle. By setting $p_{i2} < p_{i3}$, firm $i$ may thus divert a potential buyer of the bundle toward product 2. Doing so would result in less profit given that $p_{i2} < p_{i3}$. Hence, firm $i$ is always better off setting $p_{i2} \geq p_{i3}$; as a result, no customer purchases product 2. Thus, the equilibrium is the same as if $z_i = (0, 0, 1)$ and $z_{-i} = (1, 0, 0)$, as characterized in Lemma A-12. A symmetric argument applies to the case where $z_i = (1, 0, 1)$ and $z_{-i} = (0, 1, 0)$.

• If $z_i = (1, 1, 1)$ and $z_{-i} = (1, 0, 0)$: By Lemma A-2, either $D_1(p^*) = 0$ or $p_i^* = 0$. First, assume that $p_i^* = 0$. Then by (1), $\zeta_i(v, p) = 1$ if and only if $\min\{v_2, p_2\} \geq p_3$ and by (2), $\zeta_2(v, p) = 1$ if and only if $p_2 \leq v_2$ and $p_3 < p_2$. That is there is no equilibrium such that $p_2 \geq p_3 > 0$, otherwise, firm $-i$ would have an incentive to set $p_{-i,2}$ below $p_{i3}$ and make positive profit. Similarly, there is no equilibrium such that $p_3 > p_2 > 0$, otherwise, firm $i$ would have an incentive to set $p_{i3}$ equal to $p_{i2}$ and make positive profit. Hence, $\min\{p_2^*, p_3^*\} = 0$. If $p_2^* = 0$ and $p_3^* > 0$, $D_3(p^*) = 0$. And if $p_2^* = 0$ and $p_3^* > 0$, $D_2(p^*) = 0$. Next, assume that $p_i^* > 0$ and $D_1(p^*) = 0$. Because $\zeta_1(v, p) = 1$ if and only if $p_1 \leq v_1$ and $p_1 + \min\{v_2, p_2\} < p_3$, for this to be an equilibrium, we must have $p_2^* = 0$, otherwise, firm $-i$ could set its prices $p_{-i,1} + p_{-i,2}$ below $p_3$ and make some positive profit. Hence in this case, $D_2(p^*) = 0$. A symmetric argument applies to the case where $z_i = (0, 1, 1)$ and $z_{-i} = (1, 1, 0)$.

Proof of Proposition 2. Combining Lemmas A-9-A-14 leads to Tables A-1-A-3, from which we deduce the equilibria. The trembling-hand perfect equilibria are identified by maximizing each player’s payoff in case of random deviations by the other player (Selten 1975). □
\[ \text{Table A-1 } \text{Firms' Payoffs under 2CS when } v \geq 1/3 \]

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<th>(0,1,0)</th>
<th>(1,1,0)</th>
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*Note:* The shaded cells correspond to bundling equilibria.

\[ \text{Table A-2 } \text{Firms' Payoffs under 2CS when } 1/4 < v \leq 1/3 \]

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</tbody>
</table>

*Note:* The shaded cells correspond to bundling equilibria.

**EC.4. Single-Product Offerings under a General Distribution of Valuations**

*Proof of Proposition 3.* We analyze the best response of firm \(i\) to the bundling strategy of firm \(-i\). Let \(z_i(z_{-i})\) be the set of firm \(i\)'s best responses to \(z_{-i}\).

First, we show that, when firm \(-i\) offers either nothing or a single component, a weakly dominant strategy for firm \(i\) is to offer the bundle. Thereafter, we show that, if firm \(-i\) offers the bundle, then a weakly dominant strategy for firm \(i\) is to offer a single component.
<table>
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<th>(z_i, z_{-i})</th>
<th>(0, 0, 0)</th>
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<th>(1, 1, 0)</th>
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<tr>
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<tr>
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<td>0, 0</td>
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</tbody>
</table>

*Note.* The shaded cells correspond to bundling equilibria.

- When \( z_{-i} = (0, 0, 0) \), then by (4), \( \pi_i(p_i; p_{-i}, z) = 0 \) if \( z_i = (0, 0, 0) \), \( \pi_i(p_i; p_{-i}, z) = \max_{p_1} p_1 D_1(p_1, \infty, \infty) \) if \( z_i = (1, 0, 0) \), \( \pi_i(p_i; p_{-i}, z) = \max_{p_2} p_2 D_2(\infty, p_2, \infty) \) if \( z_i = (0, 1, 0) \), and \( \pi_i(p_i; p_{-i}, z) = \max_{p_3} p_3 D_3(\infty, \infty, p_3) \) if \( z_i = (0, 0, 1) \). By (1)-(2), for any \( p \), \( D_3(\infty, \infty, p) = \int \mathbb{I}_{|v_1 + v_2 \geq p|} dF(v) \geq \max_{k \in \{1,2\}} \int \mathbb{I}_{|v_1 \geq k|} dF(v) = \max\{D_1(p, \infty, \infty), D_2(\infty, p, \infty)\} \). Hence, \( (0, 0, 1) \in \tilde{z}_i(0, 0, 0) \).

- When \( z_{-i} = (1, 0, 0) \), then \( \pi_i(p_i; p_{-i}, z) = 0 \) if either \( z_i = (0, 0, 0) \) or \( z_i = (1, 0, 0) \) by Lemma A-2, so these two strategies are weakly dominated. Consider next the price equilibria that correspond to setting \( z_i \) to \( (0, 1, 0) \) or to \( (0, 0, 1) \):

\[
\tilde{p}_i = \arg \max_{p_i} \pi_i(p_i; \tilde{p}_{-i}, (0, 1, 0), (1, 0, 0)) \quad \forall i \in \{1, 2\},
\]

\[
\hat{p}_i = \arg \max_{p_i \geq 0} \pi_i(p_i; \hat{p}_{-i}, (0, 0, 1), (1, 0, 0)) \quad \forall i \in \{1, 2\}.
\]

For any \( p \) and \( p_1 \), we have \( D_3(p_1, \infty, p) = \int \mathbb{I}_{|v_1 + v_2 \geq p|} dF(v) \geq \int \mathbb{I}_{|v_2 \geq p|} dF(v) = D_2(p_1, p, \infty) \). Moreover, \( D_2(p_1, p, \infty) \) is independent of \( p_1 \). Therefore,

\[
\pi^*_i((0, 0, 1); (1, 0, 0)) = \tilde{p}_i D_3(\tilde{p}_{-i}, \infty, \tilde{p}_i) \geq \hat{p}_i D_3(\hat{p}_{-i}, \infty, \hat{p}_i) \geq \hat{p}_i D_1(\hat{p}_{-i}, \infty) = \hat{p}_i D_1(\hat{p}_{-i}, \hat{p}_i, \infty) = \pi^*_i((0, 0, 1); (1, 0, 0)).
\]

Hence \( (0, 0, 1) \in \tilde{z}_i(1, 0, 0) \).

- When \( z_{-i} = (0, 1, 0) \), we can use a symmetric argument to derive that \( (0, 0, 1) \in \tilde{z}_i(0, 1, 0) \).

- When \( z_i = (0, 0, 1) \), then \( \pi_i(p_i; p_{-i}, z) = 0 \) if either \( z_i = (0, 0, 0) \) or \( z_i = (0, 0, 1) \) by Lemma A-2, so these two strategies are weakly dominated. As a result, either \( (1, 0, 0) \in \tilde{z}_i(0, 0, 1) \) or \( (0, 1, 0) \in \tilde{z}_i(0, 0, 1) \).

In sum, we have shown that (a) offering a bundle is a best response if the other firm offers a single-component product (or offers nothing) and (b) offering a single-component product is a best response if the other firm offers the bundle.
Finally, we establish that the bundling firm always earns more profit: Because $(0, 0, 1) \in \hat{z}_i(1, 0, 0)$, because firms are symmetric, and because $\hat{p}_3 = \arg \max_{p_3 \geq 0} \pi_i(p_3; \hat{p}_1, (0, 0, 1), (1, 0, 0))$,

$$\pi^*_i((0, 0, 1); (1, 0, 0)) \geq \pi^*_i((0, 1, 0); (1, 0, 0)) = \pi^*_i((1, 0, 0); (0, 1, 0)) = \max_{p_1} \int 1_{[v_1 \geq p_1]} dF(v)$$

$$\geq \max_{p_1} \int 1_{[v_1 \geq p_1 \text{ and } \hat{p}_3 > p_1 + v_2]} dF(v) = \pi^*_i((1, 0, 0); (0, 0, 1)). \quad \square$$